Momentum Transfer in Fluids
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Lecture-40

So, in the previous class we have seen how Bernoulli's equation can be applied in real situations provided we add a loss term to the right. And the loss could be due to frictional losses or the loss that is encountered by a flowing fluid when it encounters a sudden change in area, when the fluid is made to flow in a different path or when it passes through flow measuring devices valves etcetera. So, there are two types of losses as I have mentioned in the previous class that one is the total loss the left-hand side is h l t where h l the t refers to the t over here refers to the total loss head loss the l stands for the loss h for head. So, total head loss is the sum of a major loss which is h l and a minor loss which is h l m. I have briefly explained what is and once again the textbook to be followed is Fox and McDonald. Now, the h l is the major loss due to frictional effects in fully developed flow whereas, h l m is the minor loss due to the various factors due to the various instruments that are used for flow to find out what is the flow rate there could be fittings in the path and there can be area changes and so on.

$$h_{LT} = h_L + h_{LM}$$

So, all those are collectively called as the minor losses. And once again it does not mean that since the word major appears in h l that h l is always going to be more than h l m. There are situations in which the minor losses could be more than the major losses and the purpose of this class is to figure out how do we evaluate how can we evaluate h l and h l m. There are certain kind of empiricisms involved in a figuring out what is h l m and h l, but they are based on many experiments they are quite safe to use and are very common in all engineering calculations involving flow of fluid.

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + gZ_1\right) = \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + gZ_2\right) + h_{LT}$$

$$\frac{p_1 - p_2}{\rho} = \frac{\Delta P}{\rho} = h_L$$

So, let us talk about the major loss first. Now, once we talk about flow fully developed flow in a horizontal pipe of constant cross section what a one can get from Bernoulli's equation which is this and over here, we understand that it is a it is a pipe of constant cross section. So, therefore, there is no question of fitting entrance area change etcetera and the minor loss therefore, is 0. So, h l t the total head loss is essentially due to the major loss only because I have not considered any change in the flow direction, any change in the flow path presence or any area change etcetera. So, h l t essentially in this equation is h l only and it is a horizontal pipe.

So, therefore, z 1 and z 2 are the same. So, these two terms will not will cancel out each other and since it is constant cross section. So, therefore, v the velocity will remain unchanged and therefore, v 2 will be equal to v 1. So, the second term from both sides will cancel each other out, the third terms will also cancel each other each other since they are they are they are horizontal and constant cross section. And we understand that h l t is simply equals h l which gives us the definition or the way to calculate the major loss.

It is essentially the pressure difference between two points 1 and 2 divided by rho. So, which is delta p by rho and this is known as the major loss. So, major loss is the pressure drop between two points when there is no elevation change and no change in the flow area. Whatever pressure drop that we get is due to frictional losses only and they are denoted by the h l the major loss. So, that is the definition of major loss in this case.

$$\Delta P = \frac{128\mu L Q}{\pi D^4} = \frac{128\mu L V\left(\pi D^2 / 4\right)}{\pi D^4} = 32\frac{L}{D}\frac{\mu V}{D}$$

$$h_L = \frac{\Delta P}{\rho} = \frac{64}{\text{Re}}\frac{L}{D}\frac{\overline{V^2}}{2} = f\,\frac{L}{D}\frac{\overline{V^2}}{2}$$

f ≡ Friction factor

Now, what is going to be the pressure drop in such a case from our Hagen-Poise equation the first part of the course in which we have we are trying to calculate what is going to be the pressure drop for such a situation, what is the velocity profile using Navier-Stokes equation in a circular pipe in where the flow is taking place because of pressure difference only. There we have seen the Hagen-Poise equation which relates the flow rate q with the length, the diameter and the pressure drop. So, the pressure drops and q the relation between the pressure drops and the volumetric flow rate in terms of the parameters the geometric parameters length of the pipe and the diameter of the pipe and the relevant property which is viscosity this is the Hagen-Poise equation which we have derived in the first part very first part of the course. So, if we do just little bit of rearrangement what we would get over here is that this is essentially delta p is 32 L by d mu v by d ok. So, h L so, this is this is the pressure drop h L the major loss is delta p by rho.

