Momentum Transfer in Fluids
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Lecture-46

In this lecture, we will see how the Bernoulli's equation in its modified form can be used in for various applications. So, the subsequent total of 6 lectures, I will show you how the equation can be used for engineering calculations. So, the first of that is for a simple situation, but before I get into that just a recap of quick recap of what we have covered in the past two classes about the fundamentals of Bernoulli's equation. What we know that the relevant equation in this case is the form of Bernoulli's equation, where an additional term h l t is added to make the Bernoulli's equation usable for real fluids, real situations which may include a change in the direction of the fluid, presence of a metering device, presence of a valve and so on in such cases. And we have also introduced the concept of a major loss, where major loss is f L by D v bar square that is we are talking about average velocity and this alpha 1 and alpha 2 are kinetic parameters, kinetic energy correction factors which we which also I have explained. For the case of turbulent flow, the value of alpha 1 and alpha 2 for highly turbulent fluids because of the nature of the flow because of the nature of the profile of the flow fluid flow, the value of alpha will be can be approximated to be equal to 1.

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + g Z_1\right) = \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + g Z_2\right) + h_{LT} \qquad (A)$$

$$h_L = f \frac{L}{D}\frac{\overline{V^2}}{2}, \quad \textit{major head loss}, \qquad (B)$$

So, under that conditions this equation will become even more simplified, these two alphas 1 and alpha 2 will not be present. The major loss h l t which is the sum of the major and the minor loss, the major loss is f L by D average velocity square by 2 and this f is known as the friction factor. And this friction factor can be evaluated using Moody diagram or for the case of laminar flow it is straightforward it is simply 64 by R e or we can have f from the Moody diagram, where the f the friction factor is plotted in a semi log plot with x axis being the Reynolds number. And there would be a family of curves for different values of epsilon by D, where epsilon is the pipe roughness which depends on the material of construction of the pipe.

f = 64/Re, for laminar flow

f from Moody diagram or

for smooth pipes for turbulent flow

And I have shown you how to calculate this epsilon by D for different material pipes and once I know the value of epsilon by D that is my right y axis of Moody diagram and Reynolds number which is the x axis of the Moody diagram, how can one calculate the friction factor f for the case of turbulent flow. And once I have the value of friction factor, the loss can simply be obtained the major loss can simply be obtained by a knowledge of the length of the pipe, the diameter of the pipe, the velocity and so on. Empirically purely empirically for a smooth pipe the value of f in turbulent flow is also expressed by this correlation, this is purely empirical. So, better way is to use a Moody diagram or for smooth pipes under turbulent flow the value of f can also be used in this way. So, this is all about the major losses.

How do I get the minor losses? The for the minor to evaluate the minor losses the h l m is k v square average velocity again by 2 due to fittings, bends sudden change of area and so on. And k is the loss coefficient which is once again experimentally evaluated and I have shown you the value of k for different situations. In some cases, to have parity with the evaluation method for major losses, the minor losses h l m is also expressed in the form of f L e by d v square by 2. Now look here that we have changed L to L e, this e stands for the equivalent length. So, for mostly for valves fittings and bends the value of the minor loss can be obtained f L by d where L e is the equivalent length of straight pipe.

$$h_{LM} = K\frac{\overline{V^2}}{2} \quad (\min or\ loss,\ fittings, bends, abrupt\ area\ change\ etc) \qquad (C1)$$

$$K = Loss\ coefficient\ (\exp erimentally\ \det er \min ed)$$

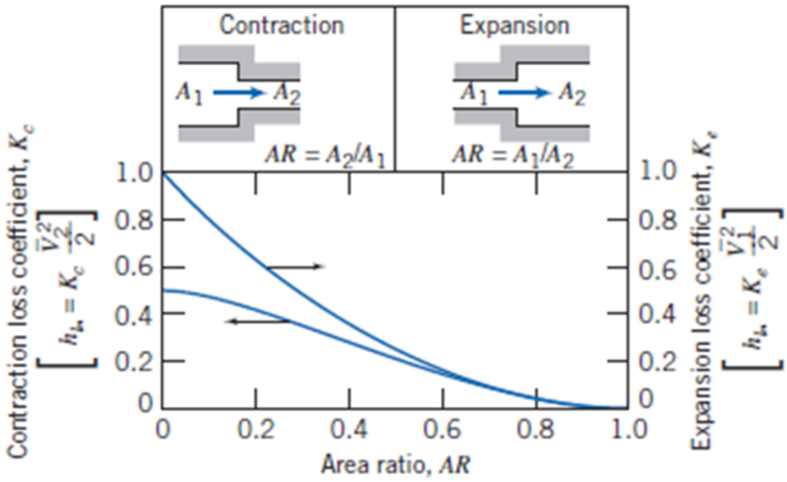
So, a valve can be replaced by some length of straight pipe and we call it the equivalent length when the pressure drop in that straight pipe is same as the pressure drop due to the valve in the path of the fluid flow. So, instead of the valve I add some more length of the straight pipe to the pipeline knowing that the pressure drops in the due to the valve and pressure drop due to the state portion state additional portion of the pipe that I have added to the pipeline are the same. If they are the same then this length is called the equivalent length for a specific type of valve, specific type of bend or something like that or any other such devices. So, L e is the equivalent length of a straight pipe. So, I can either go with k the loss coefficient or go with L e the equivalent length of the straight pipe.

