Momentum Transfer in Fluids
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Week-01
Lecture-05

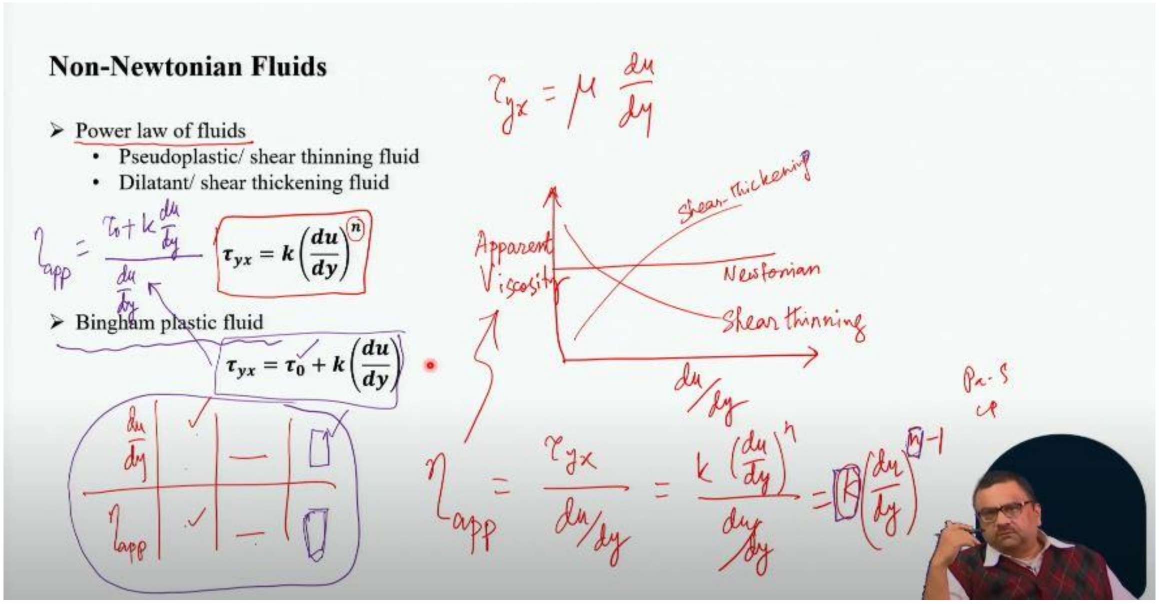
Welcome to this course of Momentum Transfer in Fluids. We were discussing some elementary frameworks. In particular, we were into the introduction of fluids that do not exactly follow the Newtonian law of viscosity. We would call them non-Newtonian fluids. So, we are going to talk about that. So, we said Newtonian fluids, said that $\tau_{yx} \propto \frac{du}{dy}$.

So, instead of that, we would be talking about non-Newtonian fluids, which are of power law type, power law of fluids. Power law is, Newtonian fluid said it is $\tau_{yx} \propto \frac{du}{dy}$. Instead of that, we are having here this, $\tau_{yx} = \mu \left(\frac{du}{dy}\right)^n$. This makes perfect sense because we said that if τ is a function of $\frac{du}{dy}$, and we said that if Newtonian law of viscosity states that we have, straight line passing through origin.

Then, if we are trying to state whether the fluid is not following a straight line but is still passing through the origin, then the power law type will tell you whether it belongs to this category or that category. So, it depends on what is the value of this exponent. This n is the exponent here. So, depending on whether n is greater than 1 or n less than 1, the shape of a certain shape of the curve comes in. What we are interested in here is instead of τ, $\frac{du}{dy}$, we tend to plot something called viscosity as a function of velocity gradient $\frac{du}{dy}$.

Though we are writing velocity gradient $\frac{du}{dy}$, it is same as the angular deformation rate. We have already established that in the previous lecture. So, viscosity as a function of $\frac{du}{dy}$, we would be expecting that Newtonian fluid then we have a viscosity which is constant. If viscosity is not going to vary with $\frac{du}{dy}$ as is given by this straight line. The slope of the line is constant, slope is not varying with $\frac{du}{dy}$ whereas, the other fluids the slope will change with $\frac{du}{dy}$.