So, if I bring the delta p by rho over here and express in terms of Reynolds number this is going to be the form of this. So, I have introduced the V bar square by 2 to to highlight that this is the this is the kinetic head and what you have then is 64 by R e L by d v square by 2. So, this is f L by d v square by 2 this f which is equal to 64 by R e in the k in laminar flow is known as the friction factor. This has been introduced to underscore that with increase in the value of Reynolds number the value of the friction is going to be different. We all understand that to make the fluid flow between two points at higher flow rates essentially requires higher pressure drop.

So, this friction factor is going to be a function of the Reynolds number. It is also going to be a function of the velocity as I mentioned with which it flows it is going to be a function of length to which length you are going to make the fluid flow. So, larger the length higher is going to be the delta p the pressure drop required and higher is going to be the losses. Smaller the value of diameter you are making the fluid flow through a wider a larger pipe then your pressure difference in that case would be is going to be smaller and consequently your losses the h L would also be small. So, this f friction factor if I could find this f friction factor for the case of laminar flow is straightforward it simply going to be 64 by Reynolds number.

$$\Delta P = \Delta P\left(D, L, e, \overline{V}, \rho, \mu\right)$$

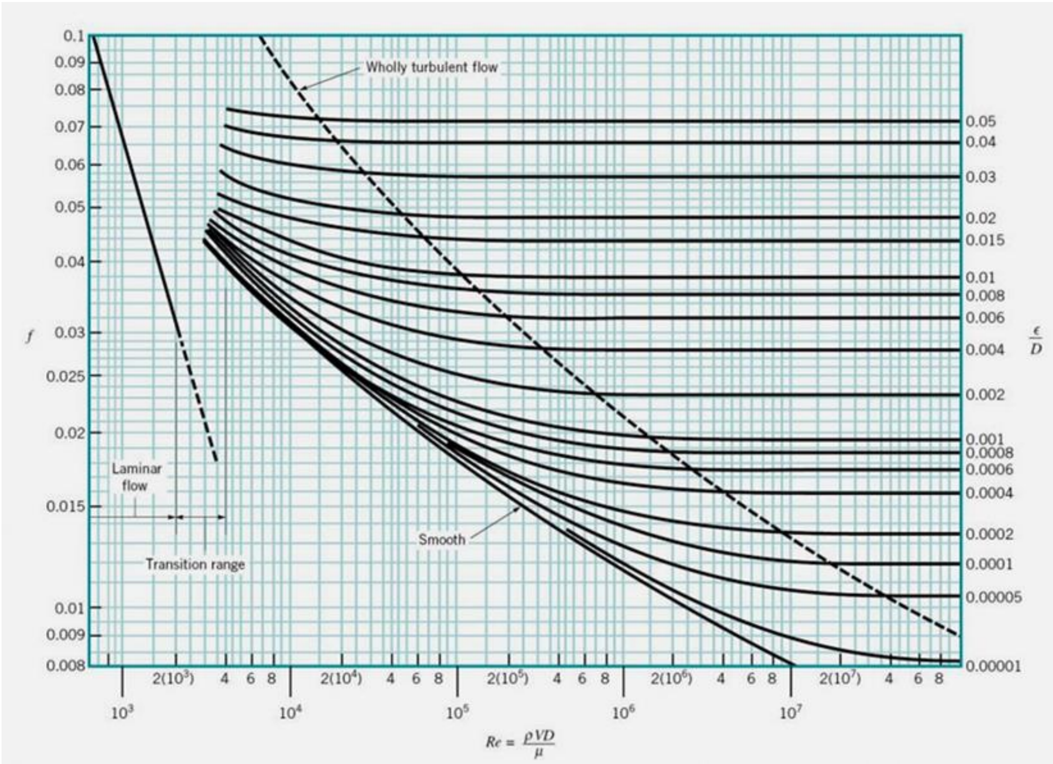
$$h_L = f\frac{L}{D}\frac{\overline{V^2}}{2}$$

