Momentum Transfer in Fluids
Prof. Somenath Ganguly
Department of Chemical Engineering
IIT Kharagpur
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I welcome you to this lecture on momentum transfer in fluids. We are going to discuss something which goes by the name dimensional analysis and similitude. Dimensional analysis why it is important? You need to appreciate this is in fact, this is when it comes to this momentum transfer operation. This is an extremely important analysis. Though it appears very simple and very trivial, it has immense significance, immense importance in handling our data and our analysis process. First of all, think of a simple situation where, let us say, I have a flow through a pipe. I have a pipe and I have a flow through a pipe.

So, I measured the Δp across this, the fluid is flowing and I have velocity of the fluid, density of the fluid, viscosity of the fluid, diameter of this pipe, length of this pipe. Let us say I have some roughness here and then I call this roughness as let us say ϵ in and I put it is with a length unit the feature size of the roughness that means here I have a rough wall. So, what would be the height of that rough feature? So, I can have that as one parameter and we end up with seeing that this Δp which is the pressure drop across this pipe as the flow takes place Δp is a function of all these. So, then you have now if someone wants to study this someone does not want to go by the theory and someone wants to perform experiment take a pipe measure the flow rate and then say what would be the pressure drop they measure it and then from there they can come up with they want to come up with some empirical relation.

So, then what they need to do is they need to measure velocity, density, viscosity all these for a particular situation and get a Δp. But then we need to know how the pressure drop Δp depends on any change in velocity or any change in density. So, when we want to know this so, then one has to perform experiments where the density has to vary. So, they have to work with let us say 5 liquids of different densities and then same thing will happen with viscosity. So, they need fluids whose density will remain same, but 5 different viscosities not only that they will and they need to have pipes of different diameters different lengths.

So, you can see if someone wants to establish from experimental point of view how pressure Δp the pressure drop across the length of that pipe depends on these parameters. So, experimentally, if they want to come up with some information, they have to perform so many different experiments, and those are some of them are very difficult to achieve when we have density constant and viscosity varying to 5 different values. These are not that straightforward or you maintain the velocity to be same and increase the diameter things like that. So, this is if someone wants to do this that is that is very that is not very straightforward. Instead of that if someone comes with a relationship which gives you let us say the relationship is something like this

$$\frac{\Delta P}{\rho V^2} = f\left(\frac{\rho v D}{\mu}, \frac{L}{D}, \frac{\epsilon}{D}\right)$$

We note that this $\frac{\Delta P}{\rho V^2}$ this does not have any unit. If you check the unit of pressure is Pascal ρ has kg per meter cube v unit is meter per second and you put these you will find that this does not have any unit it is dimensionless. Similarly, this is not dimensionless this is not dimensional this is not dimensional this is not dimensional. So, there is no dimension attached to it what is the advantage? The advantage is that when you perform an experiment, let us say my aim is to find out how this dimensionless number depends on this dimensional number and these dimensionless numbers here. So, I work only with one density one viscosity of the fluid and then simply change the velocity by tweaking with the by tinkering the pump I change the velocity 5 different velocities.

So, if I work with 5 different velocities then I will have 5 different values of these number and I want to know how this number changes by this number. So, 5 different velocity means the I will get 5 different numbers. So, this number it has a name to it these dimensionless numbers typically they are given names this is known as Euler number this is known as Reynolds number like this. So, I will immediately see if I simply change velocity I can see how Euler number varies with Reynolds number and then tomorrow someone comes up with a different fluid whose density got changed. So, then they want to know what would be the impact.

I will simply ask them find out what is the Euler number and what is the Reynolds number and relation between Euler number and Reynolds number is already established by someone already who had changed the velocity and found a relationship. So, if someone agrees to work with this dimensionless form then it would be much easier much less experiments are required, but useful information can be obtained. Another point you need to note here is that if someone wants to do some empirical treatment here person has to do Δp is equal to let us say I will write a v to the power b ρ to the power c μ to the power d, d to the power e, l to the power f like this ϵ to the power g. So, you will have this is an empirical relation and then maybe someone has done really well in varying each of these parameters and then they found out what are the various exponents that satisfies the experimental data. Then the relation that they come up with these values of b, c, d, e, f, g etcetera these are this is first of all these equation is a dimensional equation.

