Momentum Transfer in Fluids

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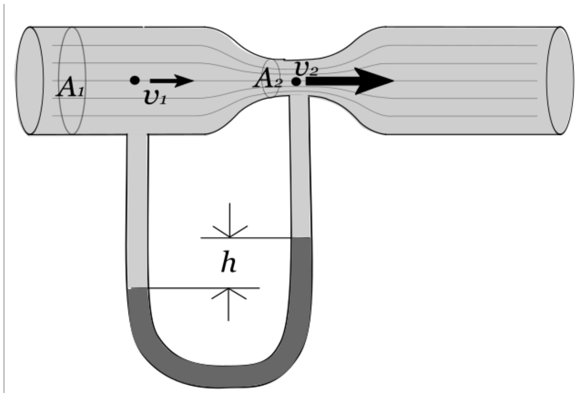
I welcome you to this lecture on Momentum Transfer in Fluids. We were discussing about flow metering. In particular, at the end of last class, we were discussing about Venturi meter and some of the areas of concerns there. Venturi meter, as I mentioned that Venturi meter would be there will be a conical, say there will be a contraction section, and then there will be an expansion section and expression at the end of the last class that the expansion section should be larger so that you can have the minimum energy loss taking place. This type of device I mean this is if someone asks you how to plot the pressure let us say, if someone asked you how to if this is the axis and if someone plots what is the pressure versus distance you may see that as I said initially the pressure gets converted to kinetic head and then kinetic head in gets converted to pressure it.



So, that means the pressure goes down, and pressure goes down up to the throat, and then pressure is supposed to recover beyond the throat. So, the pressure versus distance you have this type of a plot. So, this gives you a very unique opportunity to reduce pressure at the throat and then the question is how far it will go down. I mean if you have it depends on the ratio of A1 and A2 right what A1 by A2 you have.

Now, let us say A1 by A2 you have chosen for getting this pressure profile A1 by A2 you have chosen as 10. Suppose I use A1 by A2 as 1000. So, then what will happen to this pressure? So, this pressure will continue to go down. So, at one point, this pressure will, of course; we mentioned earlier that if the pressure is less than the vapor pressure, then there would be cavitation, but cavitation is not something that we want here. Instead what we can we can still use this device instead of putting this manometer there is another unique way we can use this device which is if we have this type of arrangement and I see that the pressure goes down at the throat and if I have an opening there then we will see that if the pressure goes below atmospheric pressure then there would be fluid there would be let us see if I open this air will be sucked in.

So, you will see that the water is entering there and what you are getting the out getting out from there is air bubbles getting in there or you will see if you connect this to connect this line to another reservoir which is airtight let us say and let us say filled with water you will find or maybe semi filled with water you will find that this there would be a vacuum created inside this vessel. So, this method of creating vacuum I mean one can produce vacuum by this means and this method of producing vacuum. So, this is only you keep one opening there, and that way, you can create a vacuum. This method is referred to as ejector this device is referred to an ejector. In many places you will find this ejector is used to generate vacuum. In fact, then in an ejector there is something called a motive fluid that comes in motive fluid the motive fluid is in many ejector motive fluid is steam.

So, steam is used steam will continue to flow and it will serve as an ejector. So, it helps in generating vacuum in another chamber that is connected to the throat. So, this principle of pressure going down the kinetic head getting converted to the pressure head has other unique applications as well, which is one aspect I need to point out. There is another aspect to it which is which also needs to be seen it is something like this. When we when we wrote Bernoulli's equation we assume first of all Bernoulli's equation you recall you may recall we talked about either the flow has to be irrotational or you have to go along a streamline etcetera otherwise Bernoulli's equation is not supposed to be applied.

But we still we apply Bernoulli's equation and add frictional loss etc which you might you must have seen in case of cavitation we have done and all those. So, those are sort of make approaches. I mean, we add a loss term, though if you go in a rigorous sense of the term, it should not be there. But it gives you the required it gives you the results that is why you are you proceed with that. Now, one problem here is when you wrote the Bernoulli's equation you wrote you said that it is $V_1^2/2$ and $V_2^2/2$. So, these are the kinetic head.