$$h_{LM} = f\frac{L_e}{D}\frac{\overline{V^2}}{2} \quad mostly\ for\ valves\ fittings\ and\ bends \qquad (C2)$$

$$L_e\ Equivalent\ length\ of\ straight\ pipe$$

With that knowledge now we get into solving some problems which involves use of Bernoulli's equation and either the major or the minor losses are to be evaluated and then put into the Bernoulli's equation. So, what we get in here is a situation in which water at some condition flows through a circular duct that suddenly contracts the area suddenly reduces from a diameter of 50 millimetre to that of a 25 millimetre. So, there is half size reduction of the pipe through which the flow is taking place. It has also been reported that the measurements of pressure drop for such a sudden change sudden drop in the flow area is 0.347 meter of water.

What I need to find out is what is the volumetric flow rate of water through the pipe. So, the pressure drop is known in this case I need to figure out what is the velocity, what is the flow rate. To obtain the flow rate, I must find out the velocity. So, if I could relate pressure drop with velocity then the then the volumetric flow rate can be obtained for such a sudden change in area. And we must invoke here is this graph which tells us the value of the contraction coefficient because this is a case of contraction from a diameter of 50 to 25.



So, this is the contraction coefficient. The area ratio is going to be equal to A 2 by A 1 smaller by the larger. So, area ratio is going to be half or 0.5 since that is the that is the diameter of the two ones.

So, from 0.5 I go all the way up and find out what is the contraction coefficient that is how one should approach this problem. So, this graph then tells us what is the value of the contraction coefficient or conversely what is the value of the expansion coefficient for similar situation. So, this is the expansion one, but please note that in all cases the velocity to be used is always the velocity of the smaller area. Here also if you see how this is expressed it is v 1. So, it is the velocity of the smaller area.

So, in all such calculations the value of the head loss is k c the coefficient times the velocity of the smaller area square of that divided by 2. So, we will use this curve to find out what is the pressure, what is the unknown velocity and unknown flow rate for a fluid that is flowing through a 50 percent change in its in its in its in its diameter I mean diameter becomes half and where the pressure drop is known. So, we are going to first use the equation of continuity and the Bernoulli's equation and as we know that it is going to be steady incompressible flow that is uniform at each cross section. So, so that v bar essentially tells us the flow velocity which is constant which would happen only when the flow is turbulent. So, this is the equation the major loss the minor loss and k being the loss coefficient the major loss we are going to disregard the major loss.

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+gZ_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+gZ_2\right)+h_{LT} \qquad (A)$$

$$h_L = f\frac{L}{D}\frac{\overline{V^2}}{2}, \quad major\ head\ loss, \quad h_{LM} = K\frac{\overline{V^2}}{2} \quad (\min or\ loss)$$

$K = Loss\ coefficient$

Assumptions:
1. $Z_1 = Z_2$
2. $h_l = 0$

So, h L is 0 we are also going to say that the contraction the there is no change in elevation which is a reasonable assumption because it is just a contraction nothing else. So, there is no change in the elevation. So, if there are no changes in the elevation this z 1 is going to be equal to z 2 and there are no major head losses the principal pressure drop is due to the minor losses only. So, therefore, h L t will contain only h L m h L would be equal to 0. So, these are the two assumptions very reasonable assumptions which are which are considered for solving this problem.

