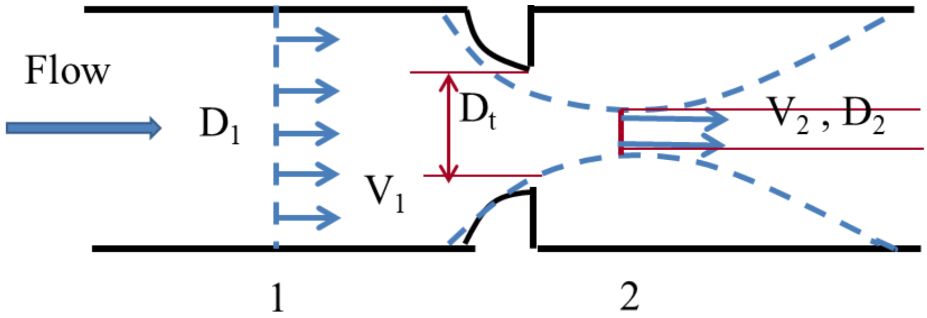
Momentum Transfer in Fluids
Prof. Sunando DasGupta
Department of Chemical Engineering
IIT Kharagpur
Week-11
Lecture-52

In the two lectures that are remaining, I am going to talk about the flow meters, their working principle and something about the pumps. So, how are the pumps characterized? Is there any way you can we can decide about the optimum condition of a pump? So, what are the characteristics of a pump that need to be known a priori before we decide about in which range of operation the pump is going to work at the maximum efficiency. So, the we are we will talk about the pump characteristics, the pump characteristic curves in the second in the second lecture, but in this one I am going to speak mostly about the different flow meters, their working principle, their merits and demerits and other associated issues. So, the flow once we discuss about the flow meters and the pumps, the last lecture on my part of this course will be a recap of what I have covered in the past 29 lectures, but today's one is going to be about flow meters. So, let us start our discussion on flow meters. We understand that the measurement of flow is an integral part of any process plant or any of any situation in which flow is taking place through a pipe, through a river, through a storm drain anywhere.

So, accurate knowledge of the flow, the quantity of flow is something that all of us would like to know. So, what are the different what are the different instruments that are being used for such a case and what are their working principles? You would see that in many situations it may not be possible to exactly figure out from theory what is going to be the flow rate. How do we connect these flow rate with something that we can measure? In fluid flow the one that we can easily measure is the pressure drop between two points. So, is there a way to connect the flow with the pressure drop and there could be a theoretical way, but you may have to add some sort of empirical factors to factor in certain situations, certain geometry associated flow issues into consideration.

So, we will slowly go through that one after the other. So, one way to measure the flow is directly figure out what is the delta p and with some equation and some numbers convert that direct measurement of delta p into the flow rate. So, the easiest way to artificially create a pressure drop in between two tapings is use a restriction in the flow path. So, if I can restrict the flow restrict the area through which the flow takes place there is going to be a pressure drop across this restriction. And the pressure drop across this restriction is going to be a function of the function of the velocity and therefore, a function of the flow rate.

So, it is this functional relationship between delta p and velocity and therefore, flow rate which is utilized for direct measurement of flow through a constriction and the many there are many devices which work on this principle. So, it looks something like this what you see over here, what you see over here is flow and the flow is assumed to be uniform that means, it has to be turbulent flow and when many of the industrial situations the flow generally the flows generally are turbulent. So, it is not too bad an assumption and there is a there is a restriction which has been provided dt sometimes is also known as the throat diameter d1 is this and then the flow gets constricted the streamlines bend they come very close to each other and then this start diverging again reestablishing the pattern what was there when it when before it encountered this restriction. So, the same flow pattern is going to get going to going to reestablish itself after certain distance and this distance obviously, would depend on the property of the property of the fluid which is flowing it will depend on the flow rate and therefore, it is going to depend on the Reynolds number. So, this region where the they come when they come very close to each other is known as the Vena contractor, but you can see that it is extremely difficult to visualize and to measure what is going to be the value of d2, but the principle with which the on which upon which these systems work these measurements are taken is that the change in the velocity leads to a change in pressure.



So, if I can figure out what is the pressure at this point and some point downstream this p 1 this p 1 minus p 2 can be correlated to can be related to v 1. So, we are going to first try theoretically obtaining the flow rates we are going to use equation of continuity and Bernoulli's equation between sections 1 and 2, but we would like to stress that the actual flow rates can be different than the theoretical flow rates. There would be some departure between the actual flow rate and the theoretical flow rate. Why is that? Because there is going to be lot of turbulence produced in here there could be back circulation there could be there could be multidimensional effects. So, what we consider theoretically as uniform flow over here and uniform flow at section 2 that may not be true.