So, you would expect that when it comes to these power law fluids, these types of fluids are referred to as power law fluids. So, then, in this case, the viscosity could be either coming down like this or going up like this. So, one is known as the shear thinning. Shear thinning means as you increase the shear, the fluid gets thin, which means its viscosity decreases. That is this class. This is known as shear thinning, but as you are imposing a higher velocity gradient, that means higher shear, then the fluid appears thinner, which means its viscosity decreases.



So, this is known as shear thinning fluid. Scientific community call it pseudoplastic; people in the industry they will call it shear thinning fluid. Whereas, some fluid with application of shear gets thickened that is referred as dilatant, and that is shear thickening fluid, and this is Newtonian because the viscosity remains constant. So, when it comes to the shear stress, I am given a sample, and we do not know what that is. So, the first thing we do is we will measure its viscosity.

Now, the fact is, what is viscosity when it comes to power law fluid? Because we have not talked about any μ here, you might have seen it is $k\left(\frac{du}{dy}\right)^{n}$. So, which one is viscosity? n is not, n is the exponent, k is it the viscosity we do not know. So, what is this viscosity when it comes to shear thickening or shear thinning fluid? So, that is why we write this as apparent viscosity and what is this apparent viscosity? This is typically written as η_{app}, or you can write $\boldsymbol{\mu}_{\text{app}}$ if you want I separated it as η. So, if we say that ok, we have no connection with $\boldsymbol{\mu}$, it is a completely different thing. η_{app} is essentially $\frac{\tau_{yx}}{(du/dy)}$.

So, that is what is η_{app}, we do not call it $\boldsymbol{\mu}$. Now, this for a Newtonian fluid η_{app} happened to be equal to viscosity. This is apparent viscosity, that means, from the viscometer reading that I get, that I am simply plotting, and we are calling it apparent viscosity. So, for example, if it is a power law fluid. So, then τ_{yx} would be $\frac{k\left(\frac{du}{dy}\right)^{n}}{(du/dy)}$.

So, that means, it would be equal to $k\left(\frac{du}{dy}\right)^{n-1}$. So, this is the apparent viscosity. So, this, you must understand that apparent viscosity, now when it comes to the viscometer. Suppose I am given a sample, and I am measuring its viscosity this is what I will measure from the viscometer. So, these will have a unit of Pa.s, these will have a unit of centipoise, etc.

So, that is the apparent viscosity I measure. So, suppose, I get a sample, and I measure apparent viscosity at different shear rates, and I get different values, then the question is what type of fluid is this? Then I will plot this. I will see whether apparent viscosity decreasing with shear rate or increasing with shear rate, then we will call it either shear thinning that is pseudo plastic or dilatant. Then what we do is we have to find out what is k and what is n. So, suppose I have η_{app} at two different shear rates. So, two different values of $\frac{du}{dy}$, unit of $\frac{du}{dy}$ just in the previous class previous lecture, we talked about that it is s^{-1}, right du is ms^{-1}, and dy is in m. So, the $\frac{du}{dy}$ unit would be s^{-1}.

So, I have the $\frac{du}{dy}$ values and I have the corresponding η_{app} values. Let us say η_{app} unit would be centipoise or Pa.s. So, let us say two different values I give you one value of $\frac{du}{dy}$ and against that I have some η_{app}, and I have another value of $\frac{du}{dy}$ I have another value of η_{app}. So, now, I have to put this η_{app} is equal to this in this equation and then I have to find out these two unknowns k and n these are the two unknowns these are the two unknowns one is k mind it another is the n. So, using these two sets of data I have two equations here and two unknowns k and n I have to find out what is k what is n and then I can project. Now I give you a third $\frac{du}{dy}$, which is some value here and then I want to know what is the corresponding η_{app}.

