Momentum Transfer in Fluids
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Lecture-48

The last class I have discussed about how to solve specific cases and use the modified Bernoulli's equation to figure out the unknown quantities. So, the major advantage of the past two cases was that major advantage that the value of the q and d both were known to us. Since q and d were known to us, we would be able to calculate what is the velocity of the fluid passing through the pipe. Now, if the velocity is known then the Reynolds number would be known and depending on what whether it is laminar flow or turbulent flow the value of the friction factor f can be obtained. If f is known then the major losses f L by d v square by 2 will be available to us. And I would also be able to calculate what are the minor losses since the contraction or the expansion coefficient k is known and the for the bends and valves the equivalent length, I can use the concept of equivalent length and the known value of f to figure out what is the total pressure drop total head loss.

$$Case\ iii)\quad \Delta P, L, D \quad known \quad Q\ unknown$$

- Combine Eq. (A) with (B and/or C)
- Results in an expression of V (or Q) in terms of f
- Assume f, based on flow entirely in the rough region, obtain V and Q
- Calculate Re, recalculate f
- As f is a weak function of Re, two iterations are sufficient

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+gZ_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+gZ_2\right)+h_{LT} \qquad (A)$$

$$h_L = f\frac{L}{D}\frac{\overline{V^2}}{2}, \quad major\ head\ loss, \qquad (B)$$

$$f = 64/_{\mathrm{Re}} - Lamminar \quad OR\ Moody\ diagram - Turbulent$$

$$h_{LM} = K\frac{\overline{V^2}}{2} \quad (C1) \qquad h_{LM} = f\frac{L_e}{D}\frac{\overline{V^2}}{2} \quad (C2)$$

$$K = Loss\ coefficient \qquad L_e\ Equiv.\ length\ of\ straight\ pipe$$

But the problem would be more complicated if we think about our type 3 problems in which delta p L d are known, but q is unknown. So, I do not I do not have the luxury of finding out what is the velocity, what is the Reynolds number and subsequently what is the value of the friction factor. I have listed here once again the equations that you would require to solve any problem which are the modified Bernoulli's equation, the major head loss h L the minor losses C 1 and C 2 equation C 1 and C 2 in terms of loss coefficient and in terms of equivalent length of straight pipe. So, how do we how do we approach? Since the velocity is not known and if I substitute this h L t by a sum of h L and h L m what you would get is that in that would result in an equation where both v and f are unknown.

So, if both v and f are unknown and I have only one equation then the only approach which is left to me is to go for an iterative solution. So, this one requires an iterative solution and I will show you through an example how this type of problems can be solved. So, the steps as I have mentioned you combine equation A with equation B and or C 1 C 2. If the if the C 1 C 2 are going to be relevant if minor losses are present in that system. This would result in an expression of v and v in terms of f.

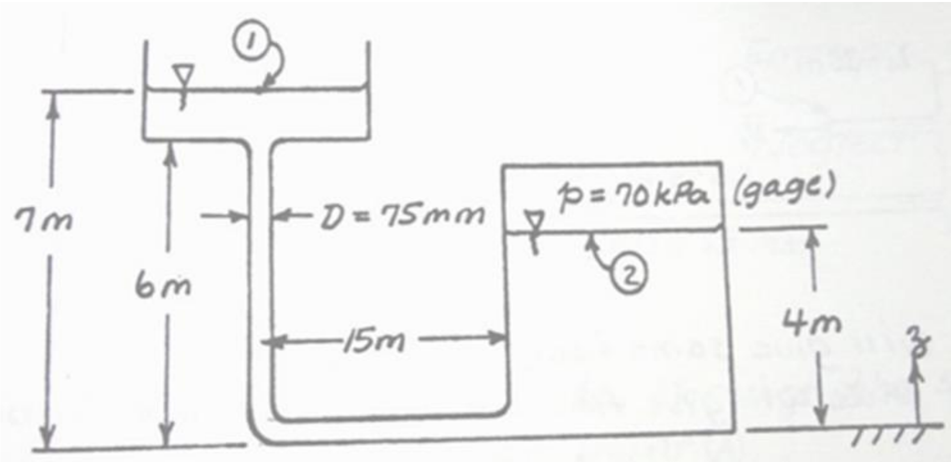
So, there will be two unknowns in the equation v and f. What you need to do is you have to assume a value of friction factor f. The value of epsilon by D is known to us provided to us. So, what you must do is if you recall the moody diagram there is going to be lines like this for different values of epsilon by D and towards the end when the flow is fully turbulent the value of f is rather invariant with the value of the Reynolds number. Let me give you let me give you show you this thing once again if I have it in here right over here.