So, to take care of such situations the empirical factors are sometimes used, but let us start with the theoretical analysis first and then we will add in the empirical factors in order to in order to discuss in order to give it something a flavour where they can be used industrially. So, we start with writing the Bernoulli's equation over in between these two in between section 1 and 2. So, the pressure differences the it is in the same the z 1 is equal to z 2. So, this is going to be the pressure difference equation and therefore, v 2 is simply going to be root over 2 p 1 minus p 2 rho by 1 minus a 2 by a 1 whole square. We understand that a 2 for a circular pipe is simply going to be pi d 1 square by 4 d 2 square by 4 d 2 square by 4 and so on.

So, 1 minus a 2 by a 1 whole square can also be equal to 1 minus d 2 by d 1 whole to the power 4. So, that is the that is the that we can directly measure and put in their p 1 and p 2 we can figure out and rho of the p 1 and p 2 can be read from a manometer or some such pressure measuring devices attached to the to section 1 and 2 and rho is the density. So, if I know the velocity by a simple theoretical analysis then m dot theoretical the mass flow rate is simply going to be rho v 2 a 2, but there are problems. So, what are the problems that we what are the things that we did not consider in this there are many and by the time I think you would be able to identify most of them even before you see what I have shown over here. First thing is steady flow, probably it will be satisfied in most of the situations.

$$\frac{p_1}{\rho}+\frac{V_1^2}{2}+g\,Z_1=\frac{p_2}{\rho}+\frac{V_2^2}{2}+g\,Z_2$$

$$p_1-p_2=\frac{\rho}{2}\left(V_2^2-V_1^2\right)=\frac{\rho V_2^2}{2}\left[1-\left(\frac{A_2}{A_1}\right)^2\right]$$

$$V_2=\sqrt{\frac{2\left(p_1-p_2\right)}{\rho\left[1-\left(\frac{A_2}{A_1}\right)^2\right]}}\qquad \dot{m}_{Theoretical}=\rho V_2\,A_2$$

$$\dot{m}_{Theoretical}=\frac{A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}\sqrt{2\,\rho\left(p_1-p_2\right)}$$

$$\dot{m}_{theoretical}\;\infty\;\sqrt{\Delta P}$$

Incompressible not too bad an assumption most of the fluids can be treated as incompressible fluids. Flow along a streamline may not be true in all the cases especially when you have highly high turbulence intermixing present in the system. This is the one of the major problems there is no friction when it passes through the constricted area. So, friction can is neglected here, but we understand that it is may not be prudent to neglect friction completely. Uniform velocity at section 1 and 2 like the velocity that you see here uniform velocity at section 1 and 2 I have noted already I have explained already why it is not right appropriate to think that the velocity is going to be uniform at section 2.

It could be uniform at section 1, especially if the flow is highly turbulent, but if the flow is laminar then even here at location 1 the velocity is not going to be uniform across the cross section. And of course, there are other problems associated at section 2 which will assume that the velocity is uniform at 2 I mean it is erroneous. So, the pressure is uniform at 1 and 2 the pressure may not be uniform at 1 and 2 there could be variation in pressure variation in pressure with the height with the height with the location where the pressure trap pressure tap is being connected to. So, the pressure at the bottom the pressure at the top of the pipe at the same axial location pressure inside near the centre line they could be different as well. And in many situations the measuring devices may not be connected horizontally.

So, what happens if there is a variation in z as well? So, all these factors are not incorporated in the simplistic representation of velocity as you can see from this. So, here the connection between the velocity and the pressure is very straightforward, but we did not take into account the problems associated mainly with assumption 3, 4, 5 and 6. So, these 3 to 6 are the major sources of error in using this formula for finding out what is going to be the mass torade and that is why we call it as m dot theoretical m dot actual could be different from this simplistic expression of velocity. So, this after putting this in there this is the expression for, m dot theoretical, but what we can see in here is a relation between the m dot the mass torade and delta p. So, we would expect that any relation or correlation that one can propose to express m dot in terms of delta p it will most likely follow the relation that m dot is proportional to root over delta p.

Assumptions:
(1) Steady flow.
(2) Incompressible flow.
(3) Flow along a streamline.
(4) No friction.
(5) Uniform velocity at 1 and 2
(6) Pressure is uniform at 1 and 2.
(7) $z_1 = z_2$

So, the pressure drop and m dot should follow this expression even though we do not know how to consider this proportionality constant. If I remove this proportionality constant what is the exact relation between m dot and delta p. So, to do that the other problems what is the actual flow area into as I have mentioned before I do not know what exactly the Vena contractor size is, what is going to be its diameter. Velocities are only uniform at high values of Reynolds number; the frictional effects are important and location of pressure taps could influence the results. So, these are the assumptions 3 to 6 that I was talking about in the previous slide which makes the use of a simplistic relation between m dot and delta p a kind of a problematic.

