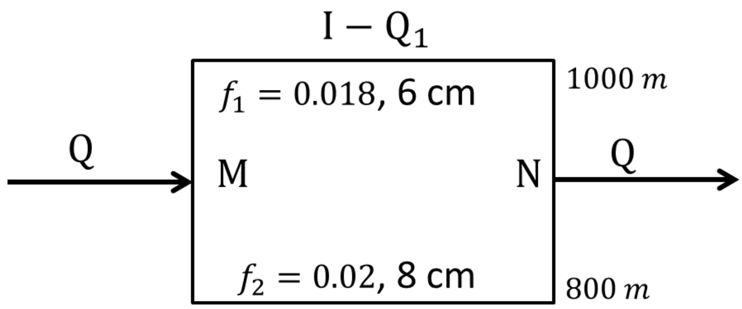
Momentum Transfer in Fluids
Prof. Sunando DasGupta
Department of Chemical Engineering
IIT Kharagpur
Week-10
Lecture-50

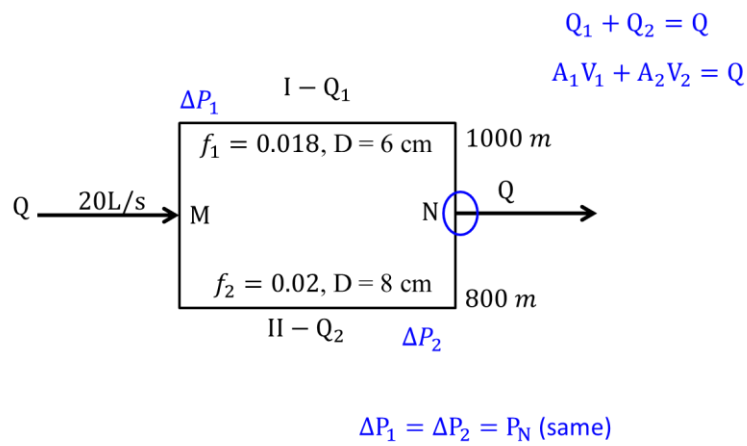
Good morning. We are going to continue the final two lectures on the application of Bernoulli's equation. We have seen how Bernoulli's equation can be used to calculate what is the pressure drop in a pipeline. And we know that the pressure losses due to the viscous nature of the fluid can be divided into two categories. One is the major loss which is the loss due to the flow in a straight pipe. And there is another category of losses collectively known as the minor losses in which the flow through the bends, flow through flow measuring devices for example, venturi meter, orifice meter or flow controlling devices like valves etcetera those are termed as the minor losses.

And the minor losses can either be expressed in terms of a loss coefficient or in terms of defining an equivalent length for the for the let us say for the valve or for the bend such that the pressure drop in a straight pipe is of certain length is going to be equal to the pressure drop through the valve or through that bend. Table that lists the corresponding values of equivalent length are that is available in the literature. And the friction factor for turbulent flow in such cases can be obtained from the Modi diagram, where the friction factor is going to be in the y axis, the Reynolds number is going to be in the x axis and the roughness of the pipe expressed in terms of epsilon, where epsilon is the numerical value of the roughness and D is the diameter, those lines are going to be going to give us the exact value of the friction factor. And once we know the friction factor the delta p the pressure drops the head loss is simply going to be f L by D v square by 2, where v is the velocity through the pipe.

And we have solved several problems and we have seen that there could be 4 different types of problem, where the pressure drop is a function of q the flow rate L the length of the pipeline and D the diameter of the pipe. So, these 4 quantities 1 or 3 would be known, 1 would be unknown and we have discussed solved and explained in detail what are what are the methodologies to be adopted for such cases. In the remaining 2 classes I am going to show you certain examples, where your understanding additional understanding of the flow physics is needed to solve the problem. So, I am not going to categorize them as type 1, type 2, type 3 problem, but I would all only like to point out the places where your input your additional input would be necessary to solve the problem. So, the first problem that we are going to solve in this as you can see that we have a pipe of 6 centimetre in diameter which is 1000 meter long and with the friction coefficient the value of which is provided to be 0.018 is connected in parallel between 2 points m and n with another pipe. So, this is the first pipe which is 1000 meter long and with a friction factor of 0.018. I have another pipe in here whose diameter is 8 centimetre and whose friction factor is known to be 0.02. And over here the total flow rate Q is 20 Liter per second and then it gets going to get divided into 2 streams, one going through 1 and the other going through pipe 2 and they rejoin at point n once again and Q starts flowing out of the piping system. So, this is a piping system in which 2 pipes are connected in parallel whose diameters are different whose friction factors are also different. We would find we would have to find out the division of flow in these 2 pipes. Essentially, we would like to know what is the value of Q1 and what is the value of Q2. So, this is to be evaluated in this problem.



