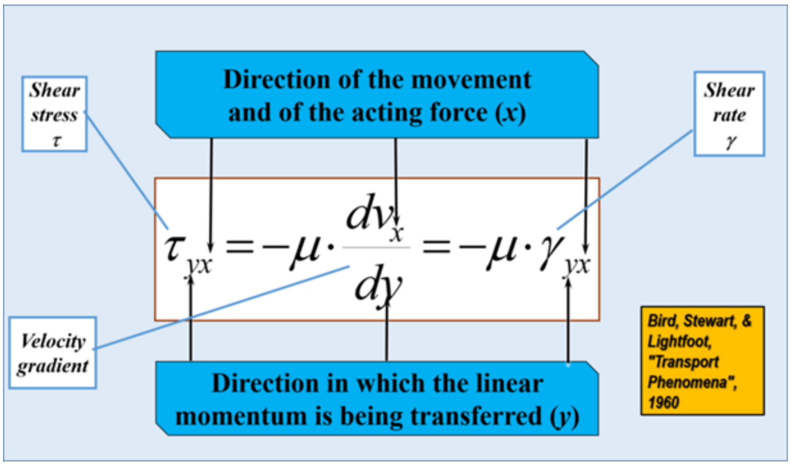
Momentum Transfer in Fluids
Prof. Sunando DasGupta
Department of Chemical Engineering
IIT Kharagpur
Week-11
Lecture-54

Welcome to the last class in my part of the course, where I am going to do a quick recap of fluid dynamics mostly what I have taught in this course about fluid flow, the interaction of fluid layers, the different equations, the relations, considerations and the calculation procedures for flow, velocity, shear stress, the pressure drop, the pump, the head loss and so on. So, in the remaining in this half an hour, I will take you through the journey that we have started with our discussion on momentum transport. And when we started our discussion on momentum transport, the equation the that we have used the first equation, the first relation, first phenomenological relation that we have encountered is Newton's law of viscosity. So, when we see the think about Newton's law of viscosity, there is a direct proportionality between shear stress which is expressed as tau y x with the velocity gradient which is dv x dy. So, we are in this slide talking about one dimensional flow in which the component of velocity v x varies only v x varies only with y v x varies only with y. So, dv x dy is the velocity gradient of the x component velocity and its gradient with respect to the y direction.



So, these two are directly proportional. Now, the proportionality constant is the is known as a viscosity which is a physical property of the liquid and it indicates the interconnection, the relation between the stress exerted by one layer of fluid when it flows past another layer with a different velocity. So, this difference in velocity results in momentum transfer from one layer to the other which is known as the shear stress. The shear stress has two subscripts x, the first subscript over here tells us about the direction of movement in this case it is the x direction and as a result in the flow in a in the x direction momentum gets transported across the direction of the flow in this case it is the y direction.

So, this one refers to the direction in which the linear momentum gets transported. So, the velocity gradient and shear stress are mostly I mean proportional to each other for many fluids the linear variation of shear stress and the velocity gradient this rule is obeyed which is known as Newton's law it defines the property viscosity and fluids that obey this law are known as Newtonian fluid. And there are there is a there are other classes of liquids are shearing thickening, shear thinning, Bingham plastic these types of fluids do not obey the Newton's law. So, it is called they are collectively called the non-Newtonian fluids and there are different categories based on the behaviour of shear stress with the velocity gradient which we have discussed in this course. Then we wanted to do a shell momentum balance momentum in a momentum balance we have defined a shell of the fluid and we figured out what is the convective momentum that comes into the control volume, what is the convective momentum that goes out of the control volume, how much of how much of shear stress the diffusive momentum is getting into the control volume and what is going out of the control volume.

