Momentum Transfer in Fluids
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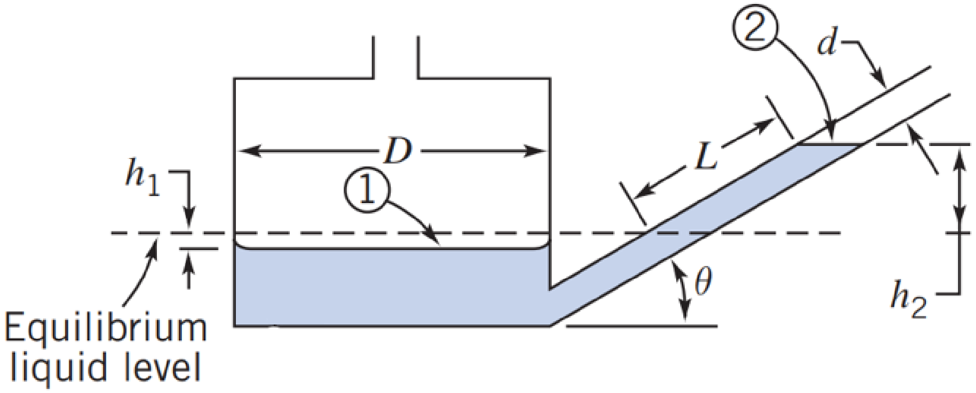
Welcome to this lecture on Momentum Transfer in Fluids. We have been discussing fluid statics at the end of the last class, and I will continue that exercise. We have discussed about manometer. We said that manometer gives us an idea how what the pressure drop is between two points. So, there is a change in height change in the level of the fluid in one of the arms of the manometer, and from there, you can find out what is the pressure drop.

Now, one concern that I raised at the end of the last lecture module is that when the pressure that we measure is very small, then how do we approach the problem if the height itself has to be, there has to be a sufficiently sufficient height difference. So, that when we make an high estimation, any that should not cause much of percentage error in that it has to be sufficiently accurate. So, we said that density of the manometric fluid can be decreased and that way it would be the height would be height change would be more. In this that is one way of doing it there is one instrument which is made which is which goes by the name inclined tube manometer where you have the where this certain changes were made specifically to capture small pressure drop.



So, here this is the device you can see. In this device here you can see that what is our traditional device how it is it is like this. So, we have this is the manometric fluid and we see this side the pressure is more. So, that is why the fluid has gone up on the other side. So, what we do in this case is you must note I mean this we should parallely see then only we can appreciate I have an equilibrium height.

So, when the pressure on both sides are same then the fluid level was at the equilibrium level. So, this is the equilibrium level. Now since the pressure is more on the left side. So, that is why the fluid level has gone up here. So, I can see that this is the level by which it has this is the height by which this liquid level on the right-hand side has gone up.

Here the changes were made is this limb is made of a larger diameter. So, I can see the difference earlier the manometer had the same diameter same cross section. Here the diameter is still D, but here it is capital D. Here it is small d lowercase d and it is uppercase D that is one thing. Second thing is there is an inclination that is why it is made inclined tube that is why it is referred as inclined tube manometer the right limb is at an angle.

So, you can expect before that let us say here manometer we always refer in terms of what is the height difference between left to right. However, we do not need to measure this in reference to the lower to the upper I mean that is not required. Similarly, we should measure the change of one of the limbs in reference to the equilibrium level, and if we get that information, we know that whatever rise is, there would be a similar fall on this side. And if you see, in this case, the cross-sectional areas are the same. So, whatever rise has taken place it would be the same.

If the cross sectional areas are different. So, whatever volumetric rise has taken place here, it has to be the same volume has to be depressed on this side; otherwise, the volumetric balance is not satisfied. So, we do not need both this information. So, since this side is much larger you will see that the depression of the level that happens here is very minuscule, but we do not bother to measure that instead we bothered we what we bother is we bother to measure this side and in reference to the equilibrium level how far the right side has right level has shifted and from there by volumetric balance I can find out how much was the depression on this side when it comes to that. So, that is what is utilized in this case.

So, what is the advantage we get here? One is first of all to do this volumetric balance I must use this is the volumetric balance.

$$\frac{\pi D^2}{4}h_1 = \frac{\pi d^2}{4}L$$

What we say here is if there is the L is the progression of this right level capital L. So, the volume we have assumed that this would be $\frac{\pi d^2}{4}$, d is the diameter here. So, $\frac{\pi d^2}{4}$ is the area multiplied by this length. So, that is $\frac{\pi d^2}{4}$ into L this is the volume and that has to be equal to $\frac{\pi D^2}{4}h_1$.