From the figure $AR = A_2/A_1 = (D_1 / D_2)^2 = ¼ \Longrightarrow Kc = 0.4$

So, what we are going to do then is from the area ratio which is A 2 by A 1 and A 2 by A 1 is the area of the smaller divided by the area of the area of the smaller divided by the area of larger for the case of contraction and you could find out that this is equal to point equal to 1 by 4. So, 0.25 0.25 somewhere around here go all the way up and this is the value of k c is about 0.4. So, your area A r the area ratio gives you from the value of the contraction coefficient to be equal to 0.4. The next step comes is this is my this is my changed equation now that the pressure the velocity the pressure heads the velocity head pressure head the velocity head and k c v square by 2. Now, from the from the continuity equation the velocity at 1 and the velocity at 2 are connected by A 1 and A 2 from that A 1 A 1. So, this is what this whole area ratio is all about from the equation of continuity.

$$\left(\frac{p_1}{\rho} + \frac{\overline{V_1^2}}{2}\right) = \left(\frac{p_2}{\rho} + \frac{\overline{V_2^2}}{2}\right) + K_c \frac{\overline{V_2^2}}{2}$$

$$\frac{\overline{V_1^2}}{2} = \left(\frac{A_2}{A_1}\right)^2 \frac{\overline{V_2^2}}{2} = \frac{1}{(AR)^2} \frac{\overline{V_2^2}}{2}$$

$$\frac{p_1 - p_2}{\rho} = \frac{\overline{V_2^2}}{2}\left[1 + K_c - \frac{1}{(AR)^2}\right] = \frac{\overline{V_2^2}}{2}[1 + 0.4 - 0.06] = 1.34 \frac{\overline{V_2^2}}{2}$$

So, if you find out what is p 1 minus p 2 by rho it is simply going to be v 2 square this one you take that common and what you have 1 plus k c. So, 1 plus k c minus of v 1 square by 2 and what is v 1 square by 2 is 1 by A r whole square v 2 square by 2. So, the pressure change is equal to v 2 square 1 plus k c minus 1 by A r whole square I will go it one more time slowly this time. I write the Bernoulli's equation between the entry over here and inside the smaller pi as the flow goes from small from large to small pi there is no change in z. So, z containing term goes away this is too small a distance for frictional forces to become important.

So, h L the major loss will also not be present what we will have been only this term which is the minor loss due to the sudden change in area. And I have I have evaluated k c knowing A r area ratio. So, area ratio is diameter ratio square which gives me the area ratio is 0.25. So, from 0.25 going up for the up to the curve which contains the contraction I get the value of k c to be equal to 0.4. So, the v square and area they are related as so, A 1 v 1 is equal to A 2 v 2. So, the velocity squares are related to A 2 by A 1 whole square. So, it is 1 by A r the area ratio whole square v 2 square by 2.

So, v 1 square by 2 in this equation is replaced by this term over here. And then therefore, the pressure drops between this point and this point higher and the pressure drop between these two points is p 1 minus p 2 by rho and I bring this v 1 square which is this to the right-hand side and what I have is the first term v 2 square by 2 which is v 2 squares by 2 k c this term is this and minus v 1 square by 2 is minus 1 by r square 1 by A r square. So, this completes my Bernoulli's equation in terms of the pressure difference and in terms of the velocity. So, I have this finally, putting the value of k to be equal to 0.4 and the 1 by A r square which turns out to be 0.06. So, this is 1.34 v 2 square by 2. Remember that we the problem has already specified me what is the pressure drop in terms of meters of water. So, I do that find out v 2. So, v 2 is the from the from the previous from the previous one I figure out what is v 2 and v 2 is simply 2 p 1 minus p 2 by 1.34 rho and this is 0.347 meters of water. So, this is height h rho g and then the rest which we get once we put in the plug in the values and calculate the value of the velocity turns out to be 2.25 meter per second. Once again one should be careful while multiplying this v 2 with area to find out the flow rate. All calculations that we have done so far are based on the smaller area A 2.

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{1.34\rho}} = \left[\frac{2}{1.34} \times 0.347m \times 999\frac{Kg}{m^3} \times 9.81\frac{m}{s^2} \times \frac{1}{999}\frac{m^3}{Kg}\right]^{1/2}$$

$$V_2 = 2.25 m/s$$

$$Q = \overline{V_2}A_2 = \frac{\pi \overline{V_2} D_2^2}{4} = \frac{\pi}{4} \times 2.25 \times (0.025)^2\, m^3/s = 1.1 \times 10^{-3}\, m^3/s$$

So, that is why v 2 is multiplied by A 2 which is 0.025 25 millimetres. So, you put that value and figure out that this flow rate is about 1.1. So, in other words you can also think of this area construction to measure the flow rate.

Using a manometer, you can find out what is p 1 minus p 2 and then the relation is straightforward if you know the value of k c. So, from this relation knowing the value of k c therefore, knowing the value of the denominator and the knowing the geometry that means, A 1 by A 2 you have a direct relation between v 2 and the pressure drop. So, from the pressure drop you can find out what is v 2 once you have v 2 you can find out what is the volumetric flow rate. So, a sudden change in area can also be used to measure what is the pressure drop between two points. So, this is a nice example to start the application part of Bernoulli's equation, but this is this is this does not stop here.