So, in this example one must realize that the equation of continuity can first be used. The equation of continuity essentially tells us that the sum of the flow rates in the 2 branches Q1 and Q2 must be equal to the overall flow rate which is 20 Liter per second. And if you divide Q1 and Q2 express Q1 and Q2 in terms of velocity and area. So, this is going to be A1 v1 plus A2 v2 is the total flow rate. So, this is nothing but the equation of continuity.



If we look at the circuit that I have drawn over here carefully, there is going to be some pressure drop delta p1 when the flow is taking through taking place through the top pipe that means, the pipe of having diameter 6 centimetre. And let us say delta p2 is the pressure drop in between m and n when we consider the flow the that part of the flow which is flowing through that 8-centimetre diameter pipe with a friction factor of 0.02. So, we have one condition in which the sum of the flow rates would be equal to the total flow rate the initial flow rate which is 20 Liter per second. Now, can we say something in terms of the pressure drop between in the 2 paths of fluid that is through 1 and through 2.

Now, when you think about it what is going to happen at point N? At the point n since the flow is coming from this side also from this side getting combined at this point and then starts to flow in this direction. So, unless and until the pressure drop through the top portion and the pressure drop through the bottom portion are equal you are you are not going to have a situation in which flow from the bottom and flow from the top combine at this point and start flowing towards this. So, the difference in pressure between m and n when the flow is through the top and the difference in pressure between m and n when the flow is flowing flow is flow is flowing when the flow is taking place through the diameter 8-centimetre diameter pipe at this junction point they must be equal. So, that is the missing condition which we need to use from our understanding of the flow situation. So, essentially what I am saying is that delta p1 and delta p2 that means, delta p1 is the flow rate through this path, delta p2 is the flow rate through the bottom path they must be equal.

So, with these two conditions that A1 v1 plus A2 v2 is equal to q which is 20 Liter per second and delta p1 and delta p2 is same we should be able to solve what is q1 and what is q2. Now, it has been mentioned that there are no minor losses the minor losses can be neglected. So, whatever pressure losses that we have in this system is due to flow the viscous resistance to the flow for when it flows through the straight pipeline. So, if this is if this is this is the case then the major losses between n and m would be F1 where F1 is the friction factor for pipe 1, L1 is the length of pipe 1, v1 is the velocity through the pipe 1 and d1 is the diameter of pipe 1. Similarly, all these quantities on the right-hand side are for pipe 2 and the other condition is A1 v1 plus A2 v2 is equal to q.

Everything here is known except v1 and v2. So, there are two equations and two unknowns you would be able to solve what is what is v1 and v2 and once you have v1 and v2 q1 and q2 are simply going to be area corresponding area multiplied by v1 or v2. So, A1 times v1 would give you q1 and A2 times v2 would give you q2. So, a simple understanding that the pressure drops when in pressure drop in the two parts of the fluid when the rejoins must be the same. So, that condition is used to solve for this problem.

$$\Delta P_1 = \Delta P_2$$

Major losses between M & N

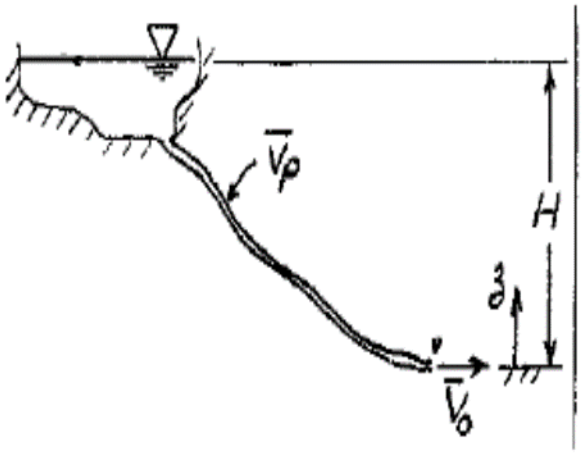
$$\Rightarrow \frac{f_1 L_1 V_1^2}{2D_1} = \frac{f_2 L_2 V_2^2}{2D_2} \qquad A_1V_1 + A_2V_2 = Q$$

$$Q_1 = 0.0063\ \frac{m^3}{s}$$

$$Q_2 = 0.00137\ \frac{m^3}{s}$$

I hope that there I have explained it clearly and I am moving on to the next problem for this part of the class. This is a case of this is a common case of hydraulic mining where a jet of water is directed towards loose rocks to dislodge them and then to mine whatever there is in the rock. So, to break relatively loose stones, rocks the hydraulic mining that means, using a high-speed jet is used to break the rocks into smaller portions or to dislodge a rock from a larger mass. So, we have a reservoir here over here and then the water is drawn through a pipe and through a nozzle the velocity out of the nozzle is v0 and it is going to strike it is going to strike the rock face which is over here. The datum starts from the rock face.

