Momentum Transfer in Fluids
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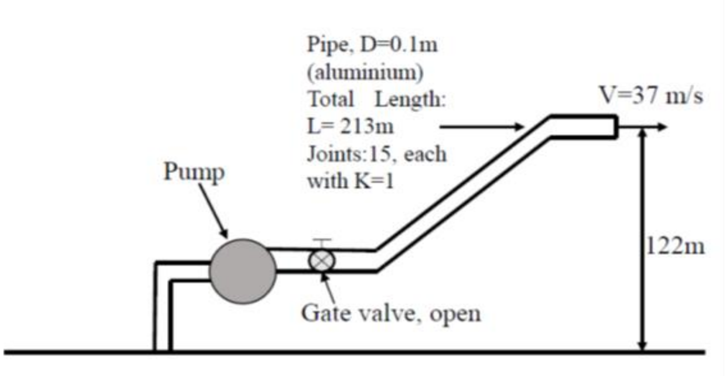
Week-10
Lecture-49

So, we are going to have a new type of problem. So, let us get into the problem quickly and what we have here is that you need to supply cooling water from a reservoir. So, this is the reservoir which is this is the reservoir from which the water is being drawn by a pump passes through a gate valve and then it goes all the way up to a height of 12.2 meters and then it gets discharged at a velocity of 37 meter per second. The pipe is 0.1 meter in diameter it is made of aluminium.



So, essentially it tells you that what is the value of epsilon the that you are that the that you must take the total length from here till here is 213 meters. There are several joints total 15 joints each with a which with a coefficient head loss coefficient equal to 1. So, not all these joints are shown in this, but there are joints after every 15 meter after some distance such that there is total 15 joints in the path. So, the pressure drops or the pressure the frictional pressure drop could be over here could be because of the gate valve and the joints which are present as well as the major loss frictional losses in the pipe.

The discharge velocity at the end of pipe is essentially you also have a nozzle that you have a nozzle at this such that to increase the discharge velocity at the end of pipe significantly from that in the pipe. So, there is some velocity with which the fluid is moving through this pipe, but right at the mouth there is going to be a small nozzle through which the velocity is going to be increased since the cross-sectional area is going to be less than that of the pipe. So, you are going to get a jet of water a high-speed jet of water at a 37 meter per second. 15 joints and the nozzle including the nozzle each with a loss coefficient k equal to 1. So, we while calculating the minor losses we must think about we have to consider the 15 joints including the nozzle with a k equal to with k value equal to 1.

What we must figure out is the minimum pressure needed at the pump outlet that is at this point. What is the minimum pressure needed at the outlet which could supply the liquid at this up to this point then through the nozzle with a velocity of 37 meter per second. Estimate the power input if the pump efficiency is 70 percent. So, we must once we know what is the minimum pressure required, we need to figure out what is the power required by the pump and the power required by the pump considering the efficiency is 70 percent. So, whatever be the requirement that must be divided by 0.7 to arrive at the actual pump power the actual pump the power to be to be to be the capable of supplying by the by the pump. The pressure at the reservoir and the outlet of the nozzle pressure over here and pressure over here are atmospheric. The value of the roughness of the pipe that means, the value of the epsilon since D is D is given the epsilon is 0.0015. So, you would be able to figure out what is epsilon by D, but keep in mind that this is in millimetre whereas, the diameter is in meters.

So, you must make sure that the units are the same. The value of the k entry at this point is 0.78 L e by D values for the gate valve this point is 8 for the 90-degree bend that means, over here L e L e by D value is equal to 30 and you also have a 45-degree bend. So, this 45-degree bend is 16. You can see intuitively that a 90-degree bend would have a higher equivalent length because the pressure drop is going to be more for 90-degree bend as compared to a 45-degree bend where it is more gradual.

So, the L e by D is 30 for 90 and 16 for 45 degrees. The water density is 1000 kg per meter cube and kinematic viscosity mu by rho is 1.17 into 10 to the power minus 6-meter square per second. So, essentially, I must calculate the minimum pressure needed at the pump and the required power input if the pump efficiency is 70 percent. So, that is essentially the problem.