Limitations

- Actual flow area at 2 is unknown
- Velocities are uniform only at very high Re
- Frictional effects could be important
- Locations of pressure taps can influence the results

So, what is done for that case we are going to rely on empirical discharge coefficients. Now, when I say empirical discharge coefficients for any type of constriction any type of flow reduction there can be huge experimental data available in the literature where people have measured what is the delta p across such a reduction in area and what is the corresponding flow rate. And they wanted to fit that flow rate with the pressure drop for a variety of fluids, for a variety of geometry and flow conditions. And what they have come up with is a sort of a number of a coefficient C as is shown by C over here where this is the actual mass flow rate which anyone can measure and the second is the theoretical mass flow rate. So, if this coefficient is known for a specific type of constriction, then we do not have to measure the actual flow rate anymore.

Empirical Discharge Coefficients $C \equiv \dfrac{Actual\ mass\ flow\ rate}{Theoretical\ mass\ flow\ rate}$

$$With\ \beta = \left(D_t / D_1 \right) \qquad \left(A_t / A_1 \right)^2 = \left(D_t / D_1 \right)^4 = \beta^4$$

$$\dot{m}_{Actual} = \frac{C A_t}{\sqrt{1-\beta^4}} \sqrt{2\rho\left(p_1 - p_2\right)}; \qquad \frac{1}{\sqrt{1-\beta^4}}\ is\ the\ velocity\ of\ approach\ factor$$

$$\dot{m}_{Actual} = K A_t \sqrt{2\rho\left(p_1 - p_2\right)} \qquad K\ is\ termed\ as\ the\ flow\ coefficient$$

I already know from theory what is going to be my mass flow rate I simply must multiply it with C and then to obtain the unknown actual mass flow rate in any condition. So, to do that my C will have to vary within a small range it should not be it should not be something which is going to vary itself by changing flow conditions and so on. So, the empirical discharge coefficient is used in this form where m dot actual is C this is the empirical discharge coefficient times the theoretical mass flow rate. We know that the theoretical mass flow rate is simply A by root over 1 minus beta to the power 4. What is 1 minus what is beta? Beta is d throat by d 1 look that it is to be noted that we have changed the Vena contractor to the diameter of the throat.

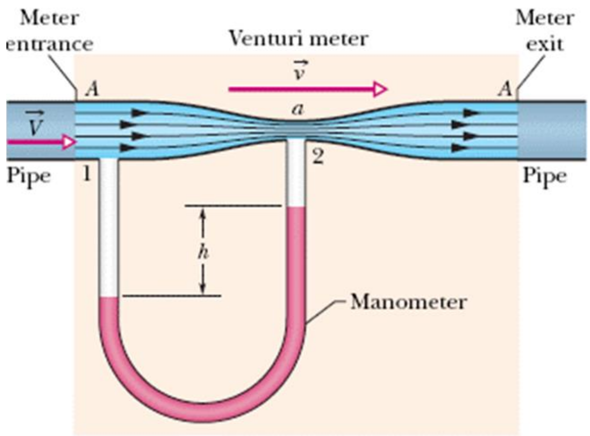
So, this is the diameter of the throat and from the construction of the flow meter this is precisely known to us. So, we are not relying on the uncertain value of d 2 the Vena contractor. We are rather expressing everything in terms of d t or the throat diameter and d 1 which is the diameter of the inlet pipe and the diameter of the outlet pipe. So, in the pipeline of known diameter d 1 I have added a restriction where the diameter of the throat diameter at this point is d t. So, both are accurately known to me.

So, I am making a change in my expression first the A t by A 1 square becomes d t by d 1 to the power 4 and this is what we call as beta we this ratio which is unique for a restriction, we need unique for a restriction is termed as beta. So, my m dot actual is A t by root over 1 minus beta to the power 4 2 rho times p 1 minus p 2 the same as the expression which you have obtained before except it is A t now and beta contains d t not d 2. So, that is the that is the change and to consider these changes in changes the C the discharge coefficient empirical in nature has now been introduced. So, C is the one which would take care of the errors associated with using A t and d t in the theoretical formulation for the mass flow rate. Sometimes this 1 by 1 by root over 1 minus beta to the power 4 is called the velocity of approach factor it is just a name and m dot actual is sometimes when you when you take this C and 1 minus beta to the power 4 and express it in terms of k just a new constant k.