Shell Momentum Balance

At steady state

Rate of MM IN – Rate of MM OUT + Σ forces $=$ 0

Boundary Conditions

No Slip Relative velocity at S-L interface, $v_{\mathrm{rel}} = 0$

No Shear Shear at L-V interface = 0, $\tau = 0$

And at steady state the rate of momentum in minus the rate of momentum out plus sum of all forces acting on the system must be equal to 0. These forces can be body force for example, gravity or electrostatic force or it can be a surface force which is due to the pressure gradient that is present in the in the flow path. Now, this equation this equation is a difference equation when we take in the limit that one of the dimensions approaches 0 in the limit, we get we got differential equation the governing equation for such systems and those governing equations essentially describe the balance of the momentum and forces at every point in the flow field. That governing equation can be solved by invoking boundary conditions and two of the most common boundary conditions which we have used extensively are the no slip where the relative velocity at the solid liquid interface would be 0 and no shear that means, at this liquid vapor interface the shear stress is going to be going to be 0 negligible or equated to 0. So, with these we have solved a number of simple problems that and obtained the velocity profile and once we obtain the velocity profile then we can also obtain what is the shear stress.

But we have in we have seen that with increasing complexity of the geometry through which the flow takes place and if the flow is a function if the velocity is a function of more than one more than one independent variable then a generalized approach is needed and the generalized approach that we have derived and we have extensively used in this course is the equation of motion. And the equation of motion the capital D on the left-hand side is the substantial derivative or the derivative following the motion. So, the left-hand side has one term which is the transient term that is del v by del t and there will be other terms where the velocity appears and in which velocity appears and they are collectively called as the convective transport of momentum. When we think of the right-hand side the right-hand side the first one is the force due to pressure, the second one is the viscous forces the tau refers to the shear stress of the viscous forces and rho g is the body force which is present in the system. So, for a system with constant rho and constant mu the equation of motion can be simplified and for a Newtonian fluid this equation can be simplified in this form.

Equation of Motion

$$\rho\frac{Dv}{Dt} = -(\nabla p) - (\nabla.\tau) + \rho g$$

For constant ρ, μ $\boxed{\rho\frac{Dv}{Dt} = -(\nabla p) - \mu\nabla^2 u + \rho g}$ Navier Stokes Equation

If viscous forces are not present $\boxed{\rho\frac{Dv}{Dt} = -(\nabla p) + \rho g}$

Euler Equation

So, this form is only for Newtonian fluid and which is known as the Navier Stokes equation. So, we have used Navier Stokes equation based on for different coordinate systems, Cartesian coordinate systems cylindrical and spherical coordinate systems. And there we have seen that based on our understanding how we can simplify the component the appropriate component of the Navier Stokes equation and arrive at the governing equation for a specific situation. So, Navier Stokes equation was found to be extremely useful in solving problems in fluid mechanics in the differential approach in fluid mechanics where by solving the Navier Stokes equation we can obtain the velocity at every point in the flow field, the shear stress at every point in the flow field and that give us some very interesting useful results. And through several examples, we have solved this Navier Stokes equation found its utility obtained the velocity profile and so on.

Now, if viscous forces are not present if you are dealing with an ideal fluid in which the viscosity approaches 0. So, if the viscous forces are not present you can easily see that the second term on the right-hand side would disappear and what you get is the Euler's equation. A Euler equation is the starting point for deriving the Bernoulli's equation which we will which we have introduced towards the end. Now, whatever we have done so far are for situations in which the information at every point is can be obtained. So, it is a differential approach the velocity expression that we obtained by solving Navier Stokes equation gives us the velocity at every point in the flow field, but there are many situations in which that information that much of detailed information is not needed what we that we can use is just an overall idea of what is the velocity the forces and so on.