So, if your if your epsilon by D is known let us say it is 0.002 you could see that from this point onwards there is almost no variation of f with Reynolds number. So, but however, for any point for any Reynolds number lesser than this this part is going to be a function of Reynolds number the value of f is going to be a function of Reynolds number. But once you cross this region when the flow is totally turbulent wholly turbulent flow the variation of f with Reynolds number is rather small. So, you are going to choose some value of f over here that that is what is prescribed in the solution methodology that you choose an f which is in the completely turbulent zone and go from there.

So, you choose this f the moment you choose this f then in the combined equation where you have v and f since f is now chosen by you should be able to figure you should be able to calculate what is the value of v. So, assuming f gives you the value of v. However, this assumption is to be checked whether it is correct or not. Now the moment I have v and I could be able to calculate q as well the moment I have v I would be able to find out what is the value of the Reynolds number. If the Reynolds number is known then using Moody diagram, I would be able to recalculate the value of f.

So, this f is a assumed value then I find out v once I find out v I find out Reynolds number and from there I recalculate the value of the f and figure and note whether there is any difference between my assumption and the value of f that I have obtained over here. If there is a deviation between these two you start from this point with the newly obtained value of f as your assumed value of f. So, let us say you must take f to be equal to 0.002 or 0.02 and here you have obtained the value of f to be equals 0.017. So, your next iteration starts with the assumption of f to be equal to 0.017 perform this calculation and see what is the value of new value of f that you get. So, this iteration will continue till your assumed value of f and your calculated value of f they are sufficiently close. You stop your iteration there and then the corresponding v is your correct value of v and if you know v and if you since you know d you would be able to obtain what is the flow rate. So, that is the correct value of your flow rate.

So, an iterative solution would give you the correct result only after 2 or 3 iterations at the most why because as you could see f is a weak function of Reynolds number. So, since f is a weak function of Reynolds number two iteration iterations are generally sufficient to arrive at close match between this calculated f and the assumed value of f and this kind of iterative approach would let you solve the case 3 type of problems where delta p L d is known, but q is unknown. So, let us take one example and see how this is achieved in practice. So, over here if you read the statement of the problem this there is an overhead tank which is open to the atmosphere and there is a closed tank where the pressure is 70 kilo Pascal gauge pressure. You have an you have a pipe which is connected the values of z that means, this is 6 meters, this is 7 meters, the length over here is 15 and the diameter is 75 millimetre and the height of the liquid in this tank is about 4 meters.



The value of epsilon by d is provided and it has one right angle bend over here and the flow can be assumed to be in the fully rough region of the Moody diagram which we can check later. The surface pressure is atmospheric over here pressure at the lower reservoir is 171.3 kilo Pascal. So, if you convert this pressure into gauge pressure it is going to be about 70 kilo Pascal. The pipe diameter is provided assume that the only significant losses occur in the pipe in the pipe and the bend for the epsilon by d is 12.

So, there are the problem states that there are no losses at entry, no losses at exit, the only losses are due to the straight pipe 6-meter, straight pipe 15 meter and a 90-degree bend over here for which the L e by d is equal to 12. So, the once again the only losses are for the straight one here and the bend the entry and the exit losses are not to be considered as per the statement of the problem. What you must find out is the direction and magnitude of the q the volumetric flow rate of water for which rho is given kinematic viscosity is known and we know that kinematic viscosity is mu the viscosity divided by rho the density. So, its value is given as 1.1 into 10 to the power minus 6.