So, z equals 0 is near the rock face and the height of the reservoir is h. So, vp is the velocity through the pipe then it comes out of a nozzle with an elevated velocity and it strikes the rock. The pipe is 900 meter long and it is from the bottom of a reservoir and the reservoir h height the height above the rock is 300 meters. The pipe has a having diameter of 75 millimetre and an epsilon by D. So, the roughness ratio is 0.01. There are many couplings in the path of this from the bottom of the reservoir to the nozzle. So, every 10 meters along the pipe there are couplings which connects one pipe to the next pipe and for each of these couplings they are going to provide additional minor losses and the equivalent length of that coupling in terms of Le by D is equal to 20. So, there are many such couplings in the flow path with Le by D. The nozzle is a nozzle is going to have the diameter the nozzle over here is going to have the diameter 25 millimetre with k equals 0.02 and this k is essentially the pressure law pressure this k considers the minor loss when from the larger pipe having velocity vp it enters a smaller nozzle which whose velocity is v naught the velocity out of the nozzle is v naught.



So, from a larger pipe you have a small nozzle for this entry from the larger pipe the pressure drops the friction the pressure drop over here is expressed in terms of the loss coefficient 0.02 as we have seen before. The water the water is directed to the vertical rock face. So, whatever the water comes out it is going to strike the vertical rock face and we can assume as what the step length of the problem that the flow is in the fully rough zone. That means, if you recall moody diagram there are lines which are parallel to each other in a region in a region of high Reynolds number.

So, it is which is called the fully rough zone. So, in that region once again thinking about moody diagram f does not vary with Reynolds number anymore, f varies with different values of epsilon by D, but f does not vary with Reynolds number. So, that region where the flow is highly turbulent it is called the fully it is called the fully rough zone and there the value of f is invariant with Reynolds number. The kinematic viscosity is provided kinematic viscosity once again is mu the viscosity divided by rho and what we must find is what is the maximum outlet velocity v naught that is this velocity and the force exerted by the jet on the face of the rock. So, what is the force exerted by the water jet on the rock? So, we will solve this problem in as a last problem in this class.

So, what we start with are essentially writing down the quantities that are known to us that means, what is the height, what is the length of the pipe and for steady incompressible pipe flow I use the modified Bernoulli's equation. So, this is the sum of all three heads on the left-hand side should be equal to the sum of all three heads on the right-hand side plus h l t where h l is the head loss t refers to the total. So, it is the total head loss. Therefore, the difference between the head at 1 and the head of head at 2 must be equal to the head loss total head loss in the piping system. Note that there are no pumps it is pumps are moving machinery in this circuit.

H = 300 m, L = 900 m

For steady, incompressible pipe flow

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{v_1}^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{v_2}^2}{2}+gz_2\right)=h_{lT}$$

So, the entire flow over here at a high speed is being is being provided by converting the hydrostatic head into velocity head, but there will be losses while the fluid flows through the pipe. So, not all the hydrostatic head gets converted into the kinetic head. So, we have to think in terms of what is going to be the losses that the flowing fluid encounters when it moves through pipe with number of couplings the entry loss may be over here, where from a large body of a large area it enters into a smaller area and what is the what is the loss over here when from the pipe it goes to the nozzle from a larger diameter pipe to a smaller diameter nozzle what is going to be the pressure loss over there. So, all those frictional losses are to be considered while solving the problem. So, let us start with first identifying what we need to what is what are the things that we can we can neglect over here.

