Momentum Transfer in Fluids
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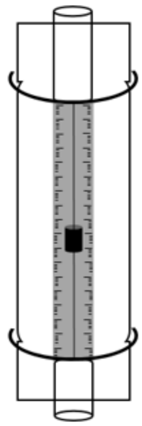
I welcome you to this lecture on flow metering. We are discussing about flow metering and we will continue this discussion on flow metering. This is part of our general course on momentum transfer in fluids. You may recall in our last lecture we talked about Venturi meter and orifice meter. There are two important metering devices which I am going to discuss now.

One is known as rotameter. So, this rotameter is very widely used a very simple device and it is reasonably accurate it gives a very good visual. So, it is a very popular device. So, you can you will see in many places I mean think of it in steam engine in case of in any even in operation theaters everywhere wherever there is a flow involved you will see these rotameters sitting there and you get a very good visual direct display direct direct display of flow rate.

So, you can get information directly you do not have to perform calculations like you have for a manometer you have to note down the the level of manometric fluid and from there you there are equations to be used to find out what is Δp and then from there what is the flow rate, but here in this case you get a direct display and more importantly visual if somebody does not know how to read things at least he will get the flow rate that it is that there should be there the flow rate it is supposed to be you are operating at half of that flow rate you get a visual even without knowing without even if someone is illiterate. So, these aspects these advantages make these rotameters very popular and another important feature is it is very inexpensive. So, rotameters are quite common in process industries, and as I said, I mean be it even in hospitals in in in name it any place, you will find this rotameter sitting there which is measuring the which is giving you real-time information on the flow rate. So, what we have here in this in this slide is that this is the picture of a rotameter. You might have seen this type of device in many different places flow enters from the bottom.



Rotameter

So, we see here that the flow enters from the bottom from here flow enters and typically there is a valve. So, you can open the valve and then only the flow takes place. Let us say I assume the valve is open valve is fully open and you have the flow going in here. So, you have this is called a float of a rotameter this is called a float of a rotameter and you have this graduated cylinder the graduations are shown here. So, this is a transparent tube.

So, you can see where the location of the float and against that what is the corresponding display of this graduation you can find out where the float is located. Now, the fact is that this float as the flow rate is increased, the float is expected to go up and settle at some location, and at that location, you have to read from this scale where this float is settling, where the float has stabilized, and it is sitting there. So, from that location you can find out this is the flow rate. Let us say these graduations are telling you these graduations are telling you how much of let us say ml per hour is the flow rate ml per h. So, that is the flow rate you have. Typically, in some case some cases, you need an ultra-low flow rate, and in some cases, milliliter per hour is an ultra-low flow rate.

So, when it comes to let us say someone administering anesthesia gas then that flow has to be that there has that has to be an ultra low flow rate whereas, if you are having water flow going into someplace there someplace in a boiler or in other process utilities that the flow rate would be very high. So, then so, these rotameters are meant so, accordingly it has to make adjustments. So, how does the rotameter work? I mean, why will it go up, and why will the float go up and settle at a location that needs to be a little bit? We have to dig into that a little bit. The way it works is suppose when the flow rate is 0 when we do not introduce anything. So, flow rate is 0.

So, at that time the float would be simply sitting here at this location at the bottom most location. So, this is the place where the float is, and then another clue here is that this tube, though it looks very much horizontal we have actually there is it has a slant that is not noticeable by the naked eye. This so, you have the float is of this is this is the float then let us say this is the float. So, this is the float and the tube is slightly inclined that is not I mean if you see naked eye it will appear like this it will appear like a it will appear like a straight parallel I mean it as if the diameter remains constant all through, but the fact is diameter continue to increase as you go up. So, what you have here is at a flow rate equal to 0, the float is sitting and perfectly fitting to the diameter here.

So, the diameter of the tube that we are talking about here. So, the diameter of these tubes is these tube at the lower part, the diameter of the tube, and the diameter of the float perfectly match. So, float is sitting there no flow you have not introduced any flow. Now, suppose you have a valve and you have closed the valve. Now, you open the valve slightly you will see that there is a flow going in.