So, these values say for example, when I say b the b is some number let us say experimentally someone found out it would be 0.398, but 0.398 that has some 0.398 is specific it would be 0.398 if Δp is expressed in so and so unit, v is expressed in so and so unit, ρ is expressed in so and so unit something I mean you have to go through that exercise.

Whereas, when you work with a dimensionless number since they do not have any dimension. So, what unit as long as you have the consistent unit they will cancel out. So, what unit, let us say if Euler number if someone comes up with Euler number is equal to the Reynolds number to the power a and some other L by D is equal to the power b, etcetera and some other, let us say q here outside some constant. So, if someone comes up with this type of a form here this a, b they are not dependent on the units because these numbers itself these parameters within the bracket they do not have any dimension. So, it does not matter here.

So, you can see there is an immense help provided when you work with dimensionless number. Now, the question is how will you find out I mean I told you that this is called Euler number or that is called Reynolds number that is there that is in the book that is already developed by researchers. How will I know that Δv ρ and v they will form a dimensionless number? How will I know some unknown set from this unknown from this set of v ρ μ D L ϵ and Δp what which one to combine and come up with what dimensionless number. So, there are certain rules specified and we are going to discuss this. In fact, that rule that we are going to discuss in this class today is known as Buckingham π theorem and we will discuss very briefly how Buckingham π theorem is supposed to work in this context.

So, what we have here is the force acting on a sphere is given as it is written as a function of diameter of the sphere, velocity of the fluid and density of the fluid and viscosity of the fluid. This force is referred as a drag force. You might have already heard of Stokes equation which is typically used for a for drag force. It is if a sphere is traveling through a fluid and if its velocity if spheres velocity is v in reference to the static fluid or the fluid velocity is v and in reference to a static sphere. Let us say it is traveling at a velocity v.

So, there would be a drag force acting in the other direction and Stokes law says that the drag force would be equal to 6 π μ r. Here we are writing in terms of d let us say then we change it to d 3 π μ d and then v. So, this is the drag force acting on this particle. Let me quickly see what the drag force is. Drag force is typically we have two types of drag forces we consider one is known as skin drag the other is known as form drag and this skin drag is when the surface is parallel to the flow.

Skin Drag

When the surface is parallel to the flow

$$F_D = \int_{surface} \tau_w \, dA$$

Form Drag

When the surface is normal to the flow

$$F_D = \int_{surface} p \, dA$$

That means, I have this is the object and surface is parallel to the flow and the flow is taking place. Whereas, form drag means when you have this is the object and you have this is the flow. So, when the surface is normal to the flow. So, here in this case when surface is parallel to the flow drag force is integration τ_w w dA. So, you pick up a differential element the shear stress that is acting on it take all such dA's and integrate it over the entire surface you get drag force.

Similarly, all such pressure forces because since it is normal pressure forces multiplied by dA integrate all such dA elements and then that you get the drag force which is the form drag. Generally, this drag force is for a more general terms Stokes law is for low velocity case, but for a general description of drag force something called a drag coefficient is defined. Drag coefficient is it is written as

