Momentum Transfer in Fluids
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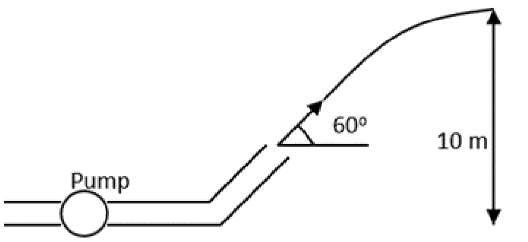
Lecture-51

The last problem on the application of Bernoulli's equation is an interesting problem in which involves a fire and a fireman who is going to deliver water at a certain height. So, there are two firemen present, two fire personnel present and they were discussing about what would be the nozzle size, should they use a nozzle, should they just use the hose or should they use a hose with a smaller diameter to douse the fire. And they are one of the fire personnel, he or she would like to test the knowledge of fluid mechanics which she or he obtained while attending this course. So, let us see what is the problem all about. This is the situation in which water must be delivered at a height of 10 meter. This portion has been exaggerated.



So, one we can assume that the jet starts from the level and it must reach the 10-meter height building. So, for do not consider this part the height of this part. So, we can assume that the water has to reach to a height of 10 meters. So, a fire personnel mistakenly picked a nozzle whose diameter is the same as that of the hose.

What would be the implication of that? The length of the hose is 30 meters, its diameter is 8 centimetres and the roughness of the hose is 0.0008 meter. He must deliver a jet of water at 60-degree angle, this angle is 60 degrees to the horizontal to a height of 10 meter above the ground. So, this height is 10 meters, this angle is 60 degrees, the length, diameter, and the roughness all are provided in here. We need to calculate as someone who studied fluid mechanics, what is the head which is to be developed by the pump and is it just is it a mistake that the fire person took the nozzle having the same diameter as that of the hose and can there be something an improvement to his or her choice which can be made.

So, what happens next is that fellow fireman who claims to have studied fluid mechanics has suggested that the head requirement of the first part can be drastically reduced by attaching a reducing nozzle. So, the first person uses the nozzle as these with the having the same diameter as that of the pipe. The second fire person she is suggesting that the reducing to use a reducing nozzle. So, the nozzle diameter is going to be smaller than the diameter of the pipe. And the diameter that she has chosen is 2.5 centimetres, previously it was 8-centimetre pipe, 8 centimetres nozzle. Now it is 8 centimetres pipe, but 2.5 centimetres nozzle size. Do you think that the second fire person has really studied fluid mechanics? So, you must analytically find out which one is correct and by how much, whether the second fire person whether she is correct in giving her friend a reducing nozzle and if so by how much. So, this is what we have to we have to we have to figure out and we are going to neglect any minor losses.

So, we are only going to talk about the major losses in this problem not the minor losses. So, we are going to solve first this problem where the nozzle and the pipe are having the same diameter. So, the velocity when it comes out of this there is a vertical component and there is a horizontal component. Now, the vertical component is going to experience the returning force due to the due to gravity. So, gravity is going to gravity is going to going to reduce the velocity, reduce the vertical component of the velocity.

But if we do not if we do neglect the air resistance what we can say is that the horizontal component will remain unchanged on the flight of water in the vertical direction. So, the vertical component will keep on decreasing till it reaches the top and the at the ideal condition the vertical component of the velocity out of the nozzle out of the nozzle from the fire personnel would be equal to 0 at this point. While the horizontal component will remain unchanged because we are neglecting the resistance provided by the air. So, the vertical component would be just sufficient to reach the top and at the top its velocity is equal to 0 that is the ideal condition. And the head developed by this pump is then obtained from the velocity head at the nozzle and the head loss in the hose.

So, whatever be the head loss in here and whatever be the head the velocity head provided at this point that much must be supplied by the pump in absence of any minor losses that is the problem. So, we know that the velocity the vertical component becomes 0 horizontal component remains unchanged. So, the horizontal component is whatever be the nozzle velocity times cos 60. So, that is going to be the only remaining component of velocity at this point. So, with this premise we start solving the problem.