So, to accommodate the flow to allow the flow to take place the only way that can happen is float has to go up. So, that flow can takes take place now it has some space because of this because of because the cross sectional area of this tube is increasing as you go up. So, if the float goes up, there would be some space developing here, some space developing here, the annular space surrounding this float through which the fluid can flow. Now, at this time if you look at the forces that are acting on this float you will see that there would be a force which is acting downward because of gravity. So, that would be if the volume of the float is v f, and let us say the density of the float is ρ f.

So, this is v float ρ float. So, that gives me the volume this is v capital V not I should not be small v no confusion capital V float is the volume of the float rho float density of the float material it could be a cork material it could be much heavier depending on what is flowing if the fluid is a very high density very low density. So, accordingly the rotameter when you go to the supplier of the rotameter they will select the right material for the float. So, you have the volume of the float density of the float product would be mass of the float volume volume in meter cube multiplied by kg per meter cube meter cube and meter cube cancel out. So, this gives you kg.

So, this multiplied by g this gives me the gravitation gravity force that is acting downward as far as the float is concerned and then you have to do the buoyancy correction when it is immersed in a liquid when it is not sitting there rather it is it is suspended. So, you have to subtract the volume of the float remains same when you calculate buoyancy, but this ρ would be ρ of fluid not ρ of float and g. So, that has to be subtracted. So, if you if you want to do the buoyancy correction. So, the buoyancy corrected gravity force acting downward is this and these has to be balanced by the drag force drag force that is operational because the fluid is traveling through these annular area.

So, through these annular area fluid is traveling and then these will be imparting a drag force and by now you are aware how drag force is how drag force is characterized maybe I will write the equation sign here instead of this side. Then this is this drag force would be if you may recall we have a drag coefficient C_D we defined it right that is force divided by projected area projected area divided by half rho whatever velocity is half rho u square right. So, from there you can find out what is the drag force. So, drag force would be $A_{projected} C_D \rho \frac{u^2}{2}$. So, that this is the this is a typical drag force that would be that would be operational.

So, drag force wherever the drag force balances the buoyancy corrected gravity force acting downward drag force is acting upward whenever they balance each other that is the location where the float will settle where the float will stop moving. So, moment you open the valve then fluid will start entering into it. So, this will cause the upward movement of the float. So, the float will move up, but as soon as the float moves up, the annular area that opens up will allow the fluid to flow through the side. So, initially, these are when the float has moved a little bit.

So, this annular area is small. So, you have much higher velocity in that case because the same the flow rate is Q you have fixed. So, think of it when the float is at this location. So, when this is the location of the float. So, think of the annular area that you have.

So, this is the annular area surrounding the float it is a much bigger area. So, if the same flow rate is flowing through this. So, here the decrease significantly because Q the flow rate remains same the fluid flow remains same. Now, that fluid flow takes place over a much larger area surrounding the float. So, Q divided by that area which gives you the velocity that velocity would be much smaller because area has increased.

So, velocity as the velocity decreases the drag force decreases because drag force is proportional to u square. So, the drag force decreases. So, since the drag force decreases. So, you would see that initially the drag force was much higher when you have in this location the drag force is much higher. So, you see that the drag force is greater than drag force is greater than the buoyancy-corrected gravity force.

So, because of these drag force dominates over these are the. So, the upward force dominates over the downward force. So, the float will continue to move up, but at one point you will find that the area surrounding the float has increased substantially. So, now, the velocity has decreased. So, at that point the drag force becomes equal to the buoyancy corrected gravity force and at that place simply there force equilibrium exists upward force and downward force they match each other.

So, float will be still at that location float will remain still at that location. And that location will your one has to identify. So, when you purchase a rotameter they will ask for what fluid you are doing it and what flow rate range you have in mind. So, moment you specify those. So, they plan accordingly make those tubes and floats and then they will make these graduations.

So, one has to do this experiment with that fluid and then see that if I give this much of flow rate where the float location will be and they mark that point they mark that point they mark this point as so, many milliliter per hour or so, many liter per hour. And then it the at some other even a greater velocity you will find that the float location is here. So, they mark that to be so, many milliliter per hour or liter per some flow rate. So, they mark these and they continue doing these marking and that marking is reflected here in this tube you can see these markings are indicated here. So, these using these marking so, once this marking this is called pre calibration I mean before use of this device you are trying to find out at what flow rate where the float should be.