f ≡ Friction factor, determined experimentally

Now, in turbulent flow there exists no such simple relation between delta p and the other parameters, but we realize that for the case of turbulent flow the delta p the pressure drop is going to be a function of various things. So, what those parameters are the firstly we realize that the delta p the pressure drop is going to be a function of diameter. It is going to be a function of diameter more the diameter less is going to be the pressure drop. It is going to be a function of length more the length more the pressure drops. I will skip e for the time being higher the velocity higher would be the pressure drop.

The density and viscosity if they are more you would require more pressure more pressure drop to make a viscous fluid flow from one point to the other, but there exists something else as well. So, what is that if you think about a normal pipe have you ever seen the inside of a pipe that is being used for some time there are going to be formation of scales there is going to be you could see that the inside of the pipe is no longer smooth as compared to a new pipe which is being installed. So, every pipe has certain roughness associated with it over prolonged use. So, more the roughness of the pipe or rougher the pipe the higher will be the pressure drop. So, epsilon is essentially the kind of imperfections the kind of roughness is which you would see in at the inside wall of the pipe and therefore, your delta p the pressure drop is going to be a function of the roughness of the pipe as well.

So, delta p is a complex function of d, L, E, V, rho and mu. So, h L the major loss is F L by d v square by 2. So, value F is the friction factor and for the case of turbulent flow the friction factor F is evaluated experimentally and of course, for the same pipe the roughness could be different I mean for different materials the roughness could be different and it is whatever pressure whatever value of F that we get is going to be a function of the roughness as well. So, the important point in turbulent flow is the value of the friction factor is needed in order to calculate the value of the h and subsequently the value of the pressure drop. So, how do we do that? And here comes one of the very famous diagrams of fluid mechanics which is called Moody's diagram.