Assume, $p_1=0;\ \overline{v_1}=0;\ p_2=0;\ \alpha_2=1;\ z_2=0$; Fully rough zone

$$gH=h_{lT}+\frac{\overline{v_2}^2}{2}=f\frac{L}{D}\frac{\overline{V_p}^2}{2}+f\ \times 90\frac{L_e}{D}\frac{\overline{V_p}^2}{2}+K\frac{\overline{V_0}^2}{2}+\frac{\overline{V_0}^2}{2}$$

From continuity, $A_p\overline{v_p}=A_0\overline{v_0}$;

$$\overline{V_p}=\overline{V_0}\frac{A_0}{A_p}=\overline{V_0}\frac{A_0}{A_p}\quad\rightarrow\overline{V_p}^2=\overline{V_0}^2\left(\frac{d}{D}\right)^4$$

So, of course, P 1 is going to be 0 it is open to it is open to atmosphere my point 1 is right over here or at the reservoir level point 2 is at the exit from the nozzle. So, when we think about P 1 over here, that means, the pressure above the reservoir is atmospheric therefore, the gauge pressure is equal to 0. So, my P 1 is 0 and since it is a large reservoir the fall of the reservoir level because of us drawing the water from there is not significant that is why v 1 is equal to 0. When you think about point 2 which is the exit over here this is open to atmosphere. So, since this is open to atmosphere the gauge pressure here will also be 0 and we are it is in the fully rough zone.

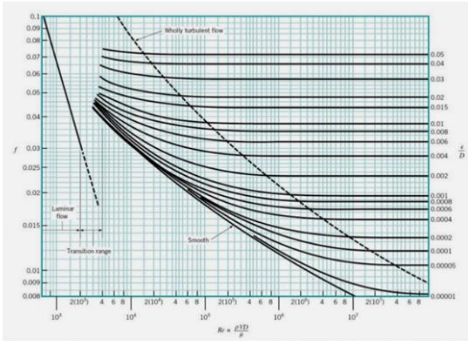
So, therefore, alpha 2 can safely alpha 2 corresponds to v 2 where v 2 is the velocity through the pipe. So, therefore, alpha 2 will be equal to 1 since this is highly turbulent and we know that the kinetic energy correction factor for highly turbulent flow approaches the value equal to 1 and z 2 the datum over here is 0. So, these are the numerical values these are the conditions which we can write based on this statement of the problem. So, if we do that the g h essentially this part this is 0, this is 0, this is 0, v 2 is unknown and z 2 is 0. So, g z 1 which is g times h would be equal to whatever be the total head loss from over here plus v 2 square by 2 where this is the v 2 square by 2 which appears over here.

Now, we are going to expand this h l t into its constituent major and minor losses. First the major loss f L by D v p square by 2 where v p this v p stands for whatever be the velocity of the fluid through the pipe not this v 0 this v 0 is the is the velocity out of the nozzle v p is the velocity through the pipe because of the dissimilar area of the pipe and that of the nozzle v p and v naught will be different. However, they are going to be connected by continuity equation as A 1 v 1 is equal to A 2 v 2 or A p v p A p being the cross-sectional area of the pipe v p being the velocity should be equals A naught v naught. This is the number of couplings that we have in this L E by D that is the equivalent length which has been provided for each of these couplings and the same value of f can be used to find out what is the loss minor loss due to the presence of the couplings in the flow path. This is what is the minor loss and over here this is going to be the other end this is the head loss on the other side.

$$\overline{V_0} = \left(\frac{2gH}{f\left(\frac{L}{D} + 90\frac{L_e}{D}\right)\left(\frac{d}{D}\right)^4 + 1 + K} \right)^{1/2}$$

In fully rough zone, $\frac{e}{D} = 0.01,\ f = 0.038$

$$\rightarrow \overline{V_0} = 28.0\ m/s$$



$$Re = \frac{\overline{V_p} D}{\gamma} = 2.60 \times 10^4$$

$$\frac{e}{D} = 0.01,\ f = 0.040$$

$$\text{New estimate, } \overline{V_0} = \sqrt{\frac{0.038}{0.040}} \times 28.0 = 27.3\ m/s$$

So, from continuity we can write that A p v p that is must be equal to A naught v naught and once we do that my v naught is simply going to be this expression where v naught is going to be this and to the power half. So, what is left right now is what is the value of the f. So, in the fully rough zone since the value of epsilon by D is provided as 0.001. So, my epsilon by D is 0.001 which is over here if we go all the way to this side the value of 0.01. So, 0.01 is over here if we go all the way from here to the left the value of friction factor is going to be equal to 0.038 that is the value of friction factor which you get over here.