So, not only you have to find out what is the volumetric flow rate of water you also must find out what is the direction of flow is the flow going from 1 to 2 or going from 2 to 1. So, this is something which we must figure out and once again since the value of q and d, are not known together, I cannot calculate I cannot figure out what is the value of the velocity v. So, this leads to this indicates then an iterative solution may be required. So, how do we start the problem? Once again, I listed the equations the modified Bernoulli's equation the major loss F being the friction factor minor losses using C 1 and C 2 k is the loss coefficient L e is the equivalent length of the bend or something which is provided in the flow circuit. So, we will start with this.

So, value of alpha is mentioned in the problem to be equal to 1 and the first thing is I have to figure out what is the direction of flow. So, I assume the flow is from 1 to 2. So, from 1 to 2 and start my calculation and see whether it leads to an absurd situation an absurd result that would indicate that would tell me whether my assumption of flow being from 1 to 2 is correct or not. So, my I start the problem with flow from 1 to 2. So, what is h L t from 1 to 2? I have this is my then this is my pressure drop the pressure from here to here and this flow is from this side to this side and this is my hydrostatic head.

So, if I figure this together if I put this together when the flow is from 1 to 2 the head loss is going to be equal to minus 40.6-meter square per second square. So, but we know that head loss cannot be negative, there cannot be any negative loss you cannot gain a head when the flow is from 1 to 2 head loss will always have to be positive. So, a quick calculation of the losses. Just the major losses would give you that the flow cannot be from 1 to 2.

$$\alpha = 1$$

Assume, Flow From 1 → 2

$$h_{LT,1-2} = -70 \times 10^3 \frac{N}{m^2} \times \frac{1}{10^3} \frac{m^3}{kg} + 9.81 \frac{m}{s^2} (7 - 4) m$$

$$h_{LT,1-2} = -40.6 \frac{m^2}{s^2}$$

$h_{LT,1-2}$ cannot be negative Flow From 2 → 1

$$h_{LT,1-2} = f \frac{L}{D} \frac{V^2}{2} + f \left(\frac{L_e}{D} \right) \frac{V^2}{2} = f \left(\frac{L}{D} + \frac{L_e}{D} \right) \frac{V^2}{2} = 40.6 \frac{m^2}{s^2}$$

So, the flow must be from 2 to 1. So, what is the flow from 2 to 1? The losses from 2 to 1 when we consider I am going to have the friction factor, the length of the straight pipe, the diameter of the pipe, the velocity through the pipe. So, the other is going to be the other loss is going to be L e where L e is the equivalent length due to this bend by d v square by 2 and since this L has the same equivalent length has the same diameter as the main pipe. So, the same F can be used over here. So, the first thing is a quick calculation choosing 1 and 2 flow from 1 and 2 finding out whether it is positive or negative.

If it is positive fine your assumption of the flow from 1 to 2 is correct if not it is having to be in the reverse direction. So, this is what you would get over here and knowing the values this is what going to be the total loss in this case ok. Now, you go to use the Modi diagram to figure out corresponding to the value of the Reynolds number and the epsilon by d what is the value of F. So, this if you know then we proceed to the next one. The straight path is 21 meters the L e by d is 12.

So, you must figure out what is the velocity. So, we know that this in the equation 1 when I substitute the value of the losses, I have an expression which contains both F and d both F and d. So, I must go for an iterative solution. The first thing is I assume F to be in the rough region. So, I assume F to be equal to 0.023. So, corresponding to L e by d equals 12 if I go back to the Moody diagram for corresponding to an epsilon by d equal to 12 the and a sorry epsilon by d not equal to 12 the epsilon by d is provided in the statement of the problem the epsilon by d is equal to 0.002. So, I go to Moody diagram. So, 0.002 in the rough region I take this value to be equals 0.023. So, that is my initial choice of the friction factor. So, with this I find out what is the value of the velocity from the expression that we have obtained before. So, this is 3.48 meter per second. Since the velocity is known to me then I can calculate what is the Reynolds number.