I will move into the pipe flow problems which will constitute mostly which will constitute the more useful calculations, practical problems, the problems that we encounter every day and which requires not only the knowledge of Bernoulli's equation, but also the concept which we have derived before starting with the differential analysis and integral analysis as well. So, this is the culmination the example problems are going to be the culmination of whatever we have studied in this entire course. I first start with the Bernoulli's equation where there is a presence of a pump in the flow circuit because we all know that to have fluid move from one point to the another, we need to have a pump in the flow circuit ok. Be it the transportation of crude oil from one point to the other or just household water supply from one location to another location for agricultural purposes or water supply for crude oil transportation for any transportation of fluid that you can think of you have to have a pump. So, how do I incorporate the presence of the pump in the flow circuit? So, presence of a presence of a pump adds more head to the flowing fluid such that it can overcome the friction, it can overcome the elevation or it can overcome the bends, the valves and the other devices which are present in there.

$$\text{Head at 1} + \text{Pump Head} = \text{Head at 2} + \text{Losses}$$

$$\dot{W}_{in} = \dot{m}\left[\left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + g Z_2\right) + h_{LT} - \left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + g Z_1\right)\right]$$

All terms are energy per unit mass

So, whatever potential that I had on the left-hand side of the Bernoulli's equation which I am going to equate to the right-hand side of the Bernoulli's equation that is potential at 2 plus losses. The Bernoulli's equation that we have understood so far are the potentials at 1 is equal to the potentials at 2 plus all the losses that the fluid has encountered while it moves from 1 to 2. The moment I have another potential being added to the fluid that must come then to the left-hand side. So, whatever be the potential the pressure head, the velocity head, and the gravity head I must add to it the pump head which is being added which is adding energy to the right to the left-hand side of the Bernoulli's equation. On my right-hand side, it remains the same to a distant location what is the pressure head, the velocity head, and the gravity head.

The difference of head in between these two points is essentially the losses that the fluid encounters in its path. So, a pump head is to be added to the left-hand side of the Bernoulli's equation and that is why I wrote over here that head at 1 plus pump head is equal to head at 2 plus the losses which are present in the flow circuit. So, this is the in terms of the work whatever be the mass flow rate I multiply this the mass flow rate P 2 the pressure the sum of all this plus the losses minus the whatever be the head over here that is the work which is done by the pump on the fluid. So, this is what the W dot in is all about all terms are in energy per unit mass. So, the pump head what we call as pump head is W divided by m dot.

This is the definition of pump head its unit is in meter square per second and the power of the pump in watts is whatever be the mass flow rate multiplied by the pump head this is going to be in watts. So, if you are going to buy a pump from the market for transportation of some fluid from point A to point B, what you need to do is figure out what is the pump head from the calculation over here. How much of fluid you would like to transport per unit time? So, that is your row time skew the product is going to give you the power of the pump that you want. So, we are going to see how we can figure out the power of the pump for certain specific geometry of pipeline, what kind of elevation that it must encounter, what is the geometry of the path is it going to go straight one bend, second bend, third bend so on. How many fittings are to be added to hold the pipes because you are not going to get a pipe of 10 kilometre long, you are going to get smaller pipes which are to be which are to be attached to one another to make a long pipeline.

$$Pump\ Head = \frac{\dot{W}_{in}}{\dot{m}}\left(in\ \frac{m^2}{s^2}\right), \quad Power = \rho Q \times Pump\ Head, (W)$$

$$\Delta P = \phi\left(L, Q, D, e, \Delta Z, \text{system config.}, \rho, \mu\right)$$

All this would give you the row times the pump head and this pump head is to be multiplied by row q. So, there will be explanations for that. Now, what this delta p is going to be for the case of let us say turbulent flow, it is going to be a function of length, more the length higher is going to be the value of delta p. Q the flow rate higher the flow rate more will be the value of that pressure drop. Diameter of the pipe higher the diameter lesser will be the delta p requirement to make the fluid flow.

Epsilon the roughness higher the roughness I am talking about turbulent flow here, laminar flow it is not going to be not going to make any significant difference. Higher the value of reference higher will be the value of delta p, delta z the change in elevation if obviously, if the elevation is more, you will have higher value of delta p. The system configuration, how are you going to lay your pipeline between 0.1 and 0.2, how many change how many changes in directions you must incorporate to bypass certain other structures present in your chemical plant.