So, it becomes more compact it is k times A t and the pressure difference square root of pressure difference. So, this k is termed as the flow coefficient. So, your flow coefficient or C must be known to you for any geometry any area constriction that you have added in the path of the flow. If you know that for a wide range of Reynolds number for an that means, for several fluids number of velocities and so on then you would be able to correlate the mass flow rate correctly precisely with the measured value of p 1 minus p 2. So, you only need to have the pressure drop data for this kind of meters and find out what is the actual mass flow rate taking place in such a system.

So, empirical relations as I mentioned C and k are available as functions of the meter bore that means, what is the size the pipe diameter what is the size at this point pipe diameter and Reynolds number. So, with this knowledge which are available in the literature you would be able to find out what is the actual mass flow rate. Now, the flow meters which work on this principle essentially the better the flow meter if the value of C is going to be closer to 1 that means, it is going to mimic the ideal situation as far as it is going to mimic the ideal situation if the value of C is closer to 1. So, if the value of C is closer to 1, we get less head loss due to this pipe due to the restriction in the flow. So, the flow is going to recover the pressure is going to recover completely when you come to this location and that means, the frictional and other losses over here are going to be very small.

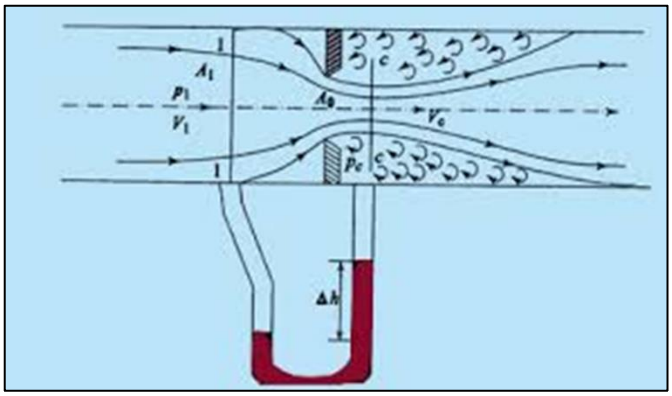
So, if the frictional and other losses are going to be very small if the pressure is recovered over here then the value is C value of C is going to be close to 1. So, any flow meter that has a value of C close to 1 are generally very accurate desirable and so on. So, one such example of a flow meter with a value of C close to 1 is venturi meter. It is widely used and the it is what you can see is the gradual lessening of the area very slow gradual change in the area and at this throat and from the throat it again changes very smoothly and becomes equal to the diameter as the approach 1. So, the streamlines are going to bend and the flow is going to be uniform less pressure drop because of the presence of the throat as you have a smooth change in the flow path not an abrupt change a smooth change in the in the flow path and then the pressure is recovered.



C = 0.98 – 0.995, Low head loss, high cost, excellent recovery of pressure

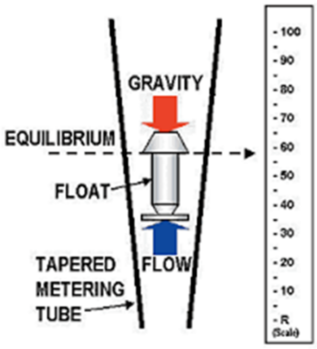
So, in such a case a venturi meter should have a value of C very close to 1 and indeed when we talk of when we measure the flow coefficient or the discharge coefficient C its value turns out to be between 0.98 and 0.995 which show which means that you have a very low head loss. So, whatever is the pressure it is completely recovered, but you can see that the machining necessary to have such a small a change in the flow area comes with a high fabrication cost. So, even though the excellent recovery of pressure, but it requires high cost and it takes up quite a bit of area to install such a venturi meter in a pipeline.

So, if you want accurate prediction of the flow rate by noting the pressure difference venturi meter is your choice, but if you cannot if the cost is prohibitive and if the space available is not sufficient then you must adjust with less accurate flow meters and one such less accurate flow meter is known as the orifice meter. There is no gradual change in the flow area it just has a restriction in the flow path. So, the flow is going to be highly turbulent over here mixed over here and there can be circulation back circulation of flow which has been shown by these arrows. So, what is going to happen is that this sharp change in area in flow area introduces considerable pressure drop in this system. So, and the back mix this kind of back mixing if this contains suspended particles the suspended particles tend to deposit over here and at some point, it may try to clog the entire flow.



