Momentum Transfer in Fluids
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Week-10
Lecture-47

Good morning. What we are going to start today will be the applications of Bernoulli's equation. The last time we have solved a problem in which there was a sudden expansion and we knew the pressure drop and we wanted to correlate that pressure drop with the velocity of the fluid which is flowing and thereby calculate the mass flow rate, volumetric flow rate. We will have direct applications of Bernoulli's equation in all our subsequent lectures. The just a quick recap if you would like to solve the pipe flow problems, we know that the pump head at 1 pump head must be added to the head at 1 and then only it is going to be equal to whatever be the head sum of all heads, the pressure head, the velocity head, and the gravitational head. Normally if we do not have the pump then the head at 1 would be equal to head at 2 plus losses due to friction, due to the bend in the path, due to the presence of flow measuring devices like a venturi meter or an orifice meter and due to the presence of valves or flow controlling devices present in its path.

Now, in addition if we have now a pump which is supplying an additional head to deliver the fluid at a certain volumetric flow rate to another location. So, the role of the pump is to overcome the losses which are going to take place while the fluid is flowing through the circuit and to ensure that the same amount of fluid is delivered either at a higher pressure, at a higher elevation or at a higher velocity when the flow is through a nozzle. So, therefore, the pump the additional head created by the pump is to be added to the head at 1 and only then it is going to be equal to head at 2 plus it will account for all the losses in its flow path. So, the work done work that is provided by the pump is simply going to be the algebraic sum of all these where h l t is essentially the total head loss.

So, h l is the loss and t signify the total losses and we know by now that the head losses are of two types one is a one is the major loss which is frictional flow in straight pipes, frictional losses in straight pipe and the minor losses which are due to the presence of bends, orifice meters, venturi meters, valves, and other such devices which other such situations in which the flow direction is being changed. So, the pump head and we have already seen how to calculate the major losses using the Moody diagram finding out what is the value of the friction factor and the pressure loss is simply going to be F L by D v square by 2. So, this F is to be evaluated from the Moody diagram which is nothing, but a plot of F on the y axis, Reynolds number on the x axis in logarithmic scale and on the right-hand side we have epsilon by D where epsilon is the roughness and D is the diameter of the pipe. So, we will get a series if you recall we will get a series of lines for different values of epsilon by D. So, the way to calculate the friction factor F is to figure out where what is the what is the Reynolds number go all the way up to the curve that corresponds to the right value of epsilon by D for your system and then read from the left y axis what is the value of the friction factor.

We will have more examples on the evaluation of friction factors in the later part of this lecture. The pump head is W divided by m dot where m dot is the mass flow rate and the power is rho q rho times q is the mass flow rate times the pump head and it is going to be it is going to be in terms of watts. And this delta p the pressure drop is going to be a function unknown function of length the flow rate q the diameter D the roughness of the pipe E the height to which the fluid is to be pumped. So, which is delta z the system configurations how many bends are there in how many bends are in there in their path how many valves are there how many flow reducing systems or flow or are incorporated in its path. So, that is what I mean by system configuration and of course, it is going to be a function of rho and mu where rho and mu are the corresponding thermophysical property of the liquid which is being pumped.

So, the way to calculate the minor losses is simply k times v square by 2 where k is the loss coefficient and the loss coefficient is generally evaluated experimentally. So, what you must do is you have to figure out from the data provided and from the graph charts relations which are available what is the value of the loss coefficient k. Sometimes the minor losses are also expressed in terms of an equivalent length which also I have explained in the last class. So, let us say I have a valve in its path and of course, there is going to be some pressure drop across this valve. Now, let us try to figure out what length of straight pipe of the same dimension as the inlet and the outlet pipe, what length of the straight pipe would result in the same pressure drop as that caused by the valve.

So, when the pressure drop caused by the presence of the valve in the flow path is equal to the length of a straight pipe where the pressure drop is going to be identical then this length is called the equivalent length of that valve. So, if we have if we know the equivalent length then we should be able to express the minor loss as f L L e by d v square by 2. So, this L e is known as the equivalent length. So, similar such equivalent lengths are there for gate valve, globe valve, a bend, L bend and so many other flow devices. So, if we think a little bit more about how are you going to solve the pipe flow problem, this delta p as I mentioned is going to be a function of all these.