So, I give you a new $\frac{du}{dy}$, you have to first find out k and n, and apply those k and n to find out what is the corresponding η_{app}. So, this is this is how you can find out the apparent viscosity. So, this is apparent viscosity mind it if you are given this standalone problem you are given a sample and someone has given you the $\frac{du}{dy}$ value the velocity gradient in s^{-1}, and the apparent viscosity in centipoise let us say two sets of values, and you are asked to find out what is the new $\frac{du}{dy}$, what is the value of apparent viscosity. The first thing you have to check is whether there is a possibility of a Newtonian fluid. So, that you can check by first seeing whether η_{app} is varying with $\frac{du}{dy}$ or not.

If η_{app} is not varying then you know it is Newtonian. So, it will in a different $\frac{du}{dy}$, and η_{app} will remain same. If you see that the η_{app} has varied between these two sets of data then you know that the η_{app} is changing. So, η_{app} is changing, it may increase with $\frac{du}{dy}$ or decrease in $\frac{du}{dy}$, but that is reflected in the exponent *n*. So, that does not matter at this point, but then you set two equations two unknowns, *k* and *n*, find out *k* and *n*, and then extend it.

There is a possibility that even the fluid does not belong to this class, which is the third class that you have, that is, the third one is the Bingham plastic fluid. To implement this, you need first of all, there is a threshold shear stress, which is τ_o. Threshold shear stress one has to overcome to get to that movement. So, an example of Bingham plastic fluid is the toothpaste. You are putting some shear stress, but you are seeing that there is no velocity gradient.

So, once you cross a certain shear stress value, that means you squeeze it really hard and cross that τ_o, then only you will find that there is a velocity gradient setting in and the toothpaste starts moving and coming out from that container. But beyond this point here it is linear. So, this is a threshold τ_o, and beyond this, τ_o is again following a straight line, but not a straight line passing through (0, 0), passing through the origin. So, in that case, this Bingham plastic fluid is defined by this equation.

$$\tau_{yx} = \tau_0 + \mu\left(\frac{du}{dy}\right)$$

You can see there is this threshold stress τ_0, and then it is written as $k\left(\frac{du}{dy}\right)$ still we are not writing it as μ. So, suppose I have a case where, I have these two sets of data, and third set third value of $\frac{du}{dy}$ I have given and asked you to find out η_{app}. So, there is a possibility that the fluid that you are working with is Bingham plastic. So, that information has to be given to you. Suppose if this information is given that the fluid is Bingham plastic, tell me for a new $\frac{du}{dy}$ what should be the apparent viscosity.

So, in that case, what you do is here, in this case, what is the apparent viscosity or viscosity for that matter? This is not *k* again; apparent viscosity in this case would be $\eta_{app} = \frac{\tau o + k\left(\frac{du}{dy}\right)}{(du/dy)}$. So, that would be your η_{app}. So, the value that is given in centipoise is essentially this and not *k*. So, that you have to keep in mind. So, now, here again, I have two sets of data and two sets of unknowns. One is τ_0, another is *k*, and you have to find out that two τ_0 and *k*.

And then extend with those known values of τ_0 and *k* putting this new value of $\frac{du}{dy}$, you have to find out what is the apparent viscosity. So, this is how we need to proceed. So, the whole purpose of this exercise here is this, the apparent viscosity is plotted. So, the viscometer will give you the apparent viscosity as a function of $\frac{du}{dy}$. They have a simple knob which you can tweak and change the shear rate, change the $\frac{du}{dy}$, and you measure corresponding viscosities.

So, many centipoise, or so many Pa.s, and then you have to plot that as a function of $\frac{du}{dy}$, and from that point on, you will get an idea whether it is dilatant, whether it is pseudo plastic, or whether it is bingham plastic. So, then you have a full control, and now suppose you have studied at 2 or 3 or 5 different shear rates, and in the plant or in the production unit or in the actual field of application, the shear rate is different. So, now, you need to know, at that shear rate what would be the viscosity. So, you have to find out these values of k, n etc., and accordingly apply that. Then you can say that this would be the apparent viscosity. So, now, this can be plugged in to find out what would be a pressure drop, what would be forces, etc.