We are having significant minor losses due to the gate valve, due to the all these joints 15 joints which are there in the flow path, the entry of water at this point and so on. And in addition, we have a pump over here that we must that we have to consider and we need to first the what is going to be the pressure at this point that we must figure out. So, let us start the solution of the problem. The Q is provided Q is given the efficiency is 0.7 the kinematic viscosity of water is known to us.

The calculating the losses the value the whenever you are going to calculate the losses the major and the minor losses for flow in the pipe, the velocity here in the pipe is to be considered not the exit velocity. Because if you remember from the statement of the problem the water in the pipe passes through a nozzle with which constricts the area and enhances the velocity. So, this velocity that you see here is not the velocity of water flowing in the pipe, but while calculating the losses the velocity in the pipe is needed not velocity through the nozzle. So, that is the first caution we need to employ to get the correct values. So, how do I find this? We need to use equation of continuity over here and here to figure out what is the velocity that we need to use.

$$Q = 0.378\ \frac{\mathrm{m}^3}{\mathrm{s}}\ , \eta_{\mathrm{pump}} = 0.7, \vartheta_{\mathrm{water}} = 1.7 * 10^{-6}\ \frac{\mathrm{m}^2}{\mathrm{s}}$$

And we are going to write the Bernoulli's equation between this point and between this point. So, the reservoir and the nozzle outlet. So, what I have here is then the sum of all heads at 1 with velocity v 1, sum of all heads at 2 and the delta h the head to be provided by the pump must be equal to all the losses. So, sum of all heads plus head provided by the pump is equal to the losses plus whatever be the head at whatever be the sum of all heads at 2. Now, once again this v 2 the velocity is going to be through the pipe and this v 2 is velocity through this pipe whereas, this v 1 is the velocity with which the reservoir level is falling.

And we understand that for a large reservoir when you suck the liquid out of this the level falls at a very slow rate as compared to the velocity through the pipe. So, v 1 essentially is going to be 0. At the same time both p 1 and p 2 they could also be 0 we will see if that is true because they are open to atmosphere. The value of z 1 and z 2 the difference is already given to us. So, it is this distance is given to us and we know that the h L and h L m the major loss contains f L by d v square by 2 and minor loss contains all the summation of all the contraction or expansion factors and f L e by d v square by 2.

$$\left(\frac{P_1}{\rho}+\alpha\frac{v_1^2}{2}+gz_1\right)-\left(\frac{P_2}{\rho}+\alpha\frac{v_2^2}{2}+gz_2\right)+\Delta h_{pump}=h_{LT}$$

Velocity through the pipe

$$h_{LT}=h_L+h_{LM}$$

$$h_L=f\frac{L}{D}\frac{v^2}{2}\qquad h_{LM}=\frac{v^2}{2}[(\sum K)+\sum f\left(\frac{L_e}{D}\right)]$$

Velocity through the pipe

So, what is f L e by 2 where does this come from? They are for the 90-degree bend and for the 45-degree bend and for the gate valve. The gate valve will also have an L e by d value. These three are the minor losses expressed in terms of the equivalent length. Whereas, this k is for the joints which are provided 15 or so joints which are provided in the flow path the value of the k is provided. But once again I note here that these velocities are velocities to the velocities to through the pipe.

$$V_1=0\,,\alpha_1=\alpha_2=1,P_1=P_2=P_{atm}$$

$$\Delta h_p=gz_2+\frac{v_2^2}{2}+f\frac{L}{D}\frac{v^2}{2}$$
$$+\frac{v^2}{2}\left[K_{entry}+ADD\ GATE\ VALVE+f\left(\frac{L_e}{D}\right)_{90}+2f\left(\frac{L_e}{D}\right)_{45^0}+15K\right]$$

So, that is to be kept in mind. Now, let us see once again the velocity over here is going to be equal to 0 alpha 1 alpha 2 all are going to be equal to 1 p 1 p 2 the pressures at both points are going to be equal to the atmospheric pressure. So, the delta h is going to be the sum of heads over here the gravity head and the velocity head the value of alpha being equal to 1 the loss major loss due to the straight length of the pipe and the k entry the gate valve the L e by f L e by d 90 degree twice f L e by d 45 degree because we have one 45 degree bend over here another over here plus 15 k 15 number of the joints which are added in this case. So, this tells gives me the complete picture of the delta h which is which is present which you would encounter between point 1 and point 2. Now, the velocity the velocity is the flow rate is given the diameter is known to us.