So, the velocity as I have mentioned the vertical component of the velocity becomes 0 at this top the horizontal component remains unchanged and the velocity is simply going to be v z cos 60 which was the velocity when it came out of this nozzle which is essentially v j by 60. So, the velocity becomes half compared to the velocity over here because the vertical component becomes equal to 0 only the horizontal component is remaining in such case. So, we are going to apply the energy equation which in this case is the Bernoulli's equation between the nozzle exit and the highest point at the nozzle exit and at the highest point we are applying the Bernoulli's equation. So, once we do that so, this is v j square that is the velocity out of the nozzle and then we have included the atmosphere term, but we know that the pressure to be used in Bernoulli's equation is always the gauge pressure. So, this term would disappear.

Similarly, at the top the pressure is also atmospheric. So, this term will also disappear since my gauge pressure will be equal to 0 and the velocity here was v j the velocity at the top since only the horizontal component of the velocity is remaining it is going to be v j by 2 as you can see over here. So, it is v j square by 2 by twice J plus 10-meter, 10 meter being the height to which the velocity to which the liquid has to reach. So, this clearly is the statement of the statement of the energy equation and we also understand that since it is moving through air there is going to be negligible frictional losses. So, no major or minor losses between the exit and at the top that is why in this equation I did not include any of the losses any of the h in here.

$$\frac{v_j^2}{2g} + \frac{p_{atm}}{\rho g} = \frac{\left(\frac{v_j}{2}\right)^2}{2g} + 10\ (m) + \frac{p_{atm}}{\rho g}$$

$$\rightarrow v_j = 16.17\ m/s$$

So, this once you solve this should give you the velocity out of the nozzle is 16.17 meter per second that is the velocity out of the nozzle at this point. Now, we must figure out what is going to be the head developed by the pump. So, the head developed by the pump first thing is what is going to be the head loss in the pipe. The head loss in the pipe is going to be f v square by 2 g L by D.

The head loss in the pipe,

$$h_l = f\frac{v^2}{2g}\frac{L}{D}$$

You remember you see I have specifically expressed h in this form because in some of the textbooks you would see that the definition of h is f v instead of v square by 2 it is v square by 2 g. So, nothing has changed conceptually in some texts specially in some somewhat older texts it is expressed in terms of meters of water or meters the head is expressed in meter. So, in that case the g appears in the denominator, but conceptually there is no difference you can solve the problem using the convention that we have followed in earlier problems that h is f L by D v square by 2 only the unit would be different nothing else. So, that is to that must be kept in mind. So, to do that I need to figure out what is the value of friction factor.

$$Re_D = \frac{10^3 \times 16.17 \times 0.08}{10^{-3}} = 1.294 \times 10^6$$

So, I put the value of this in here. Now, note here that since the nozzle diameter and the diameter of the supply pipe both are same diameter both are equal therefore, the velocity in the pipe will also be equal to 16.17 which is the velocity out of the nozzle. The nozzle size and the pipe size these two are equal. So, I figure out that my Reynolds number is 1.3 into 10 to the power 6. So, it is going to be somewhere over here 1.3 into 10 to the power 6 and we will have to go all the way up to the up to the case where 0.01. So, 0.01 is over here and this is 1.3 into 10 to the power 6. So, you go somewhat over here and you get the value of f equal to this point which is 0.038. So, exactly the value of friction factor can be obtained as 0.038. Since we do that now we would be able to calculate what is h L. So, this is everything over here g in this case and this is in meters, this is the head in meters that is what I was telling you before that if you bring the g in the denominator in the Bernoulli's equation. So, instead of p by rho plus v square by 2 plus g z it become. So, let me write it. So, the Bernoulli's equation in this case becomes p by rho this is the Bernoulli's equation form of Bernoulli's equation which we have used g z is constant, but the form of Bernoulli's equation which is used in here this g this g comes over here and here and this simply becomes z.