Examples

Newtonian fluid: Water, Glycerin, Motor oil, Diesel (At room temperature and pressure)

Pseudoplastic fluid (shear thinning): Ketchup, Toothpaste, Mayonnaise, Blood, Shampoo, Printing Ink

Dilatant (Shear thickening): Corn starch, Armour-gel, D3O, Artificial cartilage foam

Thixotropic fluid: Certain gels such as made from agar agar or gelatin, clay (drilling muds), Certain adhesives

Rheopectic fluids: certain type of paints

Viscoeleastic material: Rubber, Biological tissues like tendons, ligaments and cartilage, Polystyrene foams, Asphalt, Damping material

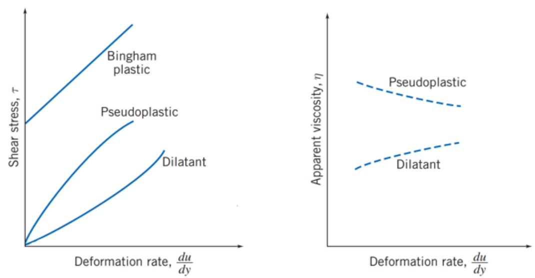
So, that is how we handle this non-Newtonian fluid. So, these are the fluids that are viscoelastic, but more towards viscous part and so, somehow doing with certain assumptions and this approach is somewhat, I would say empirical, but this approach works, this approach helps to desig the units that are workable, that are serving our purpose. So, that is why this treatment of applying power law or applying these equations works. Here are some examples of different fluids. You can see Newtonian fluids; they are written here as pseudoplastic and dilatant. The examples are given. Then there are two classes of fluids, one is thixotropic and rheopectic. These are more towards if they are not following Newtonian law in terms of time.

APPARENT VISCOSITY

Non-Newtonian fluids

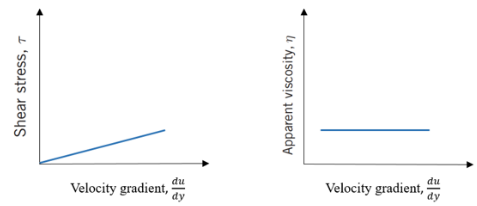
$$\tau_{yx} = k\left(\frac{du}{dy}\right)^n = k\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy}$$

$$\tau_{yx} = \eta\frac{du}{dy}$$



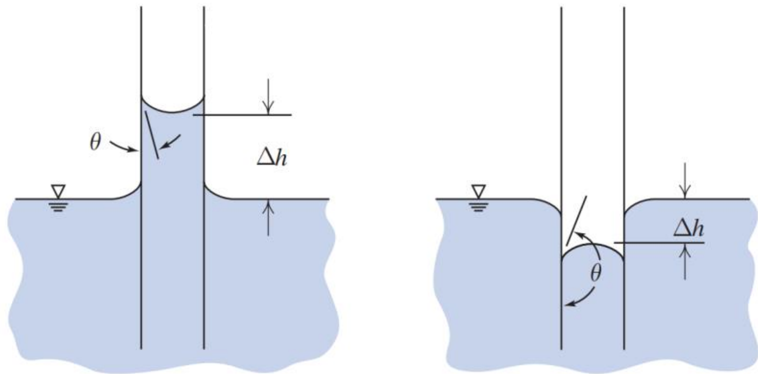
Newtonian fluids

$$\tau_{yx} = \mu\frac{du}{dy}$$



Say, for example, the shear deformation is given now, and after some time, the response comes, which is true with certain types of paints, etc. So, those are thixotropic, and rheopectic fluid, and then viscoelastic material altogether, where you have to bring in some elastic component. So, some examples are given. We have already talked about these apparent viscosity which is $k\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy}$, and then η is the apparent viscosity we have already talked about the pseudo plastic, and we have already talked about the shear stress as a function of velocity gradient and apparent viscosity as a function of velocity gradient. So, this is essentially the definition of apparent viscosity. We have already talked about. these.