$$\mathrm{v} = \frac{\mathrm{Q}}{\mathrm{A}} = \frac{\mathrm{Q} * 4}{\pi \mathrm{D}^2} = \frac{0.378 \frac{\mathrm{m}^3}{\mathrm{s}} * 4}{\pi * 0.1^2 \, \mathrm{m}^2} = 48.1 \, \frac{\mathrm{m}}{\mathrm{s}}$$

$$\mathrm{Re} = \frac{\mathrm{Dv}}{\vartheta} = 4.11 * 10^6, \epsilon = 0.0015 \, \mathrm{mm},$$

$$\frac{\epsilon}{D} = \frac{0.0015 * 10^{-3}}{0.1} = 1.5 * 10^{-5}$$

So, this is what the velocity through the pipe line that is that is provided. So, if the velocity through the pipe is known to me then I would be able to find out what is the Reynolds number would which is d v by nu the kinematic viscosity and the value of epsilon is given in the problem as 0.0015 millimetre. So, epsilon by d when you do that make sure that they are in the same units. So, epsilon b by d is 1.5 into 10 to the power minus 5. So, knowing the velocity from the known value of q and the known value of a it is 48.1 and then you have this to be the epsilon by d value. From the Moody diagram f turns out to be 0.01 you know now you know how to read Moody diagram.

From Moody diagram,

$$\mathrm{f{=}0.01}: \; \mathrm{K_{entry}} = 0.78, \mathrm{K{=}1}, \left.\frac{\mathrm{L_e}}{\mathrm{D}}\right|_{gate} = 8, \left.\frac{\mathrm{L_e}}{\mathrm{D}}\right|_{90^0} = 30, \left.\frac{\mathrm{L_e}}{\mathrm{D}}\right|_{45^0} = 16$$

$$\Delta \mathrm{h_{pump}} = 9.8 \, \frac{\mathrm{m}}{s^2} * 122 \, m + 0.5 \, (37)^2 \frac{\mathrm{m}^2}{s^2} + 0.01 * \frac{213}{0.1} * \frac{48.1^2}{2}$$

$$+ \frac{48.1^2}{2} [0.78 + 0.01 * 30 + 2 * 0.01 * 16 + 15 * 1] \frac{\mathrm{m}^2}{s^2}$$

$$\Delta \mathrm{h_{pump}} = 1195.6 + 684.5 + 24634 + \frac{48.1^2}{2} (16.4) \frac{\mathrm{m}^2}{s^2} = 45485 \frac{\mathrm{m}^2}{s^2}$$

So, I am not doing that once again and what is the value of k entry that is provided the k entry is essentially where it the liquid is entering the pipe line from the reservoir the L e by d gate valve is 8 L e by d for the 90-degree bend is 30 degree 2 45-degree bends are provided in the pipe line each with L e by d to be equal to 16. And this k equal to 1 are for all the 15 joints which are which are there in the pipe line. So, this essentially would allow me to find the whatever be the total losses whatever be the total losses in the flow path. So, delta H pump what is delta H pump is the head provided by the pump is the first the gravity heads the velocity head there is no pressure head in the system since the pressure head here is equal to 0. The losses for the for the all the losses which are considered as per as per the as per the previous one let me go to the right slide.

So, all the losses everything the values are known to me now. So, I put in all these values and the delta H pump turns out to be 45485-meter square per second square. This is the head in meter square per second square to be provided by the pump for such a system. Now the next is the theoretical power input to the pump is m dot H p where m dot is the flow rate and we also know that there is an efficiency factor which is theoretical by actual the efficiency factor is provided over here. So, to find out the actual power to be actual power is m dot H p by eta.

So, you divided by 0.7 and the actual pump power needed is 2.5 into 10 to the power 7 watt. So, it is a very large power pump that is needed to make the fluid move in this path. Now, next is what is the discharge pressure at the pump? Disturbed pressure we must find out what is the discharge pressure of the pump. The discharge pressure is essentially the pressure right at the exit of the pump.