And once you do this then you then the flow this flow meter this rotameter is ready to go then you can put it in a in a process stream and then see where the float is stabilizing at what what height the float is stabilizing and at that time you can that that time would be will give you the sorry that at that time you note down the position and from there you get so, many milliliters per hour or liter per hour that that is the flow rate. So, it is very simple that the whole concept is very simple. One thing you must keep in mind at this point is that this right hand side that you have in this equation the right hand side right hand side is not affected in this equation the right hand side is not affected by the by the by the change in position or by the change in flow rate. So, if you change flow rate the float volume is not going to change, density of float is not going to change, density of fluid is not going to change, acceleration due to gravity is not going to change. So, this right hand side remains constant.

So, essentially you have to play with this left hand side. So, left hand side is this. So, now, you work with you change the cross sectional area. So, your idea is that you change the cross sectional area by moving the float up and down and by that way you change the velocity of the fluid that is travelling through these through the side of the float and that velocity has to be remain same all the time because that velocity is defined by the right hand side.

So, when the flow rate is increased. So, if you if you retain the float at that same position the velocity will increase. So, this left hand side right hand side there will be a mismatch. So, immediately float has to go up such that the old velocity has to has to be maintained. So, velocity is something sacrosanct here and that velocity is moment there is a change in flow rate the float will move up to make sure that the velocity is velocity surrounding the float that remains same. So, the drag force exerted on that float that remains same.

So, this is how the rotameter works. In fact, you can see here that the whatever I mentioned here you can note them quickly. One is the weight of the float is given here, the buoyancy force I have said drag force given here and then the weight minus buoyancy. So, this is buoyancy corrected weight or in other words the downward force and that has to be equal to the drag force. This gravity and buoyancy force they do not depend on flow rate or velocity.

Balance of three forces

$$(i)\ The\ weight\ of\ float = (Volume\ of\ float)(Density\ of\ float)g$$

$$(ii)\ The\ buoyancy\ force = (Volume\ of\ float)(Density\ of\ fluid)g$$

$$(iii)\ The\ drag\ force\ on\ the\ float = A_f C_D \rho \frac{u^2}{2}$$

(i) - (ii) = (iii)

The drag force for equilibration for equilibration of three forces is constant. Therefore, the drag force has to remain constant. So, to get that same drag force the float has to move that is the idea. And so, you must remain constant even when the flow rate change and by some way your float has to move and provide additional cross sectional area for the flow around the float such that the velocity will remain constant. Now, one may note if the float diameter is D f because you remember this was the cross section and this was the float.

So, this is we call this diameter is D f diameter of the float and D t is the diameter of the tube, and D t is the diameter of the tube which is this extra area. So, you can see the inner diameter of linearly tapered tube is given by D f plus A h. So, that means, if h is calculated from the bottom h is calculated from the bottom then there is there is this how much is the extra here when it when it comes to sorry here the float match perfectly right this is the float this is the float right. So, this is the float and then this you are calculating h from here. So, at any location you are assuming here at this location this is the D t diameter of the tube at this location this is the D t.

So, you are saying that since this area this extra space the annular space that continue to increase you are saying that this is increasing linearly with h. So, if this is D t you are writing D t is equal to D f plus A h. A h A is the slope and h is the or A h is the h is the height here and so, you there has to be it is changing linearly. So, you it is A is accordingly the proportionality constant. So, that you can you can relate D t and this is D t the full thing and this is D f.

So, this difference between D t and D f is. So, essentially you are saying D t minus D f is proportional to h and A is that proportionality constant. That means, it is it is tapered it is linearly tapered that is that is what it means this means it is linearly tapered. So, with this understanding if someone wants to find out the flow rate you can see flow rate would be

$$Flow\ rate = u\frac{\pi\left(D_t^2 - D_f^2\right)}{4} = u\frac{\pi}{4}\left[2D_f ah + a^2h^2\right]$$

$$Flow\ rate \approx u\frac{\pi}{2}D_f\ a\ h$$

Where D_f is the diameter of the float and D_t is the inner diameter of linearly tapered tube= D_f + ah

the velocity that is something sacrosanct we said and then we have the annular area and annular area is π D t square π D t square by 4 which is the π D t square by 4 is the total area inner inner D t is the inner diameter of the tube. So, total cross sectional area out of that the area occupied by the float is π D f square by 4. So, when you subtract the two you get this expression you get this D t is written as D f plus A h.