What is this V_1? We said V_1 is Q divided by A that is Q is the flow rate and A is the area cross sectional a corresponding cross sectional area A_1 or A_2. So, that is essentially average velocity total flow rate divided by the cross sectional area that gives you average velocity. Now, V_1 is average velocity and you are multi you are taking average velocity square by 2 does it truly represent the kinetic head that question may arise. You have Bernoulli's equation the way it was thought about is it was not meant for a viscous flow. So, Bernoulli's equation did not assume that you have a velocity profile in the pipe first of all.

The Bernoulli's equation assumed that you have a constant velocity like a plug or velocity along a streamline. So, now you are applying it you are forcing it to a situation where there is a velocity profile within the pipe I mean you know velocity profile would be parabolic for a laminar flow etcetera. So, if you have such type of velocity profile and you are doing you are obtaining an average velocity and that taking a square and you are calling that energy there is a subtle issue. It is something like this where ideally if you want to find out what is the energy then you need to what you should have done is let us say I have this is my cross section and I am having the velocity profile is let us say parabolic. So, this is the velocity profile.

So, ideally if you want to find energy what you need to do is you have to take a differential element at a distance r of size dr and you have to find out what is the velocity that is prevalent there at that location for this differential element and that velocity what is the corresponding kinetic energy as far as that velocity is concerned and do all count all such differential elements and sum them together and that is your total energy. It is not just grossly taking an average velocity and square it and divided by 2 it is not rigorous. So, that is why you can see here

For an element of cross-sectional area ds, the mass flow rate = $\rho\, u\, ds$

Where each mass unit carries energy $\frac{u^2}{2}$

Total energy through ds per unit time

$$d\dot{E}_k = (\rho u\, ds)\frac{u^2}{2}$$

So, this is the total energy through ds area per unit time. So, now, this ds area happened to be for your case if you are having a flow through a pipe this ds is to the annular element we are talking about. This is the pipe this is distance r and this is the annular area of dr. So, that area would be simply $\pi(r+dr)^2 - \pi r^2$ = ds, the differential area and then we have πr^2 and πr^2 will cancel out. So, we have $2\pi r\, dr + \pi (dr)^2$ that is equal to ds and this dr as such is a differential element.

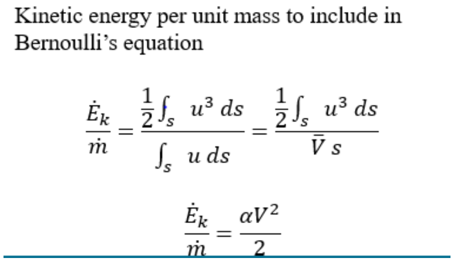
So, we do not we consider only the first order term. So, dS is equal to $2\pi r\, dr$. So, that is what we will put and then we take the energy now this is the d for total energy as far as this differential element is concerned. So, flow is taking place in this direction. So, as far as this differential element is concerned this is the energy.

Total energy through ds per unit time

$$d\dot{E}_k = (\rho u\, ds)\frac{u^2}{2}$$

For entire cross-section

$$E_k = \frac{\rho}{2}\int_s u^3\, ds$$



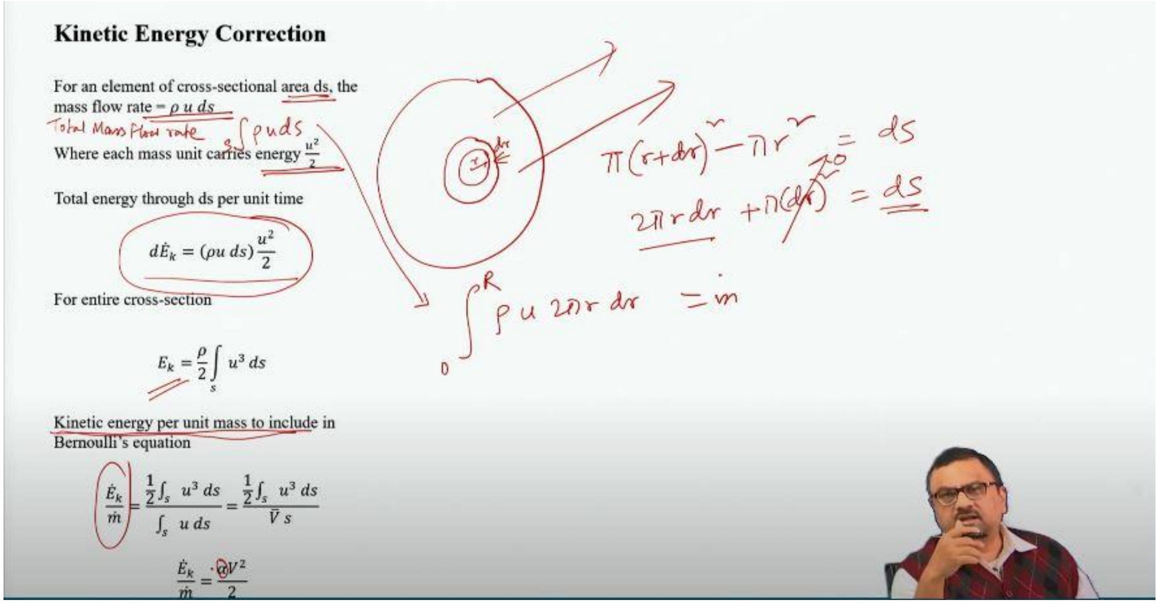
Kinetic energy per unit mass to include in Bernoulli's equation