The theoretical power input to the pump $\dot{w}_P = \dot{m}\Delta \mathrm{h_p}, \eta = \frac{\dot{w}_{Ther}}{\dot{w}_{actual}}$

$$\dot{w}_{actual} = \frac{\dot{m}\Delta \mathrm{h_p}}{\eta} = \frac{Q\rho\Delta \mathrm{h_p}}{\eta} = \frac{0.378\,\frac{\mathrm{m^3}}{\mathrm{s}} \times 10^3\,\frac{kg}{m^3} \times 45485\,\frac{\mathrm{m^2}}{s^2}}{0.7}$$

$$w_{actual} = 2.45 * 10^7\ \mathrm{W}$$

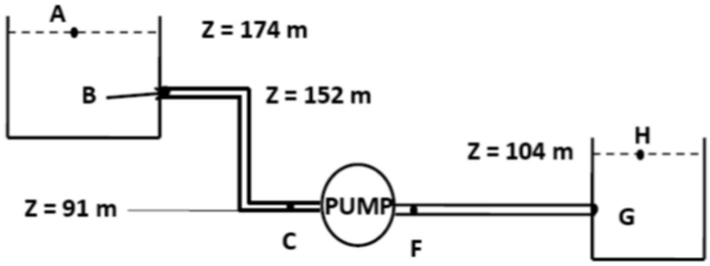
The discharge pressure from the pump is obtained using B` equation between 1 and 3 (just at the exit of the pump, neglecting losses in the inlet section, any elevation changes and kinetic energy at 3) as

$$\mathrm{P_3 - P_1 = \rho\Delta h_{pump}} = 10^3\,\frac{\mathrm{kg}}{\mathrm{m^3}} \times\ 45485\,\frac{\mathrm{m^2}}{\mathrm{s^2}} = 4.5 \times 10^7\ \mathrm{Pa}$$

So, if I go back to this point the discharge pressure is right at this point at the discharge of the pump. So, to obtain the Bernoulli’s equation to obtain that I am going to write Bernoulli's equation between 1 and 3. 1 and 3 is just at the exit of the pump and between 1 and 3, 3 is just at the exit of pump neglecting any losses in the inlet section any elevation changes and kinetic energy at 3. So, if we do that the p 3 minus p 1 is simply going to be rho delta H pump. So, between 1 and 3 between this and the exit between these 2 points there is obviously, not going to be any loss in the inter inlet section any change in elevation because they are at the same level the entry at the pump and the exit at the pump because delta H pump is known to me.

So, I can write the Bernoulli's equation the modified Bernoulli's equation between the entry of the pump and at the exit of the pump. So, p 3 minus p 1 is simply going to be rho delta H pump and rho is known to me delta H is known to me. So, the pressure at the exit is going to be the difference is going to be equal to 4.5 into 10 to the power 7. So, that much of power that much of pressure is generated by the pump at the exit to have the flow in this.

So, this is an example a problem in which the role of the pump is clearly shown and using the efficiency factor one would be able to obtain what is the total power required for such a pump. Let us move on to another problem which also involved a pump. So, what you have here is you have a tank open to atmosphere at some point you draw liquid from the pump and through the pump I mean it is the opposite way the pump draws the liquid and puts it at some height the this the all the heights are provided. So, this is z equals 152 meters and this is z equals 174 meters. So, water is pumped from a reservoir at the rate of 1310 litre per second and is being sent to another large tank.



The implication of the word large here is that the rise of the water level here and the fall of the water level at this point are going to be rather small such that the velocities at point A and velocities at point H are going to be small and the velocity heads can therefore, be neglected. We also understand that since both are open to atmosphere the pressure head is also not going to be present on either side of Bernoulli's equation. So, that with that the path of the water through the pipe is marked as BCFG. So, this is the path of the water and from B to C from here to hear the system consists of a square edged entrance over here this is a square edged entrance 760 meters of pipe 3 gate valves 4 45-degree elbows not all of them are shown in the figure, but the, but the numbers are 3 gate valves 4 elbows and 2 90-degree elbows. Gauge pressure at C is given as 197 kilo Pascal.

The system with the system between F and G between F and G contains 760 meter of pipe 2 gate valves L e by D that is known to us 4 90 degree elbows all the pipes are made of cast iron for which the value of the value of epsilon is provided 0.26 millimetre and the diameter is also provided 508 millimetres. Calculate the average velocity of water in the pipe. So, the velocity is unknown that is what we must figure out. The gauge pressure at location F and the power input to the pump of efficiency 80 percent.