Now, once you layout the pipe, once you know what is the geometry, what is the path that the fluid is going to take to reach from point A to point B, then the system configuration is known to me, the delta z is known to me and since I have already selected the pipe, then this E the friction the roughness is also known to me. And I know which liquid I am going to pump so rho and mu are also known to me. So, therefore, my pressure drop is going to be a function only of L the length, q the flow rate and d the diameter of the pipe. So, this is the function which would we would like to like to evaluate and depending on which 3 quantities are known and there will be one unknown, there can be 4 types of problem. The 4 types of problem are the first 2 are rather simple L, Q and D, L, Q and D are known delta p is unknown.

So, we need to figure out a way to calculate delta p when these 3 are known to us. The second type of problem is when delta p, q and d are known. However, the length of the straight length of the pipe is unknown. In similarly for the third and the fourth delta p, L, D are known q is unknown and delta p, L, Q are known the D is unknown. The first 2 are characterized by something which makes the calculation a bit simpler.

So, what is known here? Since my q and D both are known so, I would be able to calculate what is the what is the velocity because the velocity is nothing, but the velocity is essentially the q the flow rate is simply going to be the area which is pi D square by 4 times velocity is essentially the volumetric flow rate. So, if my q and D are known as is the case in the first 2 therefore, I would be able to find out what is the value of the velocity that is of the fluid that is flowing through the tube. And since velocity is known my Reynolds number is also known because Reynolds number is d v rho by mu. So, my d v rho mu all quantities are known to me. So, from here I should be fine I should be able to find out what is the value of the friction coefficient either from Moody diagram or if the flow is laminar F is simply going to be 64 by R e.

And once I have the value of F known to me, I should be able to calculate what is the head loss h L h L the major head loss. So, these are the advantages of having both the D diameter and q the volumetric flow rate known to us, but I do not have that luxury of knowing the velocity in these cases. So, here the q is unknown and in this case the diameter is unknown. So, I would not be able to find out what is the velocity of the fluid flowing in such systems and therefore, my calculations are not going to be that straight forward. So, what I am going to do now is I am going to solve one problem of each type and discuss the difficulty the methods the difficulties and in some cases the iterative solution that is need approach that is needed to solve this problem.

Case i) L, Q, D *known* ΔP *unknown*

- Calculate Re
- Obtain f
- Calculate h_L Eq. (B)
- Calculate h_{LM} Eq. (C1/C2)
- Calculate ΔP from Eq. A

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + gZ_1\right) = \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + gZ_2\right) + h_{LT} \qquad (A)$$

$$h_L = f \frac{L}{D}\frac{\overline{V^2}}{2}, \quad \text{major head loss}, \qquad (B)$$

$$f = 64/\mathrm{Re} - \text{Lamninar} \quad \text{OR Moody diagram} - \text{Turbulent}$$

$$h_{LM} = K\frac{\overline{V^2}}{2} \quad (C1) \qquad h_{LM} = f\frac{L_e}{D}\frac{\overline{V^2}}{2} \quad (C2)$$

K = Loss coefficient $\qquad L_e$ Equiv. length of straight pipe

So, the first case of the four cases that I have described in the previous slide is when L q and D are known the delta p is unknown. And for us for convenience, I have added this part where I have used this Bernoulli’s equation or modified Bernoulli's equation as the major head loss h L in terms of F L by D v square by 2 this is my equation b. And I have also noted down how to calculate F for the case of laminar 64 by R a and for turbulent I know I must use the Modi diagram. Then I am going to think about the minor losses either it can be expressed in terms of loss coefficient k which I call as equation C 1 or it can be expressed in terms of an equivalent length as I have explained before. So, in this case the F can be obtained the same way as laminar depending on whether it is laminar or whether it is turbulent, but I must use for all these fittings that is going to cause a minor loss as in terms of L e.