$$\frac{\dot{E}_k}{\dot{m}} = \frac{\frac{1}{2}\int_s u^3\, ds}{\int_s u\, ds} = \frac{\frac{1}{2}\int_s u^3\, ds}{\bar{V}\, s}$$

$$\frac{\dot{E}_k}{\dot{m}} = \frac{\alpha V^2}{2}$$

For entire cross section now you have to sum all such annular strips. So, you do the integration and that is the total energy. So, now, a total energy per unit mass which is essentially the Bernoulli's equation that gives you the E k dot by m that there you have to do this. So, m dot in this case would be rho integration u dS. So, you have to find out corresponding m mass flow rate.

So, what is the total mass flow rate or what is the average mass flow rate or this is the mass flow rate as far as the differential element is concerned. So, total mass flow rate would be total mass flow rate is equal to integration rho u dS over entire area dS. So, that is or if you write it for the for circular cross section it would be integral 0 to capital R, R is the radius of the pipe rho u 2 pi r dr. This way and then do the integration that will give you the mass flow rate. So, this will give you the m dot.



Kinetic Energy Correction

For an element of cross-sectional area ds, the mass flow rate = $\rho\, u\, ds$

Total Mass Flow rate = $\int_s \rho u ds$

Where each mass unit carries energy $\frac{u^2}{2}$

Total energy through ds per unit time

$$d\dot{E}_k = (\rho u\, ds)\frac{u^2}{2}$$

For entire cross-section

$$E_k = \frac{\rho}{2}\int_s u^3\, ds$$

Kinetic energy per unit mass to include in Bernoulli's equation

$$\frac{\dot{E}_k}{\dot{m}} = \frac{\frac{1}{2}\int_s u^3\, ds}{\int_s u\, ds} = \frac{\frac{1}{2}\int_s u^3\, ds}{\bar{V}\, s}$$

$$\frac{\dot{E}_k}{\dot{m}} = \frac{\alpha V^2}{2}$$

$$\pi(r+dr)^2 - \pi r^2 = ds$$

$$2\pi r\, dr + \cancel{\pi (dr)^2} \to 0 = ds$$

$$\int_0^R \rho u\, 2\pi r\, dr = \dot{m}$$

So, E k dot you find out m dot you find out E k dot by m dot that gives you kinetic energy per unit mass. And ideally this is equal to your half v square which you are using here which you are using v 1 square by 2 instead of v 1 square by 2 you should have used E k dot by m dot. So, E k dot by m dot you are not using you are instead you are using directly the average velocity square by 2. So, then there has to be so, what that means is if you use if you continue if you still choose to use average velocity then you have to put a correction factor alpha and that correction factor is referred that correction factor has to be that is given here as α and that has to be incorporated. That means, when you write Bernoulli's equation for such situation that means velocity is not velocity has a profile and you are still applying it for the overall cross section then ideally one is one has to use such correction factor.