So, to some extent it is like the problem that we have solved, but there are some variations which we must keep in mind, but whenever you solve such a problem make sure that you correctly consider all the valves the entry the exit losses which are the elbows etcetera that are properly accounted for in your calculation. So, that is very important. So, first is velocity you can calculate the velocity to be 6.46 meter per second.

Then the next thing is gauge pressure at F. So, I am going to write to obtain the pressure at F I am going to write Bernoulli's equation between this point and over here. Now, if I write Bernoulli's equation between this and so, this is what the equation is and alpha is the kinetic energy correction factor and these are the gravity heads the v f v h, v f is the velocity here, v h is the velocity at the top of the reservoir and h l t is the total loss encountered from the entry all the way up to F going through a series of valves and area changers the bends and so on. So, v h is 0 as I said the top has a velocity very small velocity p h is p atmosphere and alpha I take it to be equal to 1 and I know that alpha is equal to 1 for highly turbulent flows. So, I need to check the value of Reynolds number at the end of the problem and justify that my choice of alpha equals 1 is correct. So, we will we are going to check that towards the end of the problem.

So, the h l t the total loss is the loss the major loss the minor losses formula for major loss formula for minor loss. So, these are the equations with which you are already familiar with and then p f the pressure head at over 1 is simply the total loss plus the gravitational head minus v f square by 2. The gravitational head is whatever this points the gravity head between this and this point. The L F g the major loss in F g major loss due to straight pipe in F g the 2 gate valves 4 90-degree bends 1 exit and then you have the variation in the head and this v square by 2. So, if you if you put this in there for the gate valve you express it in L e by d bend also in L e by d exit loss is in terms of k exit the coefficient and the rest are going to be the same.

$$\text{Velocity, } v = \frac{4Q}{\pi d^2} = \frac{4\times1310\times10^{-3}}{\pi(0.508)^2} = 6.46\ \mathrm{m/s}$$

Gage Pr. At F: Bernoulli equation between F and H

$$\left(\frac{p_F}{\rho} + \alpha\frac{v_F^2}{2} + gz_F\right) - \left(\frac{p_H}{\rho} + \alpha\frac{v_H^2}{2} + gz_H\right) = h_{LT}$$

$v_H = 0, p_H = p_{atm},\ \alpha = 1$ (to be checked later)

$$h_{LT} = h_L + h_{LM} \quad h_L = f\frac{L}{D}\frac{v^2}{2} \qquad h_{LM} = \frac{v^2}{2}\sum f\frac{L_e}{D} + \frac{v^2}{2}k_{exit}$$

$$\frac{p_F}{\rho} = h_{LT} + g(z_H - z_F) - \frac{v_F^2}{2}$$

So, the values of gate valve the values of 90-degree bend L e by d values are provided the k the exit loss the coefficient loss coefficient is equal to 1. So, if that is equal to 1 then you can calculate what is Reynolds number based on the velocity that we have obtained already obtained. So, the value turns out to be this should be plus 6 3.28 into 10 to the power plus 6 it cannot be minus it is 10 to the power plus 6 and so obviously, this is highly turbulent and since it is highly turbulent, we know that the kinetic energy correction factor for highly turbulent flow will be close to 1 and it is a customary practice to have the value equal to 1 for such cases.

$$\frac{p_F}{\rho} = h_{LFG} + 2h_{LM\,Gate} + 4h_{LM\,90^\circ} + h_{LM\,Exit} + g(z_H - z_F) - \frac{v^2}{2}$$

$$\frac{p_F}{\rho} = f\frac{L}{D}\frac{v^2}{2} + 2f\left(\frac{L_e}{D}\right)_{Gate}\frac{v^2}{2} + 4f\left(\frac{L_e}{D}\right)_{90^\circ}\frac{v^2}{2} + \frac{v^2}{2}k_{exit} + g(z_H - z_F) - \frac{v^2}{2}$$

$$\left(\frac{L_e}{D}\right)_{Gate} = 8, \qquad \left(\frac{L_e}{D}\right)_{90^\circ} = 30, \qquad k = 1$$

$$Re = \frac{Dv}{\gamma} = \frac{0.508\times6.46}{1\times10^{-6}} = 3.28\times10^{-6}\ \text{(Turbulent, } \alpha = 1 \text{ checked)}$$

$$\epsilon = 0.26\ \mathrm{mm}, \qquad \frac{\epsilon}{D} = 0.0005 \rightarrow f = 0.017$$