So, I think we can proceed further. So, we have defined viscosity, and if some fluid is not following Newton's law of viscosity how to handle those. So, we have talked about that. The another area, as it comes we will talk about it, but at least we have a very quick recap on this surface tension. What surface tension does? In case of momentum transfer in fluids, surface tension will feature particularly if you dip a tube in a liquid volume, in a pool of liquid, and if the diameter of the tube is small. If the diameter of the tube is large, you do not have to bother that much, but if the diameter is small then surface tension forces are important.



(*a*) Capillary rise ($\theta < 90°$) (*b*) Capillary depression ($\theta > 90°$)

Here we have seen that if you dip a glass tube in water, and here, in this case, you are putting a glass tube inside mercury. So, here, when you dip it dip the tube in water, you will find that some amount of water rises through the tube, and the meniscus takes a shape like this. Whereas, when you dip it in mercury, you will see that there is a depression outside water outside mercury level is this much, but inside the tube the mercury level is lower, and the shape of the meniscus is in this direction. So, this has something to do with the contact angle, water wets glass that means, water makes a contact angle with glass air water. That contact angle here is less than 90^0. So, in fact, it is close to 0 degree, water wets glass whereas, in case of mercury, this is greater than 90^0.

So, how it plays, what you will see here is that this meniscus is being pulled in this direction, and the force by which to be pulled is given by σ, which is the surface tension, and the surface tension unit is N/m. You might have seen surface tension unit is N/m. So, force per unit length, and here the surface tension is acting over this entire circular line. So, this is, let us say that the radius of this tube is R or the diameter of the tube is D, then it would be σπD; because N/m into πD unit is meter. So, it is Newton, and on top of that, if you want to talk about the vertical component of this. So, if this angle is θ, then this angle will also be θ.

So, that means the vertical component of the force upward would be σπDcosθ. So, that would be the force by which liquid will be pulled upward, and then you will see of course, that this volume of liquid that is raised, there is a gravity force acting downward and that would be $\pi D^2/4$, that is the area multiplied by if the elevation is Δh. So, that would be the volume into ρ which is the density. So, this is the mass multiplied by acceleration due to gravity g. So, this becomes the gravity pulling downward, and this is the surface tension pulling upward, and what you do is you equate these two to find out what should be the Δh at which these two forces equilibrate.

So, this Δh is referred to as capillary rise. On the contrary, when it comes to mercury, the contact angle is greater than 90^0. So, you can see this is the contact angle being made. So, the force that is being pulled downward. So, this would be the vertical component that is pulling it downward, and so, you can find out in a similar way how much the depression would be.

So, in this case, now this capillary rise phenomenon in many cases you will find that this is quite important. For example, we will be discussing manometer. Though you have some preliminary background, but some special features of manometer, we will have a short discussion on manometer. You will find that if the diameter of the manometer tube is too small, then these capillary rise phenomena will come into play, and that will distort the height difference because you are measuring the height difference to find out what is the pressure difference between two points, but then if the capillary rise phenomena come in. So, there would be some amount of distortion one has to keep in mind.

So, this is one thing. Another thing is you must understand here that this capillary rise that we talked about is because of surface tension. This is a force by which, a lot of times, the fluid movements are organized. So, you can treat this as a pumping action. A lot of transport of fluids takes place by capillary action in many different applications. You do not have a pump; you simply have, for example, when this wick action, the oil flows through the wick, when you have a lamp burning. So, the oil flows through the wick because of capillary action.