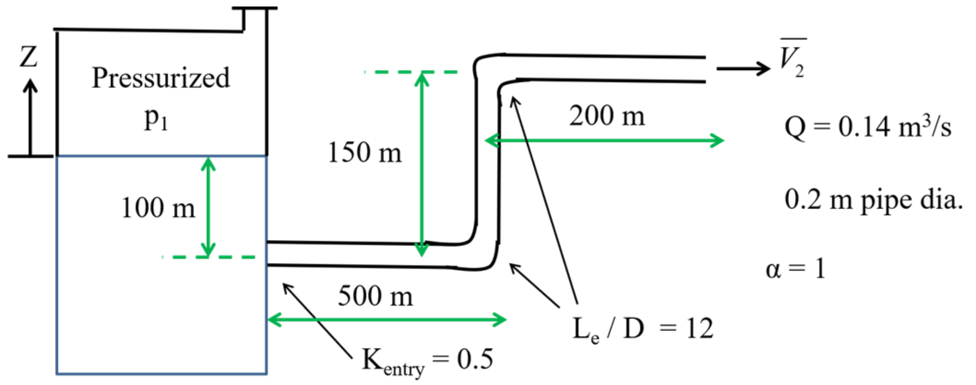
So, L e is the equivalent length of the straight pipe. So, these are the four equations which are available to me a b C 1 and C 2. Now, let us try to see how do we solve the problem when L q D are known delta p is unknown. As I said the first thing is to calculate Reynolds number. Now, in my previous slide I have shown that from q and D how can I find out what is the velocity.

Once I have the velocity then I would be able to obtain what is the value of the Reynolds number. This much I have shown you in the just the slide before. And if I know Reynolds number then depending on whether it is laminar or it is turbulent I can use straight away 64 by

R e or I can use Moody diagram if the if this is turbulent. So, F will be known to me. So, if F is known then I can use equation b to find out what is the major head loss F L by D v square by 2.

So, this comes from equation b which is this provided I know my F. And second is I must find out what is the minor law what are the minor losses. So, for that I can use either C 1 where it is in terms of loss coefficient or I can use C 2 where it is in terms of an equivalent length. So, the major loss and minor loss both can be evaluated since the value of F is known to me right now. And I can read I can find out what is the value of k from the relations or from the curves or from the graphs which are provided to me.

So, now I go now my total head loss is known to me which is h L plus h L m. I go back to this point and substitute what is the numerical value of h L t. So, what is the only unknown here velocities are known to me the elevation change is known to me the total head loss is known to me. So, what I can figure out is what is p 1 minus p 2 the delta p from equation a. So, the use of equation of just the definition of the volumetric flow rate gives me the velocity from velocity to Reynolds number Reynolds number to F from F to major loss and minor loss expressing it in terms of k or F I would be able to find out what is the total head loss and then from equation a I can figure out what is the delta p.



So, that is the methodology which we are going to take. Let us apply it to an equation. So, this is example of case 1 LQD known delta p unknown. So, I have a pressurized tank where some liquid is under unknown pressure p 1 this is closed. So, it is a closed tank the liquid is going to enter into after a height of 100 meters the liquid is a going to enter a pipe of 0.2-meter diameter and the value of alpha is taken the kinetic energy correction coefficient is equal to 1. So, the liquid is going to come changes its direction for 500 meters changes its direction changes its direction starts to move vertically up 150 meters then once again it is going to travel 200 meters in horizontally and then it is going to be discharged at the atmosphere with some velocity v 2. So, the flow rate Q is 0.14-meter cube per second the diameter is provided the properties are properties are mu.

So, it is 1.3 into 10 to the power minus 3 Newton second per meter square and since it is water we know what is its density going to be it is simply 10 to the power 3 kg per meter cube. The value of epsilon by D for the pipe material is provided as 0.0013. We need to calculate what is the gauge pressure P 1 in the in this chamber. So, let us start our step-by-step process this is my equation these are my equations which I have already described to you and then if I if I if I apply the Bernoulli's equation between this point and at the exit.

So, what I have then is this is a large reservoir. So, the liquid level is going to drop at a very small velocity as compared to the velocity exit velocity over here. So, for all practical purposes I can see that the velocity of the liquid level falling in the large reservoir is very small as compared to the exit velocity over here. So, it is safe to assume that my velocity 1 is going to be equal to 0 and I put my datum level over here. So, if this is my datum level as you can see from here.

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+gZ_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+gZ_2\right)+h_{LT} \qquad (A)$$

$$h_L=f\frac{L}{D}\frac{\overline{V^2}}{2}, \quad major\ head\ loss, \qquad (B)$$

$$h_{LM}=K\frac{\overline{V^2}}{2} \qquad (C1) \qquad\qquad h_{LM}=f\frac{L_e}{D}\frac{\overline{V^2}}{2} \qquad (C2)$$

$K = Loss\ coefficient$ $\qquad$ L_e $Equiv.\ length\ of\ straight\ pipe$

So, my z 1 is also going to be equal to 0. The liquid is going to come out in to the atmosphere. So, it opens to the atmosphere where the gauge pressure is 0-gauge pressure is atmospheric pressure the pressure at any point minus the atmospheric pressure. Since the pressure over here is atmospheric pressure. So, p 2 the gauge pressure is going to be equal to 0.