So, immediately you can see that from here you get $h_1 = L\left(\frac{d}{D}\right)^2$. So, h1 I do not need to measure as long as I measured the L and I have a priori information the what the diameters of these two limbs of this manometer I can immediately get the h1. And then if we try to find out what is Δ p, Δ p is essentially if I assume that this density of the fluid is if we ignore typically this density the low pressure when inclined tube manometer is used to measure very low pressure ultra-low pressure and low pressure arises when we have a gas as a process fluid because if water is there as a process fluid typically you measured that Δ p would be substantial you do not need to use inclined tube manometer. So, most likely the fluid that you are using here the process fluid for which you are measuring this pressure difference is a gas whose density would be several orders of magnitude less compared to even if you use oil or water forget about mercury we are not going to use mercury, but if we use oil even still several orders less. So, we can ignore that I mean we said that ρ m minus ρ f in my earlier lecture we said that it would be h into difference in density into g.

So, here in this case the ρ f the process fluid density would be several orders less. So, we can ignore that for the time being. So, what you have here in this case is Δ p would be on one hand there is a depression from the equilibrium level and here the from the equilibrium level there is this rise of h2. So, h1 is the depression here and h1 is the elevation or the rise of liquid level. So, if when it comes to Δ p, that is the pressure difference, it would be the density. Let us say the density is ρ 1 into g into this height h2, and what is h2? h2 is if this angle is θ then h2 would be equal to L sin θ.

$$\Rightarrow \Delta p = \rho_1 g\left[\mathrm{L}\sin\theta + L\left(\frac{d}{D}\right)^2\right]$$

$$\Rightarrow \Delta p = \rho_1 g L\left[\sin\theta + \left(\frac{d}{D}\right)^2\right]$$

$$\Rightarrow L = \frac{\Delta p}{\rho_1 g\left[\sin\theta + \left(\frac{d}{D}\right)^2\right]}$$

Why I am concerned with L? Because I said that I want L to be substantially large so that my percentage error in while I do I estimation of the liquid level would be less. So, when it comes to comparing this when it comes to see how effective this inclined tube manometer. So, there we have to find out, had we measured that same Δ p using a regular manometer, what would be this height? Let us say this what would be this order of magnitude of this change in liquid level as against what is the change in liquid level I see here.

So, if we compare the two because if we see that this L is 100 times multiplied by this then I know for sure that this is going to help me this inclined tube manometer this configuration is going to help me. So, what I note here is if I use density as, let us say, water as the manometric fluid here in this case, and so, in this case, this height difference here the total height difference would be total height difference $h = \frac{\Delta p}{\rho_{water} g}$. So, now, here in this case I have a length scale which is L to work with I have L to work with and here I have small h to work with when we compare these two manometers and I call that to be the sensitivity S as

$$s = \frac{L}{h} = \frac{1}{\eta_l \left[\sin\theta + \left(\frac{d}{D}\right)^2 \right]}$$

$$\left(\therefore specific\ gravity,\ \eta_l = \frac{\rho_l}{\rho_{water}} \right)$$

where L is the fluctuation liquid level in case of inclined tube manometer and here it is for the regular manometer. Of course, here the h is measured from left the difference between the left and the right limb and here the difference is in reference to the equilibrium level.

So, it would if I see this L by h we see that the Δ p will cancel out and this is the expression we end up with. So, we want this sensitivity to be high that means, L should be much greater the sensitivity of 100 let us say I talk about. So, that means, whatever I measured, let us say I measure by regular manometer, some height difference of 1 millimeter comes in. So, here, in this case, if the S value is 100, that means the L will reflect in the inclined tube manometer, and the same pressure difference will be reflected as 100 millimeters. So, in other words, 10 centimeters, which is a substantial height difference, and the percentage error would be much less.

So, when the sensitivity would be more in this case this is η_l is the specific gravity that is what we said here. Now, the specific gravity so, you want to choose sensitivity to be high. So, sensitivity to be high means the density of liquid that you choose density of density what is talking about is ρ 1. So, this has to be ρ 1. So, density that you choose has to be less.

So, that this specific gravity is less then the sensitivity would be more. Similarly, I want sin θ to be less and $\left(\frac{d}{D}\right)^2$ to be less. So, what that means is I want θ to be less as less as possible and small d to be as less as possible capital D to be as large as possible then only I will have sensitivity has increased. So, what that means is inclined I mean if I had this manometer perpendicular going up in that case θ would have approached π by 2 whereas, if it is completely flat θ equal to 0. So, ideally that that is what we want, but if the more you decrease the θ it will be difficult to measure the liquid level.