Typically for laminar flow the correction factor is already some number is specified you can you yourself can calculate that for turbulent flow some ranges are specified and in many occasions you tend to ignore the alpha or you that is an assumption if you assume alpha to be 1 then that is if alpha is assumed as 1 then you are free to use half v square where v is the average velocity and half v square is treated as the kinetic head. So, that is you get that freedom if you assume alpha is equal to 1, but you need to understand what the implications are that is important. So, this is as far as the Venturi meter and the issues are concerned we need to talk about the other flow meters as well, but let us at least see how what we have done so far. We have done for Venturi meter we can write

$$\alpha_b \bar{V}_b^2 - \alpha_a \bar{V}_a^2 = \frac{2(p_a - p_b)}{\rho}$$

$$\bar{V}_a = \left(\frac{D_b}{D_a}\right)^2 \bar{V}_b$$

So, then this relation is also another relation that is there. So, when you combine the two a nd then you write what is v b bar you see here that. So, what you are doing is instead of v a square you are writing instead of v a square you are writing d B by d a whole square v b bar square. So, if you put this in here then you can take v b bar square common. So, v b bar square if you take common then you are left with alpha B minus alpha A into d B square by d a square.

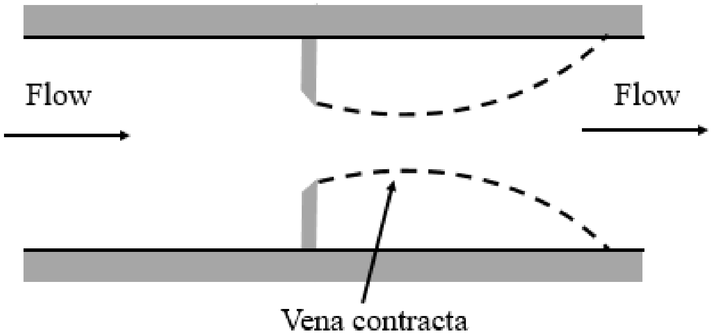
$$\bar{V}_b = \frac{1}{\sqrt{\alpha_b - \left(\frac{D_b}{D_a}\right)^4 \alpha_a}} \sqrt{\frac{2(p_a - p_b)}{\rho}}$$

So, this is going to be the master equation, but this is going to be the master equation with the understanding that there is no energy loss. That means, entire whatever kinetic head whatever pressure it got converted to kinetic head has been recovered completely. So, the net total energy at the inlet and total energy at the outlet they are same, but as I said there will be subtle losses even if you slowly diverge from the throat, but still there would be some amount of losses. So, to take into account those aspects you put this as Cv. Cv is referred as Venturi coefficient and this Cv is provided by the manufacturer.

When typically the Venturi coefficient is either 0.97 or 0.98 it is practically close to 1 and that exact value that the manufacturer will provide and the Cv signifies the loss energy loss that takes place on the from the throat to the downstream side. So,

$$\bar{V}_b = \frac{C_V}{\sqrt{1 - (\beta)^4}} \sqrt{\frac{2(p_a - p_b)}{\rho}}$$

So, that is how it works. So, Venturi coefficient comes in there. When it comes to the orifice, when it comes to the then the other form that we talked about was orifice meter and this is what a typical orifice meter looks like.



You have here we said it will be slowly the cross section is changing, but at least it is not a sudden contraction and expansion first thing and second thing is that the expansion part is you are changing it is at a very low gradient, very low angle it is the cross-section is increasing to the full cross section and at the downstream end of the throat. In case of orifice meter you do not care much about the pressure loss. What you do is you simply go for a sudden contraction and expansion.

On the pathway, you simply introduce the orifice that means, which means it is a section. This is the orifice if you see the side view here and with a small hole at the center. So, flow is being restricted here. So, there would be a sudden contraction and then there would be an expansion also. Now, this is obviously, this will lead to huge energy loss that is for certain and here I said that you do not care about what is the how much would be the energy loss. Because, this venturi meter is meant for more of a it would be a fixed apparatus whereas, these orifice meters are I mean if you want to measure the velocity for some for a special purpose you need to install then of course, you need to have a flange here that arrangement has to be there and as and when necessary you can introduce this orifice just like a compact disc you insert it and of course, this orifice will also have this and then you have the bolts and gaskets and everything.