So, this is inexpensive easy to fabricate, but comes with the high or rather low value of C high pressure head loss at this point. So, this is called the orifice meter. So, high head loss low cost suspended matters may start to build up. There is a third kind of flow meter which is a variable area meter it is known as rotameter. So, what you have is a float and in the rotameter the area diverges like this and you have flow from the bottom.

Rotameter



Summation of

drag force ↑

weight ↓

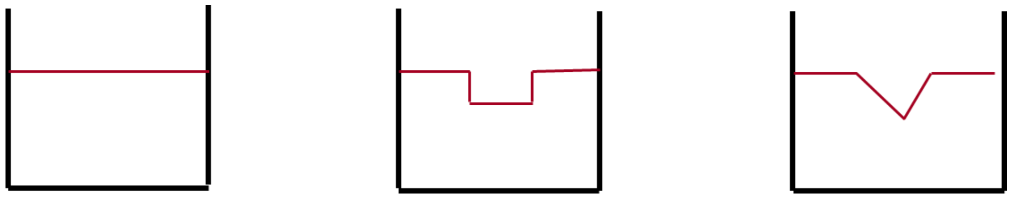
buoyancy ↑ = 0

So, if the flow from the bottom, then the flow must pass through this area and through this area. So, the if the float instead of being over here is situated at a slightly higher point, then the flow area between the float and the tapered tube would increase. So, if the float is over here then the area would be more if the area is more the velocity is going to be less. So, the upward velocity along this is going to be more if it settles down it is going to be less if it goes up. So, what is the force which is associated with the relative velocity between the fluid that is moving up and the float? What are the other forces which are acting on it? One is gravity which is trying to put it below and there is going to be some sort of buoyancy because of the density of the float itself and there is also a force due to the passage of the liquid which drags the tries to drag the fluid up.

So, when that float comes to a standstill at that point all these three forces must be balanced, they must balance each other. So, one is the upward force which is buoyancy, the second upward force is drag and the one which flows which points downward is the gravity. So, the summation of drag force the weight and the buoyancy at steady state when the float has come to a come to a equilibrium position must be equal to 0. So, of these the weight and the buoyancy does not depend on velocity drag force does higher the velocity higher would be the drag force. So, when the flow when the velocity increases the drag force also increases.

So, it takes the takes the float at a slightly at a location slightly above this to the point where the enhanced flow area causes a reduction in drag force and that drag at that location the drag force weight and buoyancy cancel each other. So, if it is at this point find out what is the flow rate. If you reduce the flow rate the drag reduces the gravity becomes more and it comes to a new position over here at which due to the decrease in the area and increase in velocity the drag force increases and that increase drag force at a lower location at a lower flow rate again makes this summation equal to 0. So, the velocity induced drag force and the location of the float are calibrated for most of the common fluids and flow rate regimes. So, this variable area meter variable area flow meter example of which is a rota meter uses the drag force weight and buoyancy and from the location position itself from the calibration curve itself you would be able to figure out what is the flow rate that is flowing through such a device.

There are certain other certain other area meter certain other meters which are used which I should I should I must mention before I close this part is how do you measure the flow rate in a municipal drain which is open one side is open the upper portion is open and it is just a rectangular channel through which the flow takes place. How do you find out the flow the amount of water which is released from a dam what is the flow rate of a river how much of water is flowing in a river. So, in this case it looks something like this is the top surface of the liquid which is flowing and what you measure what you put is a wear a wear in the flow path it could be a V-shaped as well. So, as water flows over this it is going to encounter a restriction over here and it flows over and over and above the wire therefore, looking at the height of the liquid above this above these wires they are then correlated the flow over the wire is correlated with the actual flow rate throughout this entire cross section. So, the wires with empirical discharge coefficients and there are many correlations that I am not including here you can read it from the textbook from any textbook on measuring the total flow rate in an open system using wires.



So, this more or less concludes my discussion on flow measurement devices one is venturi meter with obvious advantages and disadvantages orifice meter a variable area meter which is a rotameter which balances three forces and the fourth for measuring large volumes of flow in open channels one uses wires and the flow over the wires the rise of the liquid above the wires is correlated to the actual flow rate and thereby you can measure the flow rate through a river or when you open a dam how much of water is coming out those are calculated by wire. Nowadays there are other sophisticated ways to measure flow rates which uses different principles for example, change in capacitance and so on, but those are used in specialized conditions the very venturi meter and orifice meter are still being used in most of the industries, but the new methods do not require measurement of pressure drop per say, but they measure electrical properties of the flowing fluid as a function of flow rate and trying to convert that the change in properties to the flow rate. So, that is our discussion on the measurement of flows the flow meters and in the last class we are going to talk about pumps and their characteristics. Thank you.