So, θ there has to be some critical θ you have to maintain to measure to see the liquid level properly. That is so, you want sin θ π by 2 instead of that if you if you make it π by 4 that would be helpful that that will change the that will improve the sensitivity and by putting two different diameters and the way it is right less diameter and the left side more diameter it is going to improve the sensitivity. So, that is the purpose of using this inclined tube manometer. Then next thing what I am going to talk about is the pressure variation in a static fluid. We said that $dp = -\rho g\, dz$ that is that is something which is which we have we have talked about I mean we said that if ρ is constant then we do the integration of dP and then ρ remains outside and ρ g remains outside and then it would be simply the integration dz from there we got Δ P is equal to h ρ g equation.

Now, the situation may come where density is varying with z maybe the fluid is compressible simply the good example is air atmosphere how density varies from ground level to as you go up. Even if you do not bother about atmosphere say you are drawing some natural gas from an underground reservoir. So, there along the height the pressure varies. In fact, at the up down there I mean when at the well bore you will see that the pressure will be much higher. So, as you go up to the ground level the pressure would be much less.

So, it is a natural gas. So, this is a compressible fluid. So, you can expect the ρ to vary. So, in this case if you want to find out what would be the pressure drop I mean you cannot use h ρ g in that case. So, the one way of working is if you assume the gas to be ideal then you can use P M/RT or if it is if it is non ideal you bring in z RT I mean instead of P M is equal to n r P v is equal to n RT you bring incompressibility factor and other things. So, you those are to be incorporated in that expression.

$$p\, M = \rho RT$$

Now, for the simple case where ρ from ideal gas is P M / RT. So, if you put this so, then this ρ cannot be the ρ has to be inside ρ cannot be outside ρ has to go inside and then this ρ has to be or ρ will not go first of all this will not be what you will do is dP. $dP = -\frac{PM}{RT} g\, dz$. Now, what will happen to T? I mean T can be let us say I have an isothermal system the entire along the entire height T is constant though for atmosphere it is not the case as one goes up the temperature also changes, but if by some means you hold the entire height to be of same temperature in that case you this T will remain constant in that case you will have $\frac{dP}{P} = -\frac{Mg}{RT}dz$. Then you will have simply if you do the integration it would be ln P is equal to $= -\frac{Mg}{RT}z$ plus some constant of integration. So, you can have this form of this form of equation.

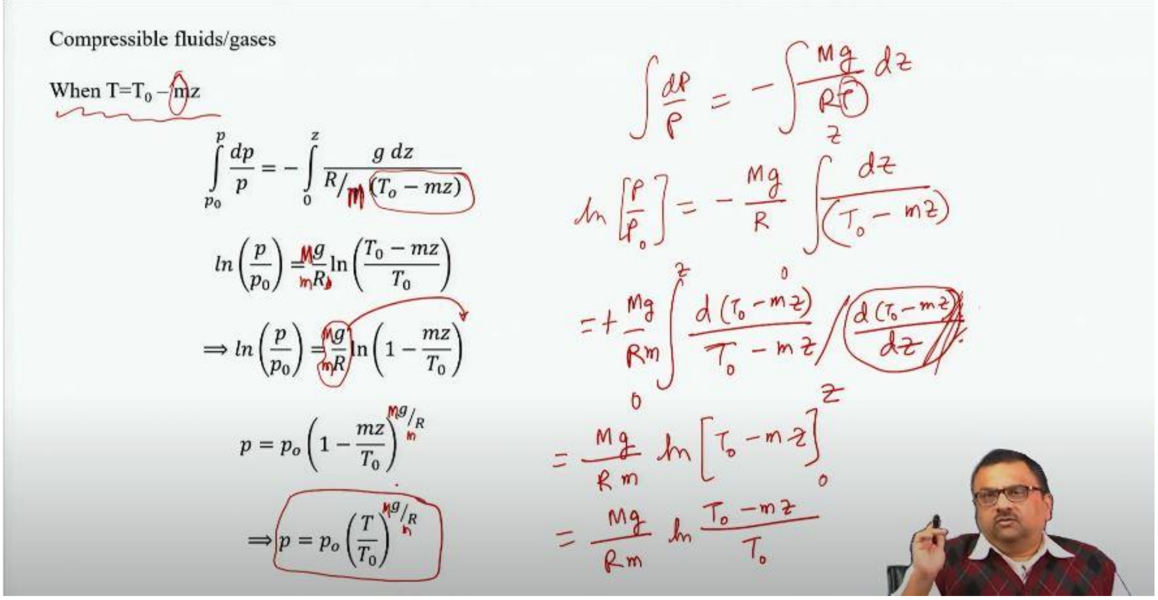
So, you can see here P will take in if you if you go by these P will take a form of P naught e to the power minus mg by RT z. So, then e to the power minus minus some expression into z. So, you will have this type of expression coming in instead of instead of Δ P is equal to h ρ g when you have a compressible system there. Now, as I said the temperature is not temperature as constant that is commonly not the case. So, temperature has to vary to simplify this one can assume for in fact, for atmosphere if you look at these measurements have been done and one found that as one goes up as the z increases temperature decreases. $T(z)=T_0 - mz$

T_0 is the temperature at the ground level and as one goes up to a higher level the temperature decreases. So, T is equal to T naught minus m z m is some factor m will have a unit of let us say Kelvin this T when I write in capital T these T that I mentioned here these T is in Kelvin. So, if you are talking about ambient temperature 28 degree centigrade ideally I have to put 298 Kelvin and R is universal gas constant 8.314 joule per gram mole or you can write it as mole Kelvin. So, that is how and M is the molecular weight in kg per mole.