So, from the differential approach we then move to the control volume form formulation and the integral so to say the integral approach. And for this the first equation that we have used the first equation that we have used is the is the control volume form formulation and you know what a control volume is control volume is defined by several control surfaces. Control surfaces do not have any mass of their own whereas, several control surfaces form a control volume which has a specific mass. So, the first equation that we have used is the dN dt N being the extensive property any extensive property eta being the corresponding intensive property. So, the relation between the time rate of change of the extensive property of the system how it is related to the time rate of the change in the time rate of change of the of inside the control volume and the efflux the net inflow and outflow of the extensive property through the control surfaces.

$$\left.\frac{\partial N}{\partial t}\right|_{SYSEM} = \frac{\partial}{\partial t}\int_{CV} \eta \rho d\forall + \int_{CS} \eta \rho \vec{V}.d\vec{A}$$

$\vec{V}$ is measured relative to the CV

Textbook: Fox and McDonald

$\left.\frac{\partial N}{\partial t}\right|_{SYSEM}$ = Total rate of change of any arbitrary extensive property of the system

$\frac{\partial}{\partial t}\int_{CV} \eta \rho d\forall$ = Time rate of change of the arbitrary extensive property within the CV

$\int_{CS} \eta \rho \vec{V}.d\vec{A}$ = Net rate of efflux of the extensive property, N, through the control surface

So, del N del t system is the total rate of change of any arbitrary extensive property of the system this is the rate of change of the extensive property within the control volume and this as I have said that is the net rate of efflux of the extensive property N through the control surfaces. So, the relation between N the extensive property and eta the intensive property is that eta is obtained by dividing N with M where M is the mass of the system. One important point to note here is that this velocity that we have used here is always measured relative to the control volume. Now, when we apply the this for the conservation of mass. So, N is equal to the mass.

Conservation of Mass N = Mass, $\eta = 1$

$$\left.\frac{\partial N}{\partial t}\right|_{SYSEM} = \frac{\partial}{\partial t}\int_{CV} \eta \rho d\forall + \int_{CS} \eta \rho \vec{V}.d\vec{A}$$

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V}.d\vec{A}$$

Incompressible Fluid $0 = \int_{CS} \rho \vec{V}.d\vec{A}$ The size of the CV is fixed

$$\int_{CS} \rho \vec{V}.d\vec{A} = \pm \left|\rho_n V_n A_n\right|$$

When uniform flow at section n is assumed

So, corresponding eta intensive property would be equal to 1 and we know that for the from the conservation of mass we can write that 0 dN dt should be equal to this and if it is at steady state and incompressible fluid then this part would also be equal to 0. So, this is going to be the algebraic sum of all inflow and outflow of mass to a control volume would be equal to 0 and that is what this relation is all about. Now, if I expand this it is simply going to be the sum of all the inflow and outflow the plus and minus sign, we have we have seen that whenever mass comes into a control volume it is to be taken as negative when mass goes out of the control volume it is going to be positive and the flow is going to be uniform at every section. So, we are not talking about the special variation of velocity across a control surface. We have assumed that the flow is going to be uniform at a control surface.

Momentum Equation for Inertial CV, N = Momentum, η = Velocity

$$\left.\frac{\partial N}{\partial t}\right|_{SYSEM} = \frac{\partial}{\partial t}\int_{CV} \eta\rho d\forall + \int_{CS} \eta\rho\vec{V}.d\vec{A}$$

$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho\vec{V}.d\vec{A}$$

$$\vec{F}_B = \int_{CV} \vec{B}\rho d\forall \qquad \vec{F}_S = \int_{A} -p\,d\vec{A}$$

So, that is why it is an integral approach and not a differential approach like what we have done before. If we now have the N to be equal to the momentum. So, the corresponding intensive property momentum divided by mass would turn out to be the velocity and we have the same equation and we could write this equation this is the vector equation which is the force the surface force which is acting on the control volume the body force it is the time rate of the time rate it is this is the unsteady term and this is the net rate of momentum being added to the system. When I said net, it is the algebraic sum of the inflow and the outflow. So, since it is a vector equation it is also possible to write the scalar component of it and these are the expressions for the body force term and for the surface force which is principally the pressure force term.