$$L = 21\text{ m}, \qquad \frac{L_e}{D} = 12, \qquad V = ?$$

ITERATION NEEDED $\qquad$ Assume $f \approx 0.023$

$$\bar{V} = \left[\frac{2h_{LT}}{f\left(\frac{L}{D} + \frac{L_e}{D}\right)}\right]^{1/2} = 3.48\text{ m/s} \qquad Re = \frac{VD}{\gamma} = 2.37 \times 10^5$$

$$f = 0.024 \rightarrow \text{Recalculate } \bar{V} \rightarrow \text{Find Re} \rightarrow f = ?$$

$$Q = VA = 0.015\ \frac{m^3}{s}$$

So, Reynolds number is V d by the kinematic viscosity kinematic viscosity is mu by rho. So, it is 2.37 into 10 to the power of use Moody diagram once again. You would see that the value of f is going to be equal to 0.024. So, I started with 0.023 and I would get 0.024. They are sufficiently close, but not too close I can do one more iteration. So, we can recalculate V find Reynolds number and find what is the value of the friction factor. Generally, one at two iterations are sufficient.

So, you put that value recalculate and find out how different it is from the assumed value of f. So, I started with 0.03 ended up with 0.024 use 0.024 over here recalculate the velocity recalculate the Reynolds number and find out what is the value of f.

Compare the result with 0.024 you would find that they are very close to each other. So, once you do that your q the flow rate is going to be equal to 0.015-meter cube per second. So, what I suggest is that you do this problem on your own and check with the answers. So, the type 3 problem essentially is requiring an iterative solution, but fortunately since f is a weak function of Reynolds number you do not have to do multiple iterations.

Just two iterations should give you sufficiently close value of your assumed f and the f that you calculate by at the end of this. So, the trick is to combine the head losses with the Bernoulli's equation and get a relation between f and V. Once you do that choose f find V find Reynolds number recalculate f compare the f that you have assumed and the f that you have obtained keep on doing this and you will see that in two iterations the values of f are going to be very close to each other. Find out what is the corresponding value of velocity and once you have you know the value of the velocity your flow rate is simply going to be V times a. So, this is what you are this is the approach that we are going to use for type 3 problems.

There still is left one type the last type and then we will we have not yet discussed about the problems which involves pumps and other devices that will come later, but the other type of problem is delta p L and Q are known, but the diameter is unknown. Once again since L and D as since Q and D, are not known together. So, you cannot calculate velocity you cannot calculate Reynolds number you cannot calculate f you cannot and cannot compare with delta p. So, an iterative solution is needed, but there is a there is another point which one must keep in mind is what is there may be different sizes of pipes which would satisfy the condition. Your pump that is present in the circuit or the flow geometry is such that the delta p that is provided and you can choose any diameter.

$$\boxed{Case\ iv)\quad \Delta P, L, Q \quad known \quad D\ unknown}$$

How to evaluate the smallest pipe size

- Assume D, find Re, ε/D and f
- Calculate head loss (Eq. B & C)
- Solve Eq. A to find ΔP
- If calculated ΔP is large, choose larger D
- If calculated ΔP is small choose smaller D
- Choose commercially available pipes

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+gZ_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+gZ_2\right)+h_{LT} \qquad (A)$$

$$h_L = f\frac{L}{D}\frac{\overline{V^2}}{2}, \quad major\ head\ loss, \qquad (B)$$

$$f = 64/\mathrm{Re} - Lamninar \quad OR\ Moody\ diagram - Turbulent$$

$$h_{LM} = K\frac{\overline{V^2}}{2} \quad (C1) \qquad h_{LM} = f\frac{L_e}{D}\frac{\overline{V^2}}{2} \quad (C2)$$

K = Loss coefficient,

L_e Equiv. length of straight pipe

If you take a larger diameter, you are not going to use that delta p which is available to you. So, a larger diameter would still give you the same would give you the required flow rate, but without utilizing the full potential of the delta p that is available to you. And of course, if you go for a larger diameter pipe the cost is going to be more. So, you must find out which is the smallest pipe size that is going to satisfy the delta p and Q and L that are provided to you. So, your job is to figure out what is the smallest pipe diameter that would solve that would satisfy the process conditions of the problem that stated.

So, once again the equations Bernoulli's equation major head loss, minor head loss in terms of loss coefficients or equivalent length of straight pipe. So, here the assumption is going to be this the starting point is going to be slightly different. I am first going to assume D. Now, the moment I assume D I am going to find out since the now my D is known and my Q is also known. So, I would be able to figure out what is the velocity.