So, the system configuration therefore, plays an important role in evaluating the value of the delta p. And finally, what is the fluid that you are what is the fluid that you are you are going to transport, what is its density, what is its viscosity. So, your delta p requirement is a complex function of L Q D E delta z system configuration rho and u rho and mu the viscosity. So, this is what I have written down once again over here, the functional form phi of delta p in terms of the system parameters, in terms of the operational parameters, in terms of the fluid property. Of this once you layout the pipeline that means, once you know that you must take the fluid from point A to point B and you must follow the certain path, you have no other choice because of certain other limitations which are not in your hand.

$$\Delta P = \phi\left(L, Q, D, e, \Delta Z, \ system\ config., \rho, \mu\right)$$

Once the pipeline layout and the fluid properties are fixed

$$\Delta P = \phi\left(L, Q, D\right)$$

So, your layout is fixed. So, if your layout is fixed then system configuration goes out of this functional form. So, delta p is then a function of L Q D E delta z and since the pipeline layout is known you know that to which height you must pump the fluid that means, your delta z is also known to you. You also know what is the pipe that you are going to what is the material of construction of the pipe that you are going to use. So, your E is also fixed. You also know what is the fluid that you are going to pump from A to point A to point B that means, your rho and mu are also fixed.

If that is the case then essentially it boils down to delta p is a function of L Q and D. So, your pressure drops for a pipeline layout that is known to you and for a fluid that is known to you the fluid that you are going to pump delta p is a function of L Q and D the length the flow rate and the diameter. So, there are these there are 4 interconnect 4 variables which relate to each other and 3 of them could be known the fourth could be unknown and based on what are known and what is unknown there can be 4 types of problem in Bernoulli's application of Bernoulli’s equation in pipes. So, what are those 4? The first case is L Q and D are known. The length of the pipeline is known, the flow rate is known, the diameter of the pipe is known you must figure out what is the delta p necessary.

Possible cases

$$Case\ i)\quad L,Q,D\quad known\quad \Delta P\ \ unknown$$

$$Case\ ii)\quad \Delta P,Q,D\quad known\quad L\quad unknown$$

$$Case\ iii)\quad \Delta P,L,D\quad known\quad Q\quad unknown$$

$$Case\ iv)\quad \Delta P,L,Q\quad known\quad D\quad unknown$$

This is the simplest problem that you would encounter while calculating the pressure drop using Bernoulli's equation. Your length velocity sorry your length Q and D are known delta p are known. I said velocity is also known because if your Q and D are known to you then your Q is the volumetric flow rate. So, your volumetric flow rate is simply I will your volumetric flow rate Q is simply pi D square by 4 times v where v is the v is the velocity. So, this is in meter cube per second the flow rate the volumetric flow rate and the velocity are connected.

So, Q and D known means you know what is going to be the velocity. So, that is one part of thing which is also true for the second part of the problem. So, Q and D being known makes life simpler because now you know velocity and if you know velocity you know what is the value of the what is the value of the Reynolds number because Reynolds number is nothing but d v rho by mu. So, your D is known v is known mu and rho those are properties of the of the fluid that you are pumping.

So, they are known to you as well. So, for case 1 and case 2 in both cases the velocity is known which is which is not going to be which is not going to be the true for the which is not going to be true for the case 3 and case 4. What is case 2? Delta p Q and D are known Q and D are known you do not know what is the length of the pipe that is necessary for such a case. So, this is the second type of problem, but it is still not as it is still having an advantage because your velocity is already known since your Q and D are known to you. The situation becomes complicated when delta p L and D are known Q is unknown. Since Q is unknown Q is unknown you do not know what is the value of v.

If the value of v is not known then Reynolds number is not known to you. If Reynolds number is not known to you then the friction factor using Moody diagram will also be unknown to you. If the friction factor is unknown then you cannot find out what is delta p. So, there is a problem of with case 3 and you probably must use iterative method to solve this. I will give you I will show you through an example how it is done.

And the fourth one is even more difficult because in this case also Q and D both are not known. So, velocity is not known. So, you have this delta p L and Q known, but the diameter is unknown. We are going to solve each one of the problems each one of the types of the problems in subsequent classes. Also, we are going to see how can one size a pump figure out what is the wattage of the pump that is necessary to make a fluid flow.

But to summarize what we have done today we have understood that delta p is a function of L Q and D and of these 4 depending on which 3 are known to me the type the solution methodology could be different. So, that is why they are categorized into 4 cases case 1, 2, 3 and 4 and the first 2 because of the knowledge simultaneous knowledge of Q and D and therefore, that of velocity it is easier to proceed move forward whereas, for the 3 and 4 they are slightly more difficult and each one of them will be clarified to you through multiple examples solved in the class as well as problems on the sizing of the pumps in next lectures. Thank you.