Scalar Component

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\,\rho\,\vec{V}.d\vec{A}$$

1. To determine the sign of $\rho\,\vec{V}.d\vec{A} = \pm\left|\rho\,V\,\,d\,A\cos\alpha\right|$

2. To determine the sign of each velocity component

$$u\,\rho\,\vec{V}.d\vec{A} = u\left\{\pm\left|\rho\,V\,\,d\,A\cos\alpha\right|\right\}$$

Since it is a vector equation the scalar component for example, here I have shown it written in the x component. So, F x F s x that is the surface force in the x direction body force in the x direction F B x and all the other terms that we see over here. And to evaluate the sign of this one rho this one is plus minus as we know before and to the sign of each component this is the component and this is the mass flow rate with the plus or minus sign being decided based on whether it is flow out or flow into the control system. So, we have solved several problems using utilizing these equations on whatever be the force which is exerted by a flowing fluid in a bend or if there is a jet which comes out of a pipe and strikes another surface what is going to be the force exerted by the jet on the on the surface. Once again, I mention here is that anything that you calculate out of these equations the forces, they are on the control volume on the fluid on whichever way you define the control volume it is on the control volume on the fluid in the control volume.

So, the force exerted by the fluid on the surface is simply going to be minus of that this convention if you if you keep in mind it is going to be very clear and you will not have any problem in using this for solving actual problems. The finally, what we have what we have introduced is the concept of Bernoulli's equation. Now, this Bernoulli's equation the last term on the right-hand side this H L t refers to all the losses which were not needed to be included for the case of an ideal fluid. So, for an ideal fluid the bracketed term on the left-hand side and the right-hand side would be the same. That means, the sum of pressure heads the velocity head and the gravity head would be a constant, but for a real fluid there is going to be some losses when the fluid flows through a pipe.

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+gZ_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+gZ_2\right)+h_{LT}$$

This equation can be used to calculate the pressure difference between any two points in a piping system, provided the head loss, h_{LT} is known.

$$h_{LT}=h_L+h_{LM}$$

Textbook: Fox and McDonald

h_L = Major loss due to frictional effects in fully developed flow

h_{LM} = Minor losses due to fittings, entrance, area changes

The losses could be viscous losses, the losses could be due to the presence of a bend because of sudden expansion or contraction, presence of a valve, presence of other metering devices and so on which would result in pressure losses. So, H L t considers all those losses which are present in the system. One more thing is that the alpha 1 and alpha 2 they are kinetic energy correction factors and we have seen that the kinetic energy correction factors are needed because we have assumed this v 1 and v 2 to be the same throughout the cross section throughout the flow cross section. But that may not be the case specially near the wall and for a laminar flow in which we know that the velocity distribution is going to be parabolic. So, the assumption of velocity being same everywhere is wrong for the case of laminar flow.

So, we could evaluate what is going to be what is going to be the expression from the expression of the known expression of the velocity we have seen that alpha 1 to be equal to 2 for the case of laminar flow and it is approximately equal to 1 for turbulent flow and almost 1 for highly turbulent flow. And in most of the flow situations encountered in engineering applications because of the turbulent nature of the liquid in the in the pipe the it is universally taken value of alpha and alpha 1 and alpha 2 are taken to be equal to 1. Now, to expand on the concept of H L T the total head loss in a piping system with a real fluid it is divided into two distinctly different types. One is the major loss which is H L and one is the minor loss which is H L M and we need to we have separately evaluated what is the major loss and what is the minor loss. So, H L is due to the frictional effects in fully developed flow.

So, since it is H L M is a minor loss due to fittings entrance area changes and so on. And the major and minor does not mean that H L is always going to be more than H L M, H L M could also be more than H L for a specific system. So, how do we calculate H L because that is something which we need to we need to figure out in order to in order to get detailed information about the flow condition and more importantly how much of head has to be provided by let us say for example, a pump in the system so as to deliver the fluid with a velocity with some velocity that means, some flow rate at a certain height and at with a given with a certain pressure. So, you want the liquid to be delivered at a certain point with a certain velocity and a certain pressure. So, we need to calculate what is the head loss from the point where we are where we are going to suck the liquid and deliver it up to that point at up to that elevated point with maybe higher pressure.