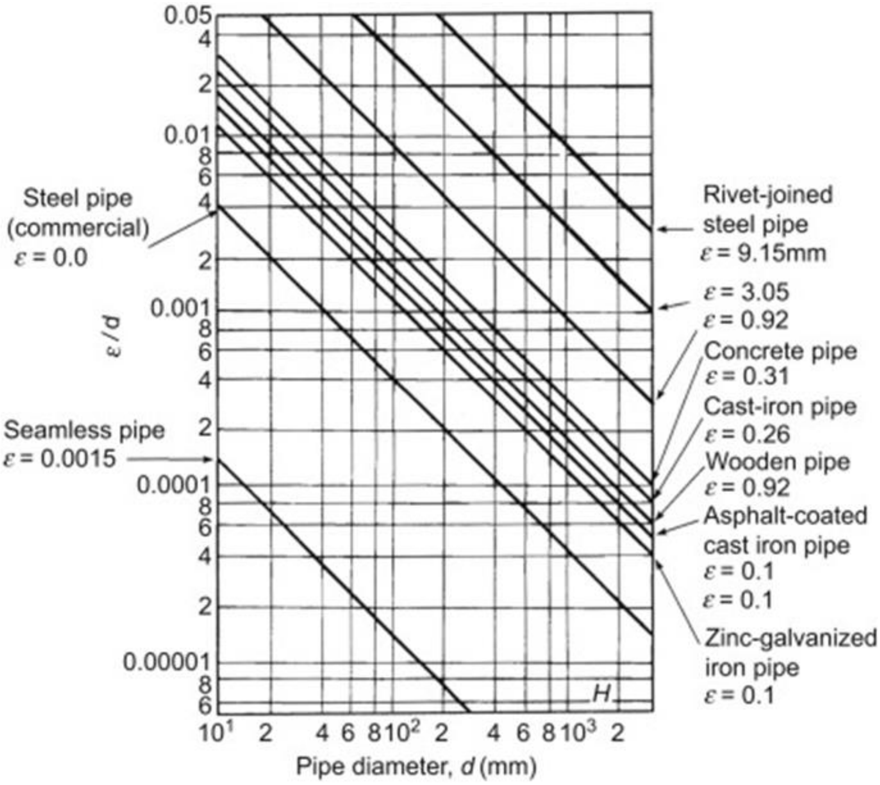


Moody's diagram gives us the value of the friction factor that is the y axis over here as a function of Reynolds number which is the x axis over here and for different values of epsilon by d where epsilon is the roughness and d is the diameter of the pipe through which the flow is taking place. If I look at this one this straight line refers to laminar flow where F is equal to 64 by Re the relation that we have seen before. So, F is equal to 64 by R e and we get the laminar flow. So, up to roughly this point the flow is going to be laminar about 2100 for pipe flow and that is going to be a transition range where the linearity still holds and 64 by R e can still be used. But when we are about 4000 or so, the value of the friction factor jumps suddenly and it becomes turbulent and if you think of the turbulent flow the lines that you see the almost parallel lines for higher values of Reynolds number are for different values of different values of epsilon by d.

So, as the value of epsilon by d increases over here the value of friction factor also increases. Friction factor is a measure of the resistance offered by the inner surface of the pipe to the flow. So, more the roughness of the pipe higher is going to be the pressure drop required to have the same flow rate. So, with increase in the value of epsilon by d the value of friction factor increases and the value of the pressure drop needed to make the flow also increases. So, Moody's diagram is something which we should understand very clearly.

Once again it gives us the value of the friction factor for different Reynolds number and you look over here and see that this is the logarithmic plot. So, this side is logarithmic plot. So, 10 to the power 4, 10 to the power 5 and so on and this side the. So, this is a semi log plot between f and Reynolds number for different values of epsilon by d. And when we consider smooth pipe this line refers to the last line refers to the smooth pipe that is that is what the value of friction factor going to be.

One more thing that one has to take into account is that initially the value of f is going to be a function of Reynolds number, but once you cross this line the dotted imaginary line the variation of f with Reynolds number is almost non-existent. What it tells us is that from this point onwards the flow has become fully turbulent totally turbulent it is so turbulent that the value of the Reynolds number is not going to make any changes in the value of f. So, it becomes invariant with Reynolds number. This specific nature of the experimentally obtained value of f will become handy while we try to solve problem of a certain type and I will discuss that I discuss it more at that time. But Moody diagram is essentially a relation between f and Re for different values of e by d and it should be well conversant about how to figure out the value of f because my whole calculation depends on correctly evaluating correctly reading the value of f from the Moody diagram.



So, I am going to tell you the steps which are necessary. So, the one that you see over here this is the plot of pipe diameter and epsilon by d. So, for different values of pipe diameter for known materials of construction of pipe. So, you have a seamless pipe, zinc galvanized pipe, cast iron pipe, concrete pipe and so on. So, for each one of these pipes what is going to be the value of epsilon by d.

So, what are they so, you need to use this figure in conjunction with Moody diagram. So, the first point is to evaluate or know what is epsilon by d and the second point is once I know the value of epsilon by d and I know the value of Reynolds number. Let us say the Reynolds number is 10 to the power 6 from 10 to the power 6 I go all the way up to the point where I reach epsilon by d let us say it is 0.004 over here. So, I go from 10 to the power 6 to 0.004 and then go to the left and read my value of a value of f is about 0.017. So, 10 to the power 6 all the way up to an epsilon by d of 10 to the power of 0.004 on this side go to the left and find out what is the value of f. So, that is the path Reynolds number epsilon by d go to the point of intersection and then find out what is the value of f.