We know v 2 and we have some idea of what is going to be z 2. Now, v 2 has been calculated as I have mentioned q by A 2. So, it is 4.46 meter per second this gives me the value of Reynolds number as 6.8 into 10 to the power 5 and epsilon by D is 0.0013. So, from Moody diagram the value of F can be obtained as 0.021 and let us see how to do this in Moody diagram. So, my Reynolds number is 6.8 into 10 to the power 5. So, it is somewhere over here at this near this point and the value of epsilon by D is 0.0013. So, here it is in between these two curves 0.0001 and 0.0002. So, it is somewhere over here. So, 6.8 let us say that 7 I go all the way up and come at this point and if I read it then it is going to be it is going to be the value of F is going to be 0.0021. So, that is how we I am going to calculate this value of the friction coefficient F. So, what is h l m the minor losses? The minor losses are going to be the loss at the entry. So, from a large area you come into a small area.

$$V_2=\frac{Q}{A_2}=0.14\times\frac{4}{\pi}\times\frac{1}{(0.2)^2}=4.46\ m/s$$

$$Re=\frac{\rho VD}{\mu}=6.83\times10^5, \frac{\epsilon}{D}=0.0013$$

$\Longrightarrow$ From Moody diagram $\rightarrow f=0.021$ $\qquad L=850\ m, \alpha_2=1$

So, this is the minor loss. I am going to have two 90-degree bends over here and over here. So, therefore, the k h l entry is k entry the coefficient k the contraction coefficient times v square by 2 and the bends are going to provide twice F L e by D where L e is the equivalent length for these 90-degree bends and it is provided in the statement of the problem. So, this is the minor loss where it is k times twice F L e by D v square by 2. This total loss is therefore, going to be the length of the straight pipe multiplied by F, F L by D v square by 2 and the two minor losses one is due to the entry the other is due to the two bends which are present in the path of the pipe in the in the pipeline.

$$h_{Lm} = h_{L,entry} + 2h_{L,bend} \quad = \left(k_{ent} + 2f\frac{L_e}{D}\right)\frac{V_2^2}{2}$$

$$h_{LT} = f\frac{L}{D}\frac{V_2^2}{2} + k_{ent}\frac{V_2^2}{2} + 2f\frac{L_e}{D}\frac{V_2^2}{2}$$

$$= \frac{V_2^2}{2}\left[f\left(\frac{L}{D} + 2\frac{L_e}{D}\right) + k_{ent}\right] \qquad k_{ent} = 0.5, \qquad \frac{L_e}{D} = 12$$

$$= \frac{1}{2}(4.4)^2\left[0.021\left(\frac{850}{0.2} + 2 \times 12\right) + 0.3\right] = 898\frac{m^2}{s^2}$$

$$p_1 = \rho\left(gz_2 + \frac{V_2^2}{2} + h_{LT}\right)$$

$$= 999\left(9.81 \times 50 + \frac{(4.46)^2}{2} + 898\right) \quad = 1.40 \text{ MPa (gauge)}$$

So, k entry is provided as 0.5 L e by D as 12. So, you we could simplify this and put the value the v value is known to us and the value of F we have read from the Morie diagram and then we could figure out what is going to be the total losses present in the system. We understand as we can see from the previous equation over here that my p 1 is simply going to be this plus this plus h L t. So, this equation A if we use equation A then it is going to be g z 2 v 2 square by 2 plus h L t the value of h L t we have already calculated over here. So, you plug in the values of rho the z 2 the z 2 over here is this is 100 this is 150.

So, if this is the datum then this is simply going to be 50. So, that is why the 50 comes over here the velocity and the h L t. So, when you add them all together the pressure at location 1 pressure in the tank which is the pressurized tank would be 1.4 mega Pascal gauge pressure. So, this is an example of how one can calculate the how one can calculate the unknown pressure if L Q D are known to us.