Once you have the friction factor known over here everything else is known in this case. So, you would be able to obtain what is the value of v naught which would be 28 meter per second. Now, we must do a check of by using this value of v naught what is the value of the Reynolds number and from the value of the Reynolds number we need to recalculate f and see how close the new value of f is to my assumed value of f for considering it is fully is fully rough zone. So, the new value of Reynolds number is 2.6 into 10 to the power 4. So, it is 2.6 into 10 to the power 4. So, it is going to be somewhere over here and if you go all the way to 0.01 right over here you could see that the corresponding value of f is about 0.04. So, the value of f is about 0.04 for an epsilon by D equals 0.01. And these two are sufficiently close 0.04 and 0.038. So, I need not do another iteration for this case. So, the new estimate for v naught is simply going to be 27.3 meter per second. Now, once you know the value of v naught over here that means, you can also find out what is the what is the v p the velocity through the pipe utilizing the continuity equation like we have done before where v p A p v p is A naught v naught since I know my A naught now, I know my v naught A p is known to me. So, I would be able to calculate what is v p in this case.

So, this shows how can one find out what is going to be the what is going to be the velocity out of the nozzle that strikes the rock. Now, with this comes the next part of the problem. In the next part of the problem, we were asked that we need to figure out what is going to be the force exerted by this jet on the face of the rock. So, the problem right now is over here, a jet coming with v naught strikes a vertical rock surface and what is going to be the force exerted by the jet on the rock. So, now we go back to the momentum balance equation that we know from before.

Apply momentum balance,

$$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho \, d\forall + \int_{CS} u\,\rho\,\vec{V}.d\vec{A}$$

No pressure forces, $F_{Bx} = 0$, Steady flow

$$R_x = u_2\{-\rho\overline{V_0}A_0\} + u_3\{+\rho\overline{V_0}A_0\}$$

$$u_2 = \overline{V_0} \qquad u_3 = 0$$

$$R_x = -\rho\overline{V_0}^2 A_0$$

The force on the rock face, $K_x = -R_x = \rho\overline{V_0}^2 A_0 = 365\ N$ (to right)

So, and I am writing it for the x component F s x the surface force in the x direction F B x which is the body force in the x direction is del del T u where u is the x component of velocity and rho v rho v v this v is essentially the volume of the it is essentially the volume of the of the fluid which is hitting this surface and this is integrated over the control surface u rho u sorry rho v d A rho v d A gives us what is the mass flow rate that strikes the rock surface mass flow rate multiplied by u this is essentially the momentum with which the momentum which comes through the control surface to the rock. So, if I assume my control surface a vertical control surface over here then this is the efflux of momentum in the x direction. It simply hits the surface with a velocity u and with a flow rate which is rho v d A. We understand that this is in this case there is no body force in the x direction nothing no body force in the x direction the body force is only in the y direction minus y direction which is due to gravity. So, F B x is 0 and it is a steady flow if it is a steady flow then nothing is a function of time anymore therefore, the first term on the right-hand side will be cancelled.

So, what we have essentially is the is the is the force on the control volume to be equal to the net rate of momentum efflux through the through the control through the control surfaces. So, my statement therefore, it becomes the r x which is the force must be equal to u 2 times whatever be the mass that comes in since the mass that comes in it is going to be negative. So, if I draw the if I draw this let me draw this. So, this is my control surface of the rock it strikes with certain velocity let us call it as u 2 and it goes out in these directions with velocity equals to u 3. So, obviously, you can see that over here some mass and momentum is coming into the control volume.

Since the momentum since the mass is coming into the control volume that is why we have this that is why we have this negative we have this negative sign and over here the mass is leaving the control volume. So, therefore, that is why it is going to be plus we must use plus. We also understand that u 2 is simply v naught this is the velocity with which it strikes the rock surface u 3 is the x component in this x component of the velocity which is leaving the control volume. The geometry tells us that u 3 will be 0 there is not going to be any x component of velocity in this direction. So, u 3 is equal to 0 u 3 refers to the x component of the x component of the velocity and of course, u 3 in that case would be 0.

So, when you plug in these values what you are going to get the r x would be minus rho v naught square u 2 is v naught v naught square times a naught everything here is known. So, since everything here is known the force on the rock face this is the force on the control volume. Remember that anything that we get out of this equation is force on the control volume. So, force the force because of the jet on the rock face would simply be equals minus of r x.

So, this is the force on the control volume. So, force on the rock face is minus of this. So, it is going to be rho v naught square a naught and this is 365 Newton and its direction is to the right it is hitting the rock face in this direction. So, the force on the rock face is in the plus x direction. So, this is a nice problem clear cut problem in which you have to consider the Bernoulli's equation on the one hand taking into account the major and the minor losses finding out what is the velocity out of the nozzle and in the second part of the problem you would see that what is going to be the force exerted by that jet on a vertical rock face that you that the calculation procedure is shown over here. So, that ends the class about this class about Bernoulli application of Bernoulli's equation.

There will be one more example problem which I would solve before we close this chapter on Bernoulli's equation in the next class. Thank you.