If your epsilon by d is somewhat somewhere in between use interpolation and find out what is the value of value of, f going to be and then I estimation would be good enough you do not need to do any further calculation. So, you just find out where you are in between 0.001 and 0.002 and then go to the left and find out what is the value of value of the friction factor. So, the first thing as I said is calculate the Reynolds number from this figure find out what is the epsilon by d for this specific case.

So, if you if you if you so, you would be able to figure out what is what is the value of epsilon by d find f from Moody diagram as I have shown you as I have explained to you for the case of turbulent flow. If it is for laminar flow f is simply going to be 64 by R e the thickness of the viscous layer is much more as compared to the thickness of the viscous layer in turbulent flow. So, laminar viscous layer is in thick in size much more than the turbulent viscous layer. So, if there are small imperfections small roughness's on the surface, they are going to affect the they are going to affect the turbulent boundary layer the turbulent friction much more as compared to the laminar flow. That is why it is not necessary not needed to consider the effect of epsilon on turbulent on laminar flow pressure drop.

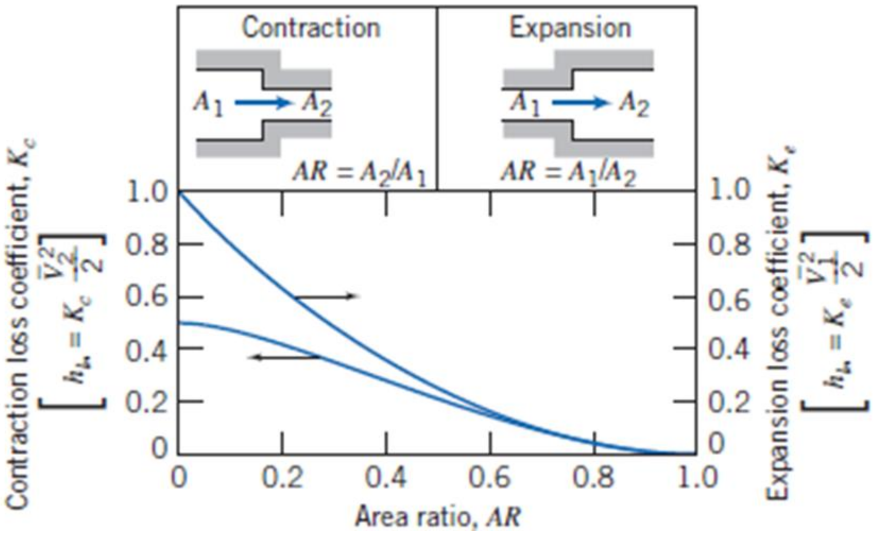
I will say it once again the thickness of the laminar for the case of laminar the thickness of the viscous sub layer is large compared to the thickness of the viscous sub layer in turbulent flow. So, if any imperfections roughness's are present on the surfaces, they do not affect the laminar flow significantly as they do the flow in the case of turbulent flow. So, that is why the value of the value of the friction coefficient f is 64 by R e and you do not do not see any in incorporation of any roughness into this expression. So, it does not matter whatever be the thickness or the case whatever be the roughness for the case of laminar flow whereas, for turbulent flow one has to consider it. So, if you have then Reynolds number epsilon by D find the value of f for laminar flow use 64 by Re for turbulent flow use this Moody diagram and then you can figure out what is going to be the value of the total head loss in such a case.

So, this is something which we are going to use further in our calculation. Now, what if the now let us talk about minor losses. So, far what we have said is for major losses now we are going to talk about minor losses. So, what are the minor losses? What you see here is a sudden contraction from a large surface area it becomes to a small surface area small flow area. For the case of expansion from a small one it suddenly expands into a larger one.