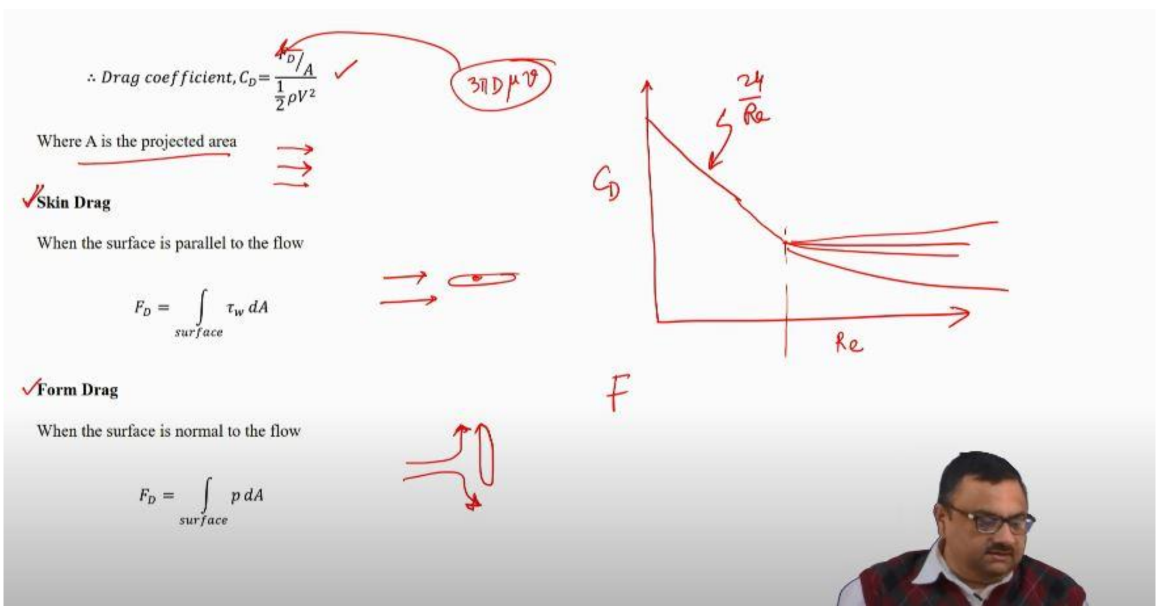
$$Drag\ coefficient,\ C_D = \frac{F_D/A}{\frac{1}{2}\rho V^2}$$

. Projected area means if we are talking about a sphere. So, the fluid sees as far as the fluid it sees the projected area of a sphere it appears to the fluid as a circle.

Similarly, if you have a cube if you are having a cube as the object and on which you are calculating the drag force the projected area would be simply square because that is the face the fluid sees. Similarly, if you have any other object that you have to find out the area that is called the projected area. So, here F_D divided by the projected area divided by half ρ v square. So, force per unit area and then that you are dividing by half ρ v square half ρ v square is typically the kinetic energy as far as that fluid flow is concerned. So, half ρ v square you divide and you call that the drag coefficient.

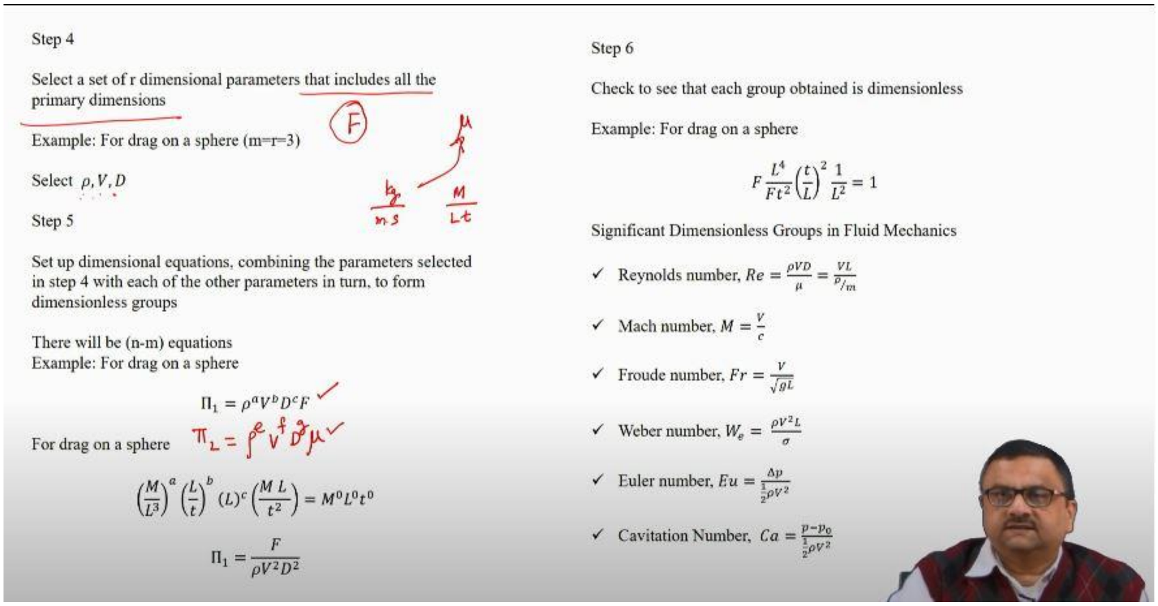
You can see here that if you if you put these F_D as 6 or F_D as 3 π D μ and v if you put that and then calculate this you will find the drag coefficient for the Stokes that is called