So, our choice of alpha equal to 1 is justified. The epsilon by d these are known Reynolds number Reynolds number is Reynolds number is also known to us. So, if the Reynolds number is known the epsilon by d is known. So, from Moody diagram I can find out what is the value of the friction factor F. So, if the friction factor F is known over here then I can put that value over there and get the complete expression for the pressure at point F and I put the value of F as 0.017 that we have obtained the velocity that we know and everything else in everything else in there.

$$\frac{p_F}{\rho} = f\frac{v^2}{2}\left[\frac{760}{0.508} + 2 \times 8 + 4 \times 30\right] + g(z_H - z_F) + \frac{v^2}{2} - \frac{v^2}{2}$$

$$p_F = \rho\left[1632f\frac{v^2}{2} + g(z_H - z_F)\right]$$

$$p_F = 999\left[\frac{1632}{2} \times 0.017 \times (6.46)^2 + 9.81(104 - 91)\right]$$

$$p_F = 705\ \mathrm{kPa}$$

So, the pressure at F is going to be 705 kilo Pascal. So, you can see that the pressure over here is 0 the pressure over here is 705 kilo Pascal and we can simply obtain the pressure at the exit of the pump to be equal to be substantially different from 705 from the atmospheric pressure that we have over here. Now then comes the last part of the problem what is the power input to the fluid by the pump. So, in order to do that the W pump the head of the pump divided by m dot. So, this is W pump by m is whatever be at the discharge and whatever be at the suction.

Power input to the fluid

$$\frac{W_{Pump}}{\dot{m}} = \left(\frac{p}{\rho} + \frac{v^2}{2} + gz\right)_{discharge} - \left(\frac{p}{\rho} + \frac{v^2}{2} + gz\right)_{suction}$$

$p_{Suction} = p_C = 197 \times 10^3$ Pa (Given)

$$W_P = \left(\frac{p_F}{\rho} - \frac{p_C}{\rho}\right)\rho A v = \left(\frac{p_F}{\rho} - \frac{p_C}{\rho}\right)Q$$

$$W_P = (705 - 197) \times 10^3 \times 1310 \times 10^{-3} = 6.65 \times 10^5 \mathrm{Nm/s^2}$$

Actual power input to the pump $= \frac{W_P}{\mathrm{Eff}} = 832$ kW

So, whatever be the condition at the suction and whatever be the condition as the discharge the difference between the two is essentially the power input to the fluid by the pump. So, P suction P C is provided in the statement of the problem. If you go back and see the statement of the problem this value of P C the pressure the gauge pressure at location C is provided. So, that is why I have written this is given and I have just now calculated what is the value of the value of the pressure over here at the discharge that is the value of the P f. So, W p is simply m dot is rho A v and P f minus P C P f by rho minus P C by rho the value of P f I have just calculated in the first part of the problem the value of P C is provided in here.

So, when you plug in the values you are going to get the power input to the fluid is 6.6 into 10 to the power by 10 to the power 5 Newton meter per second square. So, the actual power input to the in this case is going to be if you if you do the division power input to the pump is going to be this is the power input to the fluid, but the power input to the pump is going to be more than this since the pump is operating only with certain percentage only with certain efficiency. So, when you take that into account the power input to the pump turns out to be 832 kilowatts. So, that is essentially the power that is needed to the pump.

So, what we have learned in these two problems is you have to choose the points between which you are going to apply Bernoulli's equation. And whenever you apply the Bernoulli's equation always make sure that you have correctly considered the entry losses, the exit losses, the valves, the bends etcetera as well as the major loss in the in the form of straight path. And make sure that your velocity is always the velocity through the pipe in which you are making all your calculations not at the nozzle or not at some other point. The velocities of any large reservoir can be taken to be equal to 1 and whenever you are calculating pump power you also must consider what is the efficiency of that pump such that the actual power delivered by the pump and the actual power to be supplied to the pump are different by the efficiency factor. So, we are coming close towards the to the end towards the end and there are some more typical problems that I would solve in the next two classes which would complete the Bernoulli's equation application part of this course. Thank you.