So, when it comes to let us say I am talking about air let us say I come up with I take as 28 and 32 are the two molecular weights nitrogen and oxygen and let us say the average molecular weight comes to 29 I mean I assume it is not exactly 29 it is something somewhere around that value. So, in that case this is in gram mind it. So, so you have to write it as 0.029 kg per mole. So, that would be the molecular weight of air that is that is the molecular weight to be taken I mean similar sort of unit conversion has to be done.

So, in some of the texts you will find instead of universal gas constant there is it is simply written gas constant and in that case they write an expression ρ is equal to P by RT. These R is not universal gas constant that is simply gas constant. So, that gas constant it has to vary from gas to gas. So, that is for air you have a value of gas constant for nitrogen you have for so, depending on different value different gases. So, for air that gas constant has to be given to you.

So, mind it if you see this be careful that this is the gas constant that is how it is referred here we are talking about universal gas constant. So, that is what it is now m is m will have a unit of Kelvin per meter I would say then the Kelvin per meter and meter will cancel out. So, that Kelvin per meter how the earth's atmosphere how the in earth's atmosphere how the temperature changes from ground level as you go up that is that is that has been experimentally measured and m the information m will be given to you. So, if you have such a situation then when you do the integration one has to do this you have to do



Compressible fluids/gases

When $T = T_0 - mz$

$$\int_{p_0}^{p} \frac{dp}{p} = -\int_0^z \frac{g\,dz}{R/_M (T_o - mz)}$$

$$ln\left(\frac{p}{p_0}\right) = \frac{Mg}{mR}\ln\left(\frac{T_0 - mz}{T_0}\right)$$

$$\Rightarrow ln\left(\frac{p}{p_0}\right) = \frac{Mg}{mR}\ln\left(1 - \frac{mz}{T_0}\right)$$

$$p = p_o\left(1 - \frac{mz}{T_0}\right)^{Mg/_{R m}}$$

$$\Rightarrow \boxed{p = p_o\left(\frac{T}{T_0}\right)^{Mg/_{R m}}}$$

$$\int \frac{dP}{P} = -\int \frac{Mg}{RT}\,dz$$

$$\ln\left[\frac{P}{P_0}\right] = -\frac{Mg}{R}\int_0^z \frac{dz}{(T_0 - mz)}$$

$$= +\frac{Mg}{Rm}\int_0^z \frac{d(T_0 - mz)}{T_0 - mz} \Big/ \cancel{\frac{d(T_0 - mz)}{dz}}$$

$$= \frac{Mg}{Rm}\ln\left[T_0 - mz\right]_0^z$$

$$= \frac{Mg}{Rm}\ln\frac{T_0 - mz}{T_0}$$

So, this is the expression that you have for pressure when. So, in an atmosphere if you one wants to measure one wants to estimate the pressure how pressure changes with height and if this information is provided to you what is t is how the temperature changes with height and that factor is small m. And capital M here in the numerator that is the molecular weight of air which is say 0.029 kg per mole and R is universal gas constant takes the value of 8.314 R here and g is 9.8 meter per second square t is. So, at a particular height here this is irrespective of the height at a particular height temperature is so and so Kelvin and at the ground level temperature is t 0 so and Kelvin from here at the ground level pressure is one atmosphere. That means, 10^5 let us say simply in approximate terms 10^5 Pascal. So, what would be the new p at a particular height? So, it is not expressed in terms of height is expressed in terms of temperature. So, these are some of the exercises one would do with the compressible gas compressible when density varies one can work with this type of exercise one can undertake to come up with the pressure and utilize pressure temperature relationship. You can have compressibility factor you can have other factors built in.

In fact, one may come one may say that it is not it is held adiabatically or then you bring in PV to the power gamma as constant and then bring in those concepts of physical chemistry to relate pressure with density as a function of pressure and temperature and apply it there. So, that is how we perform this exercise. I will continue with some other aspects of fluid statics in the next lecture. However, the essentially the basics of fluid statics I have already talked about. In the next lecture, we will focus more on submerged objects and forces acting on a submerged object, etcetera.

That is all I have as far as the present lecture module is concerned. Thank you very much for your attention.