$$\frac{\in}{D} = \frac{0.0008}{0.08} = 0.01 \rightarrow f = 0.038$$

$$h_l = 0.038 \times \frac{(16.17)^2}{2 \times 9.81} \times \frac{30}{0.08} = 189.9 \text{ m}$$

Head of 189.9 m is lost within the hose

Now, when it simply becomes z then you can see that unit of each of these terms is meters that is why the head loss if you express head loss where the g comes in the denominator would be in meters. So, sometimes by head we mean we would like to express it in terms of meters. So, if that is the convention that is the one if some textbooks follow that then p by rho g meter is this one. However, there is no change in concept as you could see that it the same thing is obtained simply by dividing simply by dividing p by rho with g such that this z has units of meters. So, everything else has units of meter.

So, h L is 0.189 and which means that a head of 189.9 meter is lost within the hose. So, this much of loss you are going to get in the in the pipe. So, this is this is some loss which is which is which have to which have to supplied by the pump. The pump not only has to supply the loss within the pipe within the hose it is it is also connected over here to some reservoir the pump must be connected to some reservoir over here at this point.

So, the inlet is connected to a reservoir and the nozzle exit. So, I am going to figure out what is going to be the total head generated by the pump. So, the total head generated by the pump must be equal to the loss which is this and the kinetic head which is generated at this point. The kinetic head is simply v square by 2 g. So, if it is v square by 2 g and h L is the head lost due to friction in the pipe therefore, the head loss due to the velocity and head loss due to friction in the pipe essentially gives us h s a head to be developed by the pump in terms of meters because of our new definition change definition conceptually identical, but just a g coming in the denominator is going to be 203.2 meter. Now, if you consider the height to which the to which the water must reach it is only to it is only 10 meters in order to deliver water at a height of 10 meters your pump has to create a head of 203 meters. So, that is a huge value very high value considering that you are develop you are putting the water only at a height of 10 meters. So, the choice of the first fire person to have a nozzle having the same diameter as that of the pipe is a wrong one you require very high head to be developed by the pump this is not a wise decision. Now, let us see what is going to happen if according to the second fire personnel a smaller diameter nozzle is attached in front of the in front of the hose what is going to be the reduction hopefully the reduction in the head to be generated by the pump is it going to be 203 or is it going to be something else that is what we must figure out next.

Head of 189.9 m is lost within the hose

Total head generated by the pump (apply Bernoulli equation between the inlet of the pump, where the inlet is connected to a reservoir and the nozzle exit)

$$h_s = \frac{v_j^2}{2g} + h_l = \frac{(16.17)^2}{2 \times 9.81} + 189.9$$

$$h_s = 203.2 \text{ m}$$

So, if I if I have a nozzle of 2.5 centimetres my velocity remains my velocity requirement at this point remains unchanged it will still have to reach a height of 10 meters and when it reaches that height the vertical component of velocity can be 0 whereas, the horizontal component of the velocity will remain unchanged. So, using the same logic whether the water is coming out of the larger nozzle or the smaller nozzle the starting velocity the velocity of throw at this point will remain the same and that is what 16.17 is all about. However, the areas are different now the nozzle has a smaller area its diameter is 2.5 whereas, the pipe diameter is 8. So, therefore, if the velocity out of the nozzle is 16 then the velocity in the pipe using equation of continuity that A 1 v 1 is equal to A 2 v I would be able to calculate the velocity inside the hose is only 1.58 meter per second. So, if you compare previously my velocity in the hose was also 16, but now it has come down to 1.6 meter per second and since the velocity is less, we would expect that the head loss due to the flow in the pipe will also be smaller. Let us calculate that and figure it out.