So, that you can have the orifice in place without any leakage and then of course, measure the pressure drop. So, measure the pressure drop you have the manometer. Where you want to put your manometer port that is another thing where you want to put the manometer port. In this context you will find that when this sudden expansion takes place for any sudden expansion it need not have to be orifice meter. In fact, I should draw a sudden expansion problem and then it will be clear.

Let us say we have a sudden expansion here or let me correct it. Let me correct myself here you would be when you have a sudden contraction not sudden expansion, when you have a sudden contraction let us say this is the contraction. So, the streamlines that are coming here like this they are approaching I said that here there would be they are following very nicely the wall. Here, as it travels, you expect that this would be the smallest cross-section possible. The streamlines will all come here, and then they will continue, but it does not happen that way. The streamlines continue to go like this and then go this way. So, the smallest cross section where the flow takes place here it would be not exactly here at this location rather it is slightly on the right side and it because of this expansion sorry this is the place where the streamlines they are coming most close to each other.

This is not the location this is slightly on the right side because the streamlines will continue to come here this stream you are forcing the streamline to come close to each other that is what you are forcing that is what your sudden contraction is. So, when you are forcing the streamlines to come close to each other you will find that this coming close to each other continues for a short distance and then it starts again filling the complete tube it is not completely filling the tube. So, from this point on, as if the contraction is still taking place here and here, a little bit of expansion takes place, and then it fills the channel completely. So, this event this coming of streamlines close to each other at a place which is not exactly the location where the contraction takes place this is referred as Vena contracta. So, it would be contraction and expansion you will find that when it comes to loss due to contraction loss due to contraction is not I mean you might have you might have seen that we have talked about expansion losses which is substantial expansion losses fluid is allowed to freely expand and then I said that there would be what is this generating.

In case of contraction also there is some loss accounted if you go to the table you will find Ke and Kc K is expansion loss Kc is the contraction loss contraction also some loss is accounted and that loss arises because that loss does not arise till this Vena contracta point beyond Vena contracta the streamlines tend to now get some freedom to spread and fill the channel because Vena contracta here the area is slightly less somewhat less than the actual cross sectional area here. So, now the streamlines have little freedom to expand and at that time some pressure loss takes place that time some small pressure loss takes place. So, this is when it comes to contraction loss one has to measure the loss between these Vena contracta to the downstream side that is the loss that is considered as a contraction loss. Now, when it comes to this orifice meter one can see similar positioning of Vena contracta etc. So, one has to measure the pressure drop across this orifice.

So, one would be upstream of the orifice and another would be at the Vena contracta or at location now you may find that you may say that how will I find where the Vena contracta is located that is a different thing it is typically it is you are putting and putting the manometer here and a manometer there one connected to the upstream side and another to the downstream side you measure the pressure loss the Δp across this orifice that is what the bottom line is you have to measure the pressure drop across the orifice. And since we I said that there would be a big amount of energy loss you can see that just the way we have come up with this expression for the Venturi meter here also we will have a similar expression for the orifice meter and the treatment is same you are having point 1 which is the upstream of the orifice point 2 which is some point downstream of the orifice and then you are writing Bernoulli's equation, but you know there is a huge energy loss term. So, that if you want that to be accounted by a coefficient similar to C v similar to Venturi coefficient then you will have something called orifice coefficient C o, but look at the value of orifice coefficient Venturi coefficient the value was 0.97 or 0.98 against that the orifice coefficient for a turbulent flow that is Reynolds number greater than 30,000 based on the orifice diameter and or velocity at the orifice it is the C o value is 0.61 see the amount of energy loss that takes place within this orifice.

$$u_o = \frac{C_o}{\sqrt{1-(\beta)^4}}\sqrt{\frac{2(p_a - p_b)}{\rho}}$$

$$C_o = 0.61 \ for \ Re_o = \frac{\rho u_o D_o}{\mu} > 30{,}000$$

So, here you cannot expect so significant energy loss at orifice. So, here you cannot expect the orifice meter to be sitting there permanently and causing 40 percent of the energy to be or causing a significant part of the energy to be lost just for the sake of measurement. So, it is for a temporary measurements etcetera that is fine. The advantage this orifice meter provides is orifice meter is cheap you just need an orifice just like a compact disc you need a disc with a hole at the center and you can have multiple such discs stacked and you can insert the disc that you want and get the data.