There are many different applications where this capillary action, or the surface tension force provides the pumping forces to the fluid. So, this is an important concept altogether, and you must keep that in mind. Here, we have shown just the capillary rise, which means, at one point, it will stop moving. There will be an equilibration, but suppose I have a pipe, I have a tube which is held horizontally. So, then there would be a flow happening and then the progression of the front with respect to time, treating these as a pumping action. There is a good amount of leverage we can draw from these surface forces.

GENERAL CLASSIFICATION OF FLUID FLOW

Based on fluid dynamics
- Steady flow
- Unsteady flow

Based on fluid flow type
- Laminar flow
- Turbulent flow

Based on fluid physical property (viscosity)
- Newtonian fluid
- Non-Newtonian fluid

Based on fluid flow inside a confinement
- Internal flow/ Closed conduit flow
- External flow/ open channel flow

Based on compressibility
- Compressible flow
- Incompressible flow

Supercritical flow (Based on velocity of fluid)

Now, if we look at the classification of flow fluid flow, we have we have already gotten this steady flow and unsteady flow we have already defined. That means, v, the velocity vector is a function of x y and z and not a function of time, then it is a steady flow. Unsteady flow is when velocity is a function of time. Based on the fluid flow type that we will get into in a moment. Based on fluid physical property viscosity we have already discussed Newtonian fluid and non-Newtonian fluid. Fluid that is following Newton's law of viscosity, that is Newtonian fluid, and the fluids that are not obeying that it is not Newtonian fluid.

Based on compressibility, that means we have talked about density. Now, the density of the fluid can be constant, like the density of water, though water also has some very minuscule amount of compressibility, or it could be air, where density varies with pressure. So, you have a compressible flow when you are working with a compressible fluid. If it happens to be an ideal gas, you can write $\rho = PM/RT$. Pressure, molecular weight, universal gas constant, and temperature in absolute scale. So, you can have compressible flow or incompressible flow, flow of water like fluid or flow of some gases.

There could be laminar flow or turbulent flow depending on whether the flow is moving sliding one layer against the other. That means, I may quickly tell you that there is an experiment done by Reynolds in a tube where the flow is taking place, and then he introduced a color in the stream, and he found that the color travels like a straight line maybe a little bit of diffusion here a little bit of fuzziness here for that downstream. But then, as he increased the flow, he found that this line got wavy, and as he increased the flow rate even further, he found that there was a movement of the color like this. So, this he considered to be a flow of transition between laminar to turbulent. Laminar means one layer sliding against the other, the type of flow we have discussed when we talked about upper plate moving, and the lower plate is fixed, and all the intermediate layers they are one is sliding against the other.

Turbulent flow means eddies, they are having random motion. So, we can have laminar and turbulent flow that classification is there or we can have internal flow or external flow. That means, internal flow means flow through a pipe. External flow means flow through an open channel, maybe just over a flat plate. So, if it flows over an open channel over some over a substrate. So, that is external flow, whereas the channel on which the upper is not a closed one, or it could be a pipe where the flow is happening within that conduit.

So, then it is an internal flow. And of course, there is another class of fluid, which you need to remember. There is a class of fluid, which is referred to as supercritical fluid, where the pressure and temperature are above the critical values. So, these fluids, they have their unique advantages in the sense, its density and viscosity can be tuned to make sure that its viscosity is more like gas, whereas its density is more liquid like. So, it can move very easily through a porous medium, through the pores because its viscosity is gas like, viscosity is low, whereas, its density can be tuned density could be made more so that it can dissolve lot of substances in it. And since it is above the supercritical values, I mean PC and TC, those values are exceeded.

So, you do not expect an in liquid vapor interface. So, they provide some unique opportunities for this supercritical fluid, which does not follow our conventional understanding of liquid and gas. So, that can be leveraged. So, this is also one class of fluid that is important, though we are not going to discuss much, but just to make just to inform you that these are the various types of fluid flow that exist. I think this is all I have as far as this lecture module is concerned. So, from the next class, we will start working on a different type of fluid flow which we are going to refer as inviscid flow. That is all for this lecture. Thank you very much.