$$h_{LT} = h_L + h_{LM}$$

h_L = Major loss due to frictional effects in fully developed flow

h_{LM} = Minor losses due to fittings, entrance, area changes

h_L Major loss

Textbook: Fox and McDonald

For FD flow in a horizontal pipe of constant cross section (from B eqn)

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + gZ_1\right) = \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + gZ_2\right) + h_{LT}$$

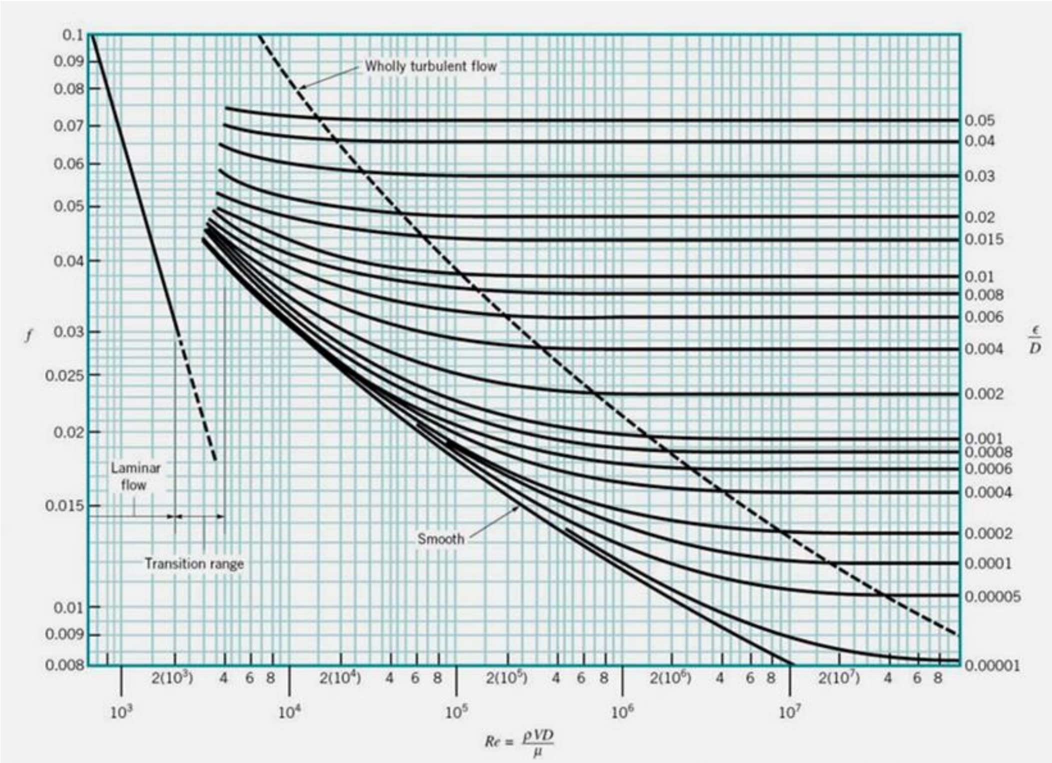
$$\frac{p_1 - p_2}{\rho} = \frac{\Delta P}{\rho} = h_L$$

So, the pump must provide all those losses which are present in the system. So, it is important that we individually calculate what is H L and what is going to be H L M. So, H L is the for a for a fully developed flow in a pipe if you use Bernoulli's equation, you are the H L is simply the losses in the straight pipe. So, if it is losses in the straight pipe of constant cross section there is no question of H L M. H L M only takes place when there is a bend or there is a there is a restriction in its flow path.

$$\Delta P = \frac{128\mu L Q}{\pi D^4} = \frac{128\mu L V\left(\pi D^2 / 4\right)}{\pi D^4} = 32\frac{L}{D}\frac{\mu V}{D}$$

$$h_L = \frac{\Delta P}{\rho} = \frac{64}{\mathrm{Re}}\frac{L}{D}\frac{\overline{V}^2}{2} = f\ \frac{L}{D}\frac{\overline{V}^2}{2}$$