If a nozzle of 2.5 cm exit diameter is used the velocity required in the hose is,

$$V_n = 16.17 \times \left(\frac{2.5}{8}\right)^2 = 1.58 \text{ m/s}$$

The Reynolds number becomes 1.2 into 10 to the power 5. Now, Reynolds number 1.2 into 10 to the power 5 is somewhere over here this line and we must go all the way to 0.1 where it is unchanged. Since we are at the beginning of the fully rough zone the value of f does not vary much even though the Reynolds number has reduced by almost an order of magnitude.

Corresponding Re number, $Re = \frac{10^3 \times 1.58 \times 0.08}{10^{-3}}$

$$Re = 1.264 \times 10^5$$

$\frac{\epsilon}{D} = 0.01$ → f = 0.038 (unchanged since the flow is fully turbulent)

$$h_l = 0.038 \times \frac{(1.58)^2}{2 \times 9.81} \times \frac{30}{0.08} = 1.81 \text{ m}$$

So, the Reynolds number changes, but due to the nature due to the fully rough zone nature of this the Reynolds number is going to the value of the friction factor will remain unchanged at

0.0 through 8 and the reason for that is the flow is fully turbulent. So, this line is horizontal. So, no matter whatever be the value of the value of this thing the Reynolds number over here the value of f does not change. So, if the value of f does not change it is the value is the same as before.

Total head developed by the pump is now,

$$h_s = \frac{v_j^2}{2g} + h_l = \frac{(16.17)^2}{2 \times 9.81} + 1.81 = 15.13 \text{ m}$$

So, now with this I should be able to calculate what is going to be my h L. So, the h L in this case the it is going to be the same as before only the v is going to be less. So, the velocity is going to be 1.58 square and the rest remains unchanged the length the diameter etcetera this is 1.81 meter. So, if you compare that with the head which we have calculated previously let us see what was the value of that the value of that was 189 one almost 190 meters. So, compared to 190 meters what we have here is just about 2 meters. So, an order of magnitude difference can be obtained when you 2 orders of magnitude can be obtained when you reduce when you have a proper nozzle fitted to the hose and therefore, the pump power requirement would be reduced significantly. So, how much would the pump power requirement decrease? So, this head loss is 1.81, but remember that the velocity here is still going to be 16.17 for it to reach the top. So, even though the for calculating the head loss in the hose I am using the reduced velocity. In terms of velocity head development, I am using the velocity out of the out of the nozzle in this case and the total head that is to be developed by the pump simply turns out to be 15.13 meters. So, the pumping head requirement can be cut down considerably and the head requirement indicated the second person second fire person has studied her fluid mechanics the Bernoulli's equation properly and she could give the right suggestion to her to her colleague about the utility of choosing a smaller diameter nozzle as compared to that of the hose. And finally, one can the other person he suggested that why not use the hose diameter itself let us reduce the hose diameter to 2.5 centimetres like that of the nozzle. So, what is going to be the harm in that my it will still deliver at the top and since you are suggesting that the nozzle diameter to be reduced the nozzle diameter is 2.5 centimetres why not make the hose diameter 2.5 centimetres as well. Let us do some more calculation and see if that is justified. So, the hose velocity in that case is going to be the same as the jet velocity whatever be the velocity at this point is going to be the velocity over here.

So, my Reynolds number is going to be going to going to increase the Reynolds number going to change the Reynolds number is going to be 4 into 10 to the power 5 my epsilon by D remains same. So, sorry my new value of epsilon by D becomes 0.032 because my diameter has changed right now.