Stokes regime the low velocity regime where the Stokes law is valid that is called Stokes regime. And when you are working with a with a high velocity regime then you are outside the Stokes regime, but still you have a drag coefficient and drag coefficient will help you to find out what is the drag force. So, you have this drag coefficient if you plot the drag coefficient you will find a very standard literature you will having C_D versus the Reynolds number you will find that this C_D versus Reynolds number it decreases and then it takes different then it takes different lines then it takes different lines depending on what what is the nature of this object whether it is a cube whether it is a sphere whether it is of cylinder or some other form. For example, cylinder will have projected area as a rectangle. So, in this case you will find that this is this part the slope is $\frac{24}{Re}$ which is direct which will directly follow from here drag coefficient is equal to F D by A divided by half ρ v square you put this as F_D you will see that this drag coefficient would take the form $\frac{24}{Re}$.



So, this is valid, this is called Stokes regime. It is a straight line, and then there is a change in slope, and you will have different lines followed. This part is outside the Stokes regime. So, you have such a mastered plot C_D versus Reynolds number available and using this master plot any at any time you can find out if you know what your Reynolds number is Reynolds number here in this case you have to if you find that Reynolds number from you for your case you can find out what would be the drag coefficient acting drag coefficient applicable and from there you can find out what is the drag force. So, you can use this master plot master chart that is made already by the work from previous research and then from there you can find out what is the drag force. So, essentially this is some structure of drag force.

Now, here if we try to find out what would be the force acting or what is the drag force if we know that the drag force would be a function of diameter, velocity, density and viscosity. So, it is drag depends on these four parameters difficult to know how to set up experiments to determine dependence is difficult to know how to present results because you have if we try to present Δp as a function of so many you have to think of hyper surface and all these. So, presentation itself is also a major issue and then you, if someone comes up with this expression $\frac{F}{\rho V^2 D^2} = f\left(\frac{\rho v D}{\mu}\right)$, is a very neat expression. So, one can work with these dimensionless numbers and it would be much easier to set up experiments easy to present results this I have already discussed. Buckingham π theorem this is one way to find out what is the how this theorem works.



First thing what you need to do is we have these these are the parameters F, D, V, ρ , μ F is the force as far as this drag on a sphere is concerned F is the force D is the diameter of the sphere V is the velocity or the fluid ρ is density and μ is the viscosity. So, list all dimensional parameters. So, I have listed these are there are five such parameters. Let n be the number of parameters n is equal to 5 in this case. Step 2 is select a set of fundamental dimensions I have to pick up what all fundamental dimensions are involved and more or less in every case it is MLT mass length and time and that is how we have defined here.

You may have some unique cases where other dimensions come in, but for the time being, the examples that I will be working on are mass length and time. So, now, what you do is list dimensions of all parameters in terms of primary dimensions. So, F is ML by T square mass length by time square force can be written velocity is length per time diameter the unit is length. So, we write this density is kg per meter cube mass per length cube. So, we wrote in terms of these primary dimensions r is the number of primary dimensions.