So, if it is a horizontal pipe of constant cross section whatever delta p that you are going to get that you will encounter that you are termed as the major loss. Using the Hagen-Poise lie equation which we have done which we have seen before the H L was expressed as f L by d v square by 2. This f for laminar flow is simply equal to 64 by R e and then we have introduced the Moody diagram which I said again and again is going to be a very important curve to obtain what is the total what is the major head loss due to friction. So, we have seen in the Moody diagram the x axis is with Reynolds number and it is a semi log plot where the x axis is the Reynolds number and y axis is the friction factor and the lines that we see are for different values of epsilon by d where epsilon is the roughness the measure of a roughness which could be in terms of microns on the pipe walls and d being the diameter. So, first evaluating the Reynolds number we go all the way up to the epsilon by d that is for the specific pipe diameter that we know and what is the value of epsilon for those kinds of pipes that we also know.

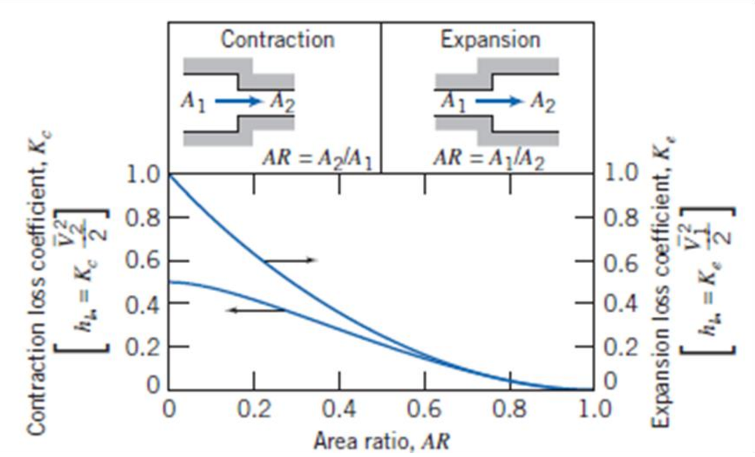


So, we find out what is the value of f. So, from a knowledge of the Reynolds number going up to the point where we know what is epsilon by d and then we calculate the value of f and once we know f the pressure drop is simply going to be f L by this from this f L by d v square by 2. So, Moody diagram tells us about the unknown f friction factor in this case. There are also the minor losses that we must consider. One of the major causes of this loss is when you have a sudden contraction that is fluid is coming and then the this is this is going to go from a larger pipe to a smaller pipe or in the case of expansion from a smaller pipe it goes to a larger pipe.

$$h_{LT} = h_L + h_{LM}$$

$$h_{LM} = K\frac{\overline{V}^2}{2}$$

$$h_{LM} = f\frac{L_e}{D}\frac{\overline{V}^2}{2}$$



K is the loss coefficient, to be determined experimentally
L_e is the equivalent length of straight pipe

So, the contraction loss coefficient and the contraction losses are simply k c v 2 square by 2. So, v 2 square by 2 expansion is going to be k e v 1 square by 2. So, if you look at these two carefully you would see that this v is always the velocity at the lower diameter pipe and the k c is this curve and k expansion is this curve. So, you would be able to figure out what is k L m, k is the loss coefficient and, in most cases, it is evaluated experimentally. In some cases, this entire contraction or the expansion is expressed in terms of an equivalent length of a straight pipe.

So, you have a contraction in the path there is going to be some pressure drop. Instead of going through this curve it may be reported that the equivalent length for this contraction is x meters ok. So, in x meters of straight pipe whatever be the pressure drop that would result due to friction would be same as the pressure drop which is which is happening in that contraction. So, the entire contraction over here can be replaced by certain length of a straight pipe which is known as the equivalent length. So, that I could use the same f that I have calculated before, but instead we use L e the equivalent length for a contraction, for an expansion, for a valve and so on.