Since the velocity is known to me, I can find out what is the value of epsilon Reynolds number. This epsilon by D is known to the f value of epsilon was known to me. Now, since the D is assumed, I can find out what is epsilon by D. Since I know Reynolds number and epsilon by D I would be able to find out what is the value of the friction factor. So, just the choice of D can give me Reynolds number epsilon by D and f.

The questions come is what diameter are you going to choose? You can choose any diameter for that matter ok. Start with any diameter and when you do iterate you are going to see that you will end up with a unique solution for such situation. What is going to be the next step? You are going to calculate the head loss equation B and C. So, my diameter is known to me, my velocity is now known to me. So, I would be able to obtain what is the major loss and what are the minor losses since the velocity and the equivalent length and the diameter are known to me now.

So, the major loss, the minor losses are known to me at this point. So, we would be able to find out what is h l t the total head loss. Since the total head loss is known to me, I would be able to figure out what is the delta p that is being predicted by equation A. I understand that my delta p is known to me, but knowing h l t and everything else in here I should be able to figure out what is the delta p that is required for the chosen value of the diameter D ok. So, what if the calculated delta p is large by using this equation A, I can figure out what is delta p and if this delta p the required pressure drop is more than the supplied pressure drop more than the available pressure drop then I must relax the flow condition by choosing a larger diameter.

That means, I have chosen a small diameter smaller diameter than that is necessary and since it is a smaller diameter it is giving me a large pressure drop larger than what is available to me. So, what should I do? I increase the diameter size. Now, the moment I increase the diameter size and repeat this calculation what I am going to get is I am going to see that the that the pressure drop calculated from equation A has reduced because it is a larger diameter. Now, the question is having it reduced below the delta p that is available to me. If it is if not then I must reduce del the diameter I have to increase the diameter further to make the required delta p even lower.

So, increasing the diameter always lowers the delta p. So, if my choice of if my choice of larger D is not good enough then I must use an even higher value of diameter such that this delta p reduces further. Not only it reduces further my correct value of diameter would be when the calculated value of delta p is going to be less than the delta p which is available to me. Then I know that yes, I have reached the condition where all conditions are satisfied. So, for the given length for the flow rate that I want I have chosen a diameter now by an iterative process which would let the flow takes place utilizing the delta p that is available to me. So, this delta p whether it is provided by hydrostatic head whether it is provided by a pump or something else that is sufficient to make liquid flow through the pipe with a Q the flow rate that is known to me.

So, that is the iterative process which you must continue by choosing higher progressively higher and higher value of D such that this match with delta p can take place. Now, if you have started with a very large D then you would see that you are not utilizing the delta p available to you. In that case you must progressively make it smaller such that your required delta p and your calculated delta p are very close to each other with the available delta p being slightly higher than the required delta p that is the procedure which one must follow. But there is still something else one must think of because if you if you if you if you calculate the D and as I have written if the calculated delta p is small, you must use a smaller D, but after all of this you end up with a diameter required diameter let us say it is about 12 centimetres, but that may not be the right solution. It may not be the right solution because manufacturers do not make diameter do not make pipes of all possible diameters.

There are some standard diameter pipes are available and if you want to make a pipe whose diameter is going to be exactly what you want it is going to be prohibitively expensive. So, you have to choose a commercially available pipe which is closest to the diameter that you have calculated and for that if you are you cannot go below your calculation because in that case the delta p will increase and which you may not be supplied by the available delta p. So, you go for the next higher commercially available pipe in order to in order to satisfy everything that your flow rate is correct your delta p is close to fully utilized, but not over shooting it and you have come across you have you are prescribing you are designing a pipe which you can buy from the from the market because that is a commercially available pipe. So, these are the 4 types of types of problems that we have discussed we have not discussed about the pump or the role of the pump that we would see in the next class, but these 4 the type 1 and type 2 can be solved easily and for type 3 and type 4 you need an iterative solution. So, that is what I wanted to cover in different types of problems and there will be more complicated ones when you have something else involved as well.

That means, you must supply a liquid at a certain height with a certain velocity or you have a pump you would like to size the pump figure out what is going to be the wattage of the pump that is necessary. So, those kinds of problems will be discussed in the next 3 lectures. Thank you.