So, if you replace this D t by D f plus. So, it would be D f plus A h whole square. So, then this D f square and D f square here one D f square they will cancel out and so, you will be left with A h whole square and 2 D f A h. So, these two terms will be there. Now, since we have seen we have we have already mentioned that this slant is the you cannot see this in naked eye. So, that means, this slanting is this value of A is very small.

So, you can ignore this A square h square term the second order term you can ignore. So, you can say flow rate is U into π by 4 into this term. So, essentially this is flow rate is almost close to these or in other words flow rate is flow rate is proportional to flow rate is proportional to h. That means, h is the position of the float from the bottom. So, you can see that if you change the flow rate the change in height where the float is stabilizing that would be linearly changing.

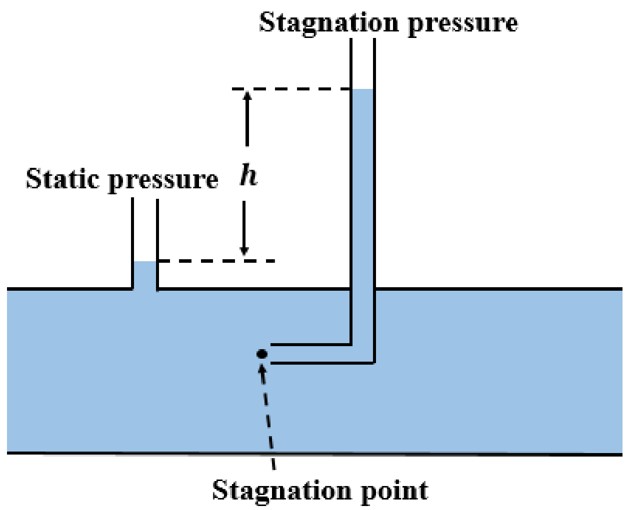
So, this is a major observation because if you look at if you may recall what we had in case of orifice meter and venturi meter there we had you may recall that this is for example, the expression that we had in our earlier lecture about the venturi meter. So, you may recall it is it is $\frac{2(p_a - p_b)}{\rho}$ and this this was expression we had worked with last time. So, here in this case p a minus p b is essentially the deflection in the manometer level and then into rho I would say rho of manometric fluid ρ manometric fluid minus ρ of process fluid. So, that into g.

So, p a minus p b is given by this quantity. So, and what you are measuring in case of venturi meter or orifice meter is that height h. So, this is what you are measuring. So, you can see the here the flow rate would be proportional to in case of venturi or orifice meter flow rate is proportional to square root of h because there is a square root sign outside. So, this so, they if there is any change in manometer level the reflection of that in the flow rate would be the dependence would be of square root type nature. So, in that case you cannot by seeing that now the height that the change in height has doubled.

So, I cannot immediately extrapolate that I have to keep in mind that there is a square root type dependence. Whereas, in this case in case of a rotameter I have flow rate proportional to h that is one major advantage I would say in using the rotameter you have the movement of the flow would be linearly affected by the flow rate that is one thing. Another thing you may recall that while discussing the venturi meter or orifice meter we said that one has to provide an entry length and exit length. So, if the if the venturi meter is located here upstream of this and the downstream of this. So, the upstream and downstream side you must have some straight length available you cannot have a bend immediately at the opening of a bench at the inlet side of the venturi meter.

Because in that case the flow there would be some amount of rotation involved in that or some amount of vortex formation due to other reasons that will affect the pressure drop. Because you are you are measuring the Δp and your Δp measurement would be corrupted. So, one has to have some straight lengths on both sides that is a requirement for venturi meter norifice meter. However, for rotameter you do not need this because at the end of the day you what is flow what is flowing by the side of that float that is what matters. So, so that that so straight pipe section at inlet and outlet is also not required.

So, these are some of the advantages that is that is one thing we wanted to talk about. So, rotameter I mentioned the other device that I have I have not talked about, but I must point this out which goes by the name Pitot tube. So, this the name of this is Pitot tube which is which is given here. So, here you can see the purpose of the Pitot tube is to find out the velocity local velocity at a point in the cross section. So, far what we discussed rotameter, venturi meter or orifice meter they are primarily for measurement of flow rate.