Representative Dimensionless Equivalent Lengths (L_e/D) for Valves and Fittings

Fitting Type	Equivalent Length,[a] L_e/D
Valves (fully open)	
Gate valve	8
Globe valve	340
Angle valve	150
Ball valve	3
Lift check valve: globe lift	600
angle lift	55
Foot valve with strainer: poppet disk	420
hinged disk	75
Standard elbow: 90°	30
45°	16
Return bend, close pattern	50
Standard tee: flow through run	20
flow through branch	60

[a]Based on $h_{l_m} = f(L_e/D)(\overline{V}^2/2)$.

So, there are two ways by which you can do one is using loss coefficient or through or two by the knowledge of the equivalent length present in a system. And the losses the minor losses the equivalent lengths of different valves, different elbows, bends etcetera we are provided and we could use that. So, the solution of relevant flow problems the equations are the equations are the Bernoulli's equation the in the modified Bernoulli's equation which is equation a. Equation b is for the major loss 64 by Re for laminar flow, f from Moody diagram or we can use f from some correlations which is only valid for smooth pipes. So, for any rough pipe scenario we must use Moody diagram.

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+gZ_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+gZ_2\right)+h_{LT} \qquad (A)$$

$$h_L = f\frac{L}{D}\frac{\overline{V^2}}{2}, \quad \textit{major head loss}, \qquad (B)$$

All terms are energy per unit mass

f = 64/Re, for laminar flow

f from Moody diagram or $f = \dfrac{0.3164}{\mathrm{Re}^{0.25}}$

for smooth pipes for turbulent flow

And the h l m the minor losses are through the loss coefficient or the equivalent length. So, C 1 and C 2 equations we have used to obtain what are the what are the losses. And after this that we have seen that there are four different types of situations. So, the variations are delta p the pressure drop is going to be a function of whatever be the length of the pipe, whatever be the volume volumetric flow rate v of the pipe and so on. So, out of this that delta p is a function of function of the length function of the function of the velocity function of the flow rate.

$$h_{LM} = K\frac{\overline{V^2}}{2} \quad (\text{min}\, or\, loss, fittings, bends, abrupt\, area\, change\, etc) \qquad (C1)$$

$$K = Loss\ coefficient\ (\text{exp}\,erimentally\ \text{det}\,er\ \text{min}\,ed)$$

$$h_{LM} = f\frac{L_e}{D}\frac{\overline{V^2}}{2} \quad mostly\ for\ valves\ fittings\ and\ bends \qquad (C2)$$

$$L_e\ Equivalent\ length\ of\ straight\ pipe$$

So, three four different types of problems are can be envisaged all problems can be can be divided into four different types where three of the parameters should be known and the force would be unknown. So, let us say if the length the velocity and the flow rate are unknown delta p is unknown. So, how to handle such a problem and through several such applications I have shown that how this the unknown let us for example, let us say delta p for a flow assembly can be obtained. And then we spoke about the pumps the head developed by the pumps and the how much of head is needed to make the liquid flow from one point to the other. So, we spoke about the pumps and I have solved number of problems to clarify the or to highlight the concepts which are involved in it.

And we also spoke about the metering devices for example, the orifice meter, the venturi meter their advantages and disadvantages, the rotameter which is a variable area meter how to obtain the flow rate for such cases once again the advantages of using a rotameter vis-a-vis that of a venturi meter. And finally, we spoke about the pumps the requirement of net positive suction head and how much of we need to know from the manufacturer how much of NPSH is required and what is the NPSH that we have based on certain equations that we have derived in the last class. So, this together would give us an idea of whether a pump chosen for a specific purpose of moving fluid from one point to the other up to certain elevation will work properly without cavitation or not. So, this in a nutshell what I have covered in my part of the course and I hope you have enjoyed it, I hope you have learned something new in a structured manner and if there are any questions through the tutorials and through the assignments and with the help of the TA, we would be able to resolve any remaining doubts that you may have. So, thank you very much and best wishes for a wonderful fluid mechanics study and I hope you have learned something new in this course. Thank you.