So, that is that is the comprehensive example of that. So, the best the way to do once again is to find out the velocity find out the Reynolds number find out F from Moody diagram figure out what are the minor losses add all add them together to get the total losses and then use equation a to obtain the unknown pressure. Now, let us come to type 2 problems the type 2 problems are delta p q d are known L is unknown. Once again since q and d both are known I should be able to obtain what is the value of the velocity. If the velocity is known the Reynolds number is also known.

Case ii) $\Delta P, Q, D$ *known* L *unknown*

- Calculate h_{LT} from (A)
- Calculate Re, Obtain f
- Solve for L using Eq. (B) and/or C1, C2

$$\left(\frac{p_1}{\rho} + \alpha_1\frac{\overline{V}_1^2}{2} + gZ_1\right) = \left(\frac{p_2}{\rho} + \alpha_2\frac{\overline{V}_2^2}{2} + gZ_2\right) + h_{LT} \qquad (A)$$

$$h_L = f\frac{L}{D}\frac{\overline{V}^2}{2}, \quad \textit{major head loss}, \qquad (B)$$

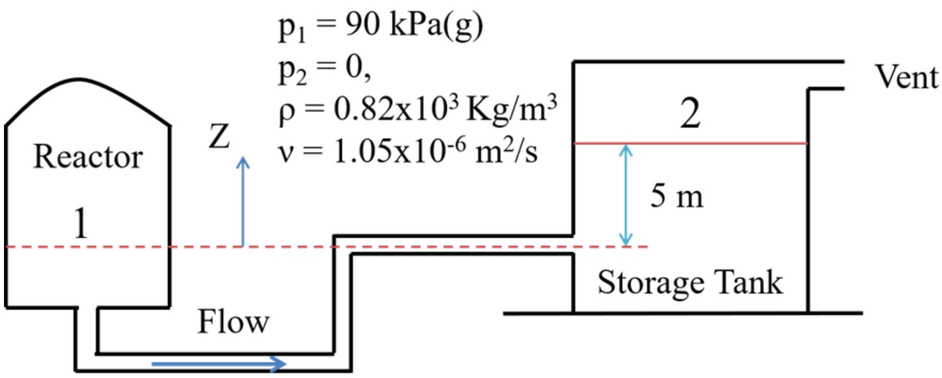
$$f = {}^{64}/_{Re} - \textit{Lamminar} \quad \textit{OR Moody diagram} - \textit{Turbulent}$$

$$h_{LM} = K\frac{\overline{V}^2}{2} \quad (C1) \qquad h_{LM} = f\frac{L_e}{D}\frac{\overline{V}^2}{2} \quad (C2)$$

$K = \textit{Loss coefficient}$ $\qquad$ L_e *Equiv. length of straight pipe*

So, if the Reynolds number is known then the rest would follow, but here delta p is known the velocity are known this z 1 z 2 would be known. So, I should be able to calculate what is the value of h L t the numerical value of h L t from over here from equation a. So, I can calculate Reynolds number as I said since my velocity is known q and d known velocity known. So, Reynolds number known I should be able to find out what is what is the value of the friction factor. Now, if the value of the friction factor is known then I can use equation b and or C 1 or C 2 if this is known then the total the major loss and the minor losses would be known to me.

And if you see that the major loss contains length L the minor loss contains L e which is known for the geometry for the type of type of things that we have valves, bends etcetera that we have we have in the system. So, if the numerical value of h L t is known then the only unknown here is length L length of the straight pipe. So, if we follow these steps the L can be obtained by knowing the value of the total frictional total frictional losses from equation a. So, follow these procedures and you would be able to obtain what is the unknown length. Now, let us try to do that using a using a specific problem.



So, you have a reactor and from the reactor the reactant is going to come pass through a geometry a complex geometry and it is stored in a tank. The reactor is closed the tank is open there is a vent which is open to the atmosphere. So, the pressure at 2 is going to be equal to 0, the gauge pressure at 2 is going to be equal to 0, the gauge pressure at 1 is going to be high and which forces the product to come to the storage tank. So, the P is given P 2 as I have explained open to atmosphere. So, the gauge pressure is 0 rho and v are rho and the and the kinematic viscosity kinematic viscosity is mu by rho.