So, if you want to get a velocity you will get an average velocity over the entire cross section. Now, here in case in case you want to know the local velocity. So, then that case you will use the Pitot tube. The way it works here is you are going to have let us say I have the flow taking place. So, let us say I have the I want to measure the flow rate at this location.

So, let us see what is going on here. I have the same it is on the same cross section I have this point here and the others on the wall I have this. So, let us say if I look at if I connect it to a manometer I would see that the pressure here would be more compared to the pressure here. So, if you put the manometer you will find that the manometer reading would be like this. Because, you have let us say a flow is taking place.

So, this is the streamline you have right. So, in that case you have what would be what would be the net if you if you if you look at pressure plus because this is going to be a stagnation point. This point would be referred as stagnation point because these fluid that comes in the fluid that is come the fluid that comes in that has to that the streamline that comes in this streamline here it will be countered by a static fluid here. So, the flow has to stop. So, whatever velocity let us say it is coming with a velocity u 0 the streamline it is having a velocity u 0.

So, there is a change of velocity from u 0 to 0. So, kinetic head gets changed from u 0 to 0. So, there would be a corresponding change in pressure head. And, here in this case the if you may you may even know you may not already that when there is a flow taking place through a pipe through a capillary through any cross section generally across the cross section you assume that the pressure is uniform. So, you may recall when we talk about Poiseuille equation when you talk about pressure drop over a length of the pipe.

So, there we have we write the pressure drop is in this direction. So, pressure in this direction pressure is higher here, and pressure is lower here, but within a cross section, we never considered any pressure gradient because if there is a pressure gradient existing, then there would be a flow also existing in this direction. However, we talked about laminar flow, etcetera. Everywhere the flow is, we assume that the streamlines are looking like this, or you have the velocity you assume if this is your x direction. So, you have only the velocity component in x direction velocity component x direction that is a function of r that is different you have at different radial positions you have this velocity changing, but you will never consider v r, v r is 0 you do not have any radial velocity because that is that is not that we do not see that that we do not have any reason to believe velocity would be from upstream to downstream in x direction. So, if the velocity is not there evidently you have the pressure would be changing from upstream to downstream, but in radial direction you do not expect any variation in pressure. So, the pressure value p would be same as this location and this location this is known as the static pressure, but here on top of that you have the streamline that is being stopped completely.

So, you have lost that kinetic head. So, that would be reflected in this manometer. So, if you relate this if you say that this pressure here is p s is referred also as static pressure and the streamline AB terminates at B point B this is the point B this is A. So, streamline AB terminates at point B. So, $\frac{u_o^2}{2}$ gets converted to $\frac{p_s - p_o}{\rho}$. Bernoulli's equation you are doing you will end up with this expression

$$u_o = \sqrt{\frac{2(p_s - p_o)}{\rho}}$$

So, by looking at this manometer reading you can find out what is the velocity at that location. Now, you will simply move this tube here you bring the tube here in this location and then you get the velocity at that location. So, using a pitot tube in fact, you can get the velocity distribution over the radius or over the cross section. So, pitot tube is meant for measurement of local velocity whereas, so far all other devices that we talked about those will give you the overall flow rate or the average velocity if you divide that flow rate by cross sectional area.

So, that is the subtle difference this pitot tube is unique in that way. That is all I have as far as today's lecture is concerned there are other devices we have already talked about Boudon gauge for measurement of pressure. We have talked about differential pressure transducers for measurement of pressure and of course, manometer is there that is a that is a very traditional measuring device for pressure. There are other measurements you may like to have as far as the flow is concerned. For example, to study the flow you may have to measure density, you have to measure specific gravity, you have to measure viscosity. Fo viscosity there are viscometers, there are capillary viscometers that means, you have a capillary and you are trying to find out how long it takes for certain volume of fluid under the action of gravity which remains fixed.

You have how long it takes for the fluid to drain and or you can have a more sophisticated viscometers where you we had on the very first set of lectures we talked about how a viscometer is supposed to work. So, there are other measurements also, but if you want to make the flow rate measurement these are the devices that I have talked about so far. So, that is all I have as far as today's module is concerned. Thank you very much for your attention.