So, r is equal to 3 in our case. So, now, we have here a set of r dimensional parameters that includes all the primary dimensions. So, I have to pick up a set of r dimensional parameters. What all dimensional parameters we have? We have force, we have diameter, we have the velocity, we have the density, we have the viscosity, we have the viscosity. We had how many parameters? 5 parameters F D v ρ μ.

So, F D v ρ μ. So, here out of these 5 I have to pick up 3 dimensional parameters which are going to be so called recurring set that is that is what is important. Recurring set means those variables I expect them to recur in let us say I have more number of dimensionless groups right I said Euler number I said Reynolds number. So, some parameter is recurring. For example, the parameter v is recurring here v is here v as well v is in Euler number v is in Reynolds number. So, they are recurring, but the pressure Δp is not recurring Δp is fixed in one place.

So, I have to pick up what are the parameters which has good chance of recurring and that is that is that is here in this case we choose ρ v and D these are the 3 parameters we have to choose. Now, there are certain rules to it when we choose these parameters. One is that these parameters within themselves should not form a dimensionless number. So, within themselves I mean ρ by v or ρ v by D we should not be able to form a dimensionless number that is one condition. Second condition is these parameters should be simple in the sense if we for example, D what is the unit just length diameter of the sphere.

So, it has a length unit v length per time ρ it is kg per meter cube. So, M per L cube why not viscosity because viscosity think of it viscosity is kg per meter second that is the unit. So, we have mass per length time. So, all 3 dimensions are involved here, and I do not expect such a bulky group such a bulky parameter to recur. If such a bulky parameter recurs, that is not advantageous to me. So, I do not want this bulky parameter to recur, which is not advantageous. I prefer D to recur. d has just a length unit. Simple, it is easy to recur.

The other thing is something which I what I want to know force is something I want to know. So, when I write this expression I prefer a dimensionless number consisting of f to be on the left hand side. So, that I whenever I talk about that dimensionless number I know that dimensionless number represents the force. Now, if the force term recurs everywhere and you your objective is to find out the force because force is your net outcome that is what you are looking for. Then if the force appears as a recurring set in every dimensionless number then it you form a very implicit relationship which is difficult to handle.

So, here I do not want the force to be or the parameter which I want to find out which I want on the left hand side in the in that expression. So, that I do not want this to want to be want to recur. So, these are these are some of the relations and the 3 that you choose these 3 within themselves they should not form a dimensionless number, but within themselves they should cover all the primary dimensions. That means, that is what I it is written here select a set of r dimensional parameters that includes all the primary dimensions.

I ensure here ρ is kg per meter cube. So, M per L cube, V is L per t and D is L. So, M is taken care of L is taken care of T is taken care of. So, all mass length and time they are taken care of. So, that is how we approach this dimensionless, making this dimensionless.

So, we picked up these variables and then what we do is we try to we find we write π_1 that is the dimensionless number here as $\Pi_1 = \rho^a V^b D^c F$.

Because if I pick up these 3. So, I have removed ρ, I have removed V, and I have removed this one. So, these 3 are already the recurring set. So, I am left with only the F and μ. So, now, I will write 2 dimensionless numbers in terms of F and μ. So, I will write

$$\Pi_1 = \rho^a V^b D^c F$$.

And similarly, I will write $\Pi_2 = \rho^e V^f D^g \mu$

So, we have now then what you have to find out is what are these a, b, c and e, f, g such that Π_1 and Π_2 they are dimensionless. So, this ρ V, D they are recorded with some exponent, and here in this case, I have F, and here in this case, I have μ, and then this equation has to be solved. So, from here I will get what is a, b, c and here I will get e, f, g. So, from there I can find out what are the dimensionless numbers involved.

I will continue this exercise in the next class. I will complete this exercise and I will show what are the implications of these dimensionless number as I proceed. That is all as far as this lecture module is concerned. Thank you for your attention.