If the hose diameter itself is reduced to 2.5 cm;
the hose velocity will be same as jet velocity ,

$$Re_D = 4.04 \times 10^5, \frac{\epsilon}{D} = 0.032 \rightarrow f = 0.059$$

$$\rightarrow h_l = 943.53 \text{ m}$$

Bernoulli equation has to be applied between the pump inlet and the nozzle exit, not between pump inlet and outlet as at the pump outlet the pressure is unknown

So, the diameter is 2.5 centimetres. So, value of epsilon by D is 0.032. So, it is going to be somewhere over here. Now, if you go from Reynolds number which has reduced 4 into 10 to the power 5. So, Reynolds number 4 into 10 to the power 5 and you go up to the point where it is 0.03 where it is 0.03 you come to this point around this point and if you go to your left, you could see that the value of F is 0.059. Once again with Reynolds number 4 into 10 to the power 5 from here all the way up to the point where it is about 0.3 point sorry 0.03 the value of the value of epsilon by D you go to the left and you would see the value is close to 0.059. Now, you recalculate what is the value of h h L in that case same as before twice the F L by twice F L by D v square by 2 g the value of F has increased considerably as compared to the previous case. So, the value of the loss is huge now it is not 190 as was as we have obtained before it is not 1.5 which is when we have a reducer as a when you have a nozzle with a reduced diameter, but when the nozzle diameter is the same as the when the host diameter is the same as the nozzle diameter of the second problem the head requirement becomes very very large. So, but this is this is something which we must keep in mind. Now, which velocity is to be used where and that is Bernoulli's equation is applied is to be applied between the pump inlet from here and the nozzle exit not between the pump inlet and outlet as the pump outlet the pressure over here is unknown.

So, whatever we are doing we are including the pump in mice in our circuit to find out what is going to be the head to be developed by pump and the pressure at this point is not known to us pressure at the pressure at pressure outlet the pressure at the pump outlet the pressure is unknown. Now, this I thought is a nice problem to end our discussion our extended discussion problem solve problem solving on Bernoulli's equation. So, just a quick recap in 2 minutes what we have done in Bernoulli's equation. We know that Bernoulli's equation tells us for an ideal fluid the sum of pressure heads the velocity head and the gravity head would be a constant, but there are several restrictions to the use of Bernoulli's equation. The major one that we this is for an ideal flow ideal fluid in which there are no pressure drops due to friction.

In the second major assumption is that we take we have to we have to we are taking that the flow to be uniform at any cross section. For the second assumption that can be achieved if the flow is highly turbulent. If the flow is turbulent then it can be assumed that the flow is more or less more or less uniform at any cross section. If it is not in order to take into account the non-uniformity of flow at any cross section the concept of kinetic energy correction factor was introduced. And we have seen that the kinetic energy correction factor for the case of purely laminar flow turns out to be 2 whereas, for turbulent flow its value approaches that of 1 that is the first thing.

Second is we are also we have we have also added terms which consider the losses the frictional losses for flow in the pipe which we call as the major loss which can be obtained for laminar flow from the knowledge that F equals 64 by Re and the head loss is going to be F L by D and v square by 2. If it is turbulent flow then this value of F is to be obtained from Moody diagram. And I have shown you again and again how to calculate the friction factor from the Moody diagram knowing the value of Reynolds number and knowing the value of what is epsilon by D which is the roughness of the pipe E being the roughness in absolute scale and D being the diameter through which the flow takes place. All the other minor losses flow through a bend, flow through a valve they can be either or flow from large diameter tube to a small diameter tube all these can be obtained either by defining a coefficient contraction coefficient expansion coefficient at or by defining an equivalent length. So, together we have an idea right now what those losses consist of and how to evaluate those losses.

Once we incorporate the losses on the right-hand side and any pump which adds more head to the flowing fluid then the head at one the sum of all heads pressure velocity and gravity head at one plus any additional head due to the pump which has been provided must be equal to the sum of all three heads at location 2 and the losses both major and minor taking place in the flow from point 1 to point 2. So, head at 1 plus pump head must be equal to head at 2 plus all losses. And the different types of problems where the delta p pressure drop could be a function of L length Q flow rate and D diameter and the various combinations when 3 or any 3 are known and 1 is unknown how to do that and the pump problems I have explained in detail in this part and I hope that you are you are now familiar with Bernoulli's equation and would be able to solve any problem that comes your way as far as the circuit flow through a circuit is concerned. That is that concludes our discussion on Bernoulli's equation. Thank you.