So, the one that you knew that you see over here is the is the kinematic viscosity. So, the kinematic viscosity is defined this nu is mu by rho and knowing the kinematic viscosity you would be able to find out what is the viscosity. So, that is how many at times it is used not just viscosity it is the kinematic viscosity that is that is used in this. The epsilon by D value is known the k entry that is at this point is equal to 0.5 the k exit over here is 1 L e by D is 12. So, L and the pipe diameter is 1.5 meter. So, you have one 90-degree bend, second 90-degree bend and third 90-degree bend. So, those are the causes of minor losses in the system. So, we must find out what is the total length of the straight pipe in this system.

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+gZ_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+gZ_2\right)+h_{LT} \qquad (A)$$

$$h_L = f\frac{L}{D}\frac{\overline{V^2}}{2}, \quad \textit{major head loss}, \qquad (B)$$

$$f = {}^{64}\!/_{\mathrm{Re}} - \textit{Lamninar} \quad \textit{OR Moody diagram} - \textit{Turbulent}$$

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$$K = \textit{Loss coefficient} \qquad L_e \ \textit{Equiv. length of straight pipe}$$

So, once again the equations A B C 1 C 2 are already known to you the Bernoulli's equation the major loss and the minor losses either in terms of k or in terms of L e. So, what we do here is then the P 1 is simply going to be g z 2 the right-hand side of the Bernoulli's equation the major loss F L by D there is going to be a minor loss at the inlet a minor loss at the exit the outlet and 3 90-degree elbows 1 2 and 3 the values of L e by D has been provided for those epsilons by D is 0.003 the kinematic viscosity is 1.5 into 1.05 into 10 to the power minus 6. So, you can calculate Reynolds number to calculate Reynolds number you figure out what is the velocity. Once you have the velocity then you find out what is the Reynolds number and from the Moody diagram for the value of epsilon by D provided to you should be able to read the value of the friction factor. So, this is what you are going to get and then the h in the minor loss due to the inlet is k in v square by 2 the minor loss due to the minor loss for the elbows are for each elbow is F L by D v square by L equivalent by D v square by 2 and the L exit minor loss over here is once again k exit v square by 2. So, this is your Moody diagram.

$$\frac{p_1}{\rho} = gz_2 + f\frac{L}{D}\frac{V^2}{2} + h_{INLET} + h_{EXIT} + 3h_{L,Elbows}$$

$$\frac{\epsilon}{D} = 0.0003, \qquad \gamma = 1.05\times10^{-6}m^2/s$$

$$Re = \frac{\rho VD}{\mu}, \qquad \mathrm{V} = \frac{Q}{A} = 2.17\ m/s$$

$$\mathrm{Re} = 3.1\times10^5, \quad \mathrm{f} = 0.017$$

$$h_{IN} = k_{IN}\frac{V^2}{2} \qquad h_{EL} = f\frac{L_{eq}}{D}\frac{V^2}{2} \qquad h_{EX} = k_{EX}\frac{V^2}{2}$$

So, for the for this specific case epsilon by D 0.003 and you are going to get the value of friction factor to be equal to 0.017 ok. So, that is that is what the Moody diagram tells you. So, read the Reynolds number over here go all the way up to the all the way up to the point where your epsilon D is 0.003. So, it is in between these two and you should be able to figure out for 3 into this over here between 0.002 0.002 and 0.004 it is going to be somewhere over here. So, that is that is 0.017. So, this is the point that you read from Moody diagram. So, from Reynolds number go all the way up then come to the right and read what is the value of the friction factor. So, this is how you calculate the friction factor and over here now you know what is z 2 what is the elevation the friction factor L is unknown, V is known, D is known, K in the contraction coefficient is known, velocity known 3 of the L both 3 of the 90 degree bends which are provided for which L equivalent is provided in the problem, F will be the same as the F that you have obtained because this is the friction this is the length of the straight pipe. So, the value of the friction factor will be the same V D are known, K exit the expansion coefficient is known.

$$\frac{p_1}{\rho} = gz_2 + f\frac{L}{D}\frac{V^2}{2} + k_{IN}\frac{V^2}{2} + 3f\frac{L_{eq}}{D}\frac{V^2}{2} + k_{EX}\frac{V^2}{2}$$

$$L = 212\ \text{m}$$

So, V is known everything here including p 1 is also known to you. So, the only unknown is length L which you should be able to evaluate and that is 212 meters. So, this is the example of type 2 problems in which p, q, D are known and L is unknown. So, we are going to solve problem types 3 and 4 and some more problems in the subsequent classes. Thank you.