In case of Venturi meter you can think of the size of the apparatus and that has to be cast. If you look at the fabrication from the fabrication point of view you take a plate and make a hole at the center making a hole at the center that is that is a completely different mechanism. You take a plate and apply. I mean, create a hole, many from one plate, you can make many such orifice many such discs that would serve as an orifices. Whereas, when you make such converging and diverging section etcetera that has to be cast you have to go through a molding and casting and similar methods or you can take a plate and weld and there could be other that I am not getting into the fabrication part of it, but we you must admit that this the fabrication of Venturi meter involves a large amount of materials and of course, fabrication costs. And you have to keep see you want to measure flow rates for certain fluids for some particular fluid and some particular because ρ is involved in this equation you can see ρ is there.

So, for some particular fluid at some where the flow is taking place at certain velocity you have one Venturi meter. For some other fluid at some other velocity you need to have another Venturi meter. So, one has I mean if you want to measure the flow rate within certain range if the flow is taking place. So, you have certain Venturi meter if the flow rate is at a different range then you may see that the pressure drop would be excessive and you cannot do that measurement you need another Venturi meter where the dA by dB or d1 by d2 that ratio is different. So, dA and dB at what ratio you are putting this Venturi meter what dA by dB.

So, there has to be many such Venturi meter in the stock and one has to use them. So, Venturi meter and the orifice meter these are two different ball games all together orifice meter is easy to install do the do your measurement and leave it you can keep a stack of such disks and whichever you want to use you can use it, but Venturi meter it is expensive it needs you have to maintain the inventory and find out which one is applicable etcetera, but obviously, the advantage of Venturi meter is there is no I mean very minimal energy loss. So, these issues are there. Another major issue that I told you before I moved to other there are other flow metering devices is that there has to be an entry and exit length. So, you have a Venturi meter let us say I have this as the Venturi meter.

This is the Venturi meter. So, if this is the Venturi meter I need to have the pipe that is connected on this side and this side there has to be some straight length I cannot afford to have a bend here I cannot afford to have a bend here connected to the Venturi meter because when the fluid enters I have to make sure the streamlines are all straight as they approach the throat, but if I have a bend immediately next to the upstream end or next to downstream end we will see that the streamlines are bent streamlines are not following the way I wanted them to. So, that will cause some extra energy loss that I have not accounted. So, we want a straight section on both upstream and downstream side that is a requirement for Venturi as well as orifice meter. So, that is one thing which is a requirement one must note. The another point is that the velocity is proportional to square root of pressure drop and pressure drop is reflected by the change in manometer height let us say.

So, whatever manometer height I see velocity is not directly it is not a linear relationship it is a square root type relationship. So, you cannot directly you cannot expect some linear chart to help you obtain velocity from directly from the difference in manometer liquid level that is one thing. And other thing is I mentioned that there are various obstruction meters that are there which are proprietary the point there the what they have what the manufacturers have done is they said I am putting an orifice that is giving some kind of pressure drop because that is obstructing the flow and across the obstruction there is a pressure drop. So, why to put an orifice and all instead I have let us say a flow taking place through a pipe and I have some arbitrary obstruction let us say this is the obstruction and then I measure the pressure drop across this I measure the pressure drop across this. So, then also I will get a pressure drop and if I have a priori information for this flow rate for this fluid I will get this much of pressure drop and then we using that as a using that for pre calibration for an unknown flow rate I can immediately find out.

So, this can also be a device as such if you have the pre calibration available and one can sell this product as a flow meter with the pre calibration for certain fluid. So, this type of proprietary devices are also there it need not have to be only orifice or eventually an obstruction is required and we because of that obstruction there would be a pressure drop as long as you have a priori calibration and calibration is sound you can have and most likely these devices are used for a particular fluid. So, they have studied this this obstruction they have studied these for several flow rates they have good amount of data the seller of this product and still it works. So, this is also another way of doing the flow measurement. I want to stop at this point however, I will continue this discussion on flow metering for at least for few more minutes because I have not there are few other topics for example pitot tube and rotameter we have we could not get a chance to talk about in this lecture.

So, I will continue this discussion in the next class as well next lecture module as well. That is all as far as this lecture this discussion this lecture module is concerned. Thank you very much for your attention.