The coefficients the minor losses in such cases are expressed as k which is a contraction coefficient times v bar square by 2 where v bar is the average velocity. Now, the figure over what you see is that this line the line at below is for the is the contraction loss coefficient. The line at the top is the expansion coefficient and what you need to find out is what is A r, how is A r defined for the case of contraction it is the smaller area divided by the larger area A 2 divided by A 1. So, you find out from the geometry of the contraction what is the value of your A r.

Let us say this is point my A r is 0.4. So, I go all the way up here and find out that my value of k is about 0.3 k c is about 0.3 contraction coefficient. So, I go into over here and use this value of 0.3 and figure out what is going to be my minor losses.

And this k c here you see that it is a v 2 the velocity at the smaller area that is to be used for finding out what is the minor losses. So, for minor losses find out from the area if it is contraction figure out what is the contraction coefficient and use v 2 square by 2. On the other hand, if this is about expansion. So, A 1 goes to A 2 in that case the loss is going to be once again k c. So, I read what is the value of my k e the expansion coefficient read that value of expansion coefficient put it in there and here it is v 1 v 1 means it is the smaller area the velocity in the smaller area.



$$h_{LT} = h_L + h_{LM}$$

$$h_{LM} = K\frac{\overline{V}^2}{2}$$

$$h_{LM} = f\frac{L_e}{D}\frac{\overline{V}^2}{2}$$

So, for expansion I use the top curve for contraction I use the bottom curve and always use this the velocity at the higher of the two velocities the velocity through the smaller one and over here also the velocity through the smaller one. So, this is how the contraction or expansion losses sudden contraction or sudden expansion losses are evaluated and k is the loss coefficient which is determined experimentally. And in some cases, the minor losses are also in order to have some parity in between the minor losses are also expressed in terms of F L E d v square by 2 and this F is to be evaluated the same way we have done for the case of major losses. But here look that instead of L I have used L E this E stands for equivalent length. So, what is equivalent length? So, if I could figure out that for this expansion or this contraction it is I can find an equivalent length that equivalent length if I use in here then I can find out what is the value of friction what is the value of friction coefficient using Moody diagram and then find this out.

So, I can use either one of these two approaches if I know the equivalent length I can use this or I can use the contraction coefficient and find out what is H L M both approaches are used in the literature for this minor calculation. So, what is equivalent length? Let me draw this and it would be clearer to you. Let us say in the pipeline I have a valve and obviously, since I have a valve in here there is going to be some delta p around these two points from the inflow and outflow because of the presence of the valve there is some delta p pressure drop in this. I will instead I will draw a line a straight pipe like this where I am adding this additional length L such that the pressure drop in this length of the pipe is the same as this delta p this is what I call as L E or the equivalent length. So, delta p for the valve is the same in both cases where instead of the valve I have added some length to it such that the pressure drop remains the same.

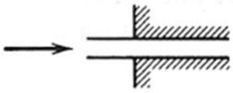
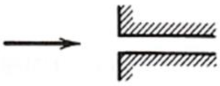
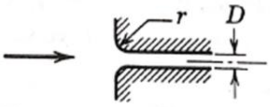
If that condition is satisfied then this is called the L E. So, over here I have this is the L E this is the L E that I am going to that I am going to use for the case this is the equivalent length that I am going to use if I could figure out what is F from Moody diagram. So, both methods are in practice the concept of equivalent length I have explained over here where the valve or any such construction in the path is expressed in terms of some length L E. And what are the equivalent lengths? So, these are some of the common valves, some of the elbows which changes the direction, some where it changes if there is a T in the flow path all these for the case for each case what is the equivalent length that is provided in this table the source for which is once again Fox and McDonald. So, for a gate valve which is open or close the value of the equivalent length is small as compared to the gate valve I am sorry which compares to gate valve and globe valve and other valves all these are provided.

Representative Dimensionless Equivalent Lengths (L_e/D) for Valves and Fittings

Fitting Type	Equivalent Length,[a] L_e/D
Valves (fully open)	
Gate valve	8
Globe valve	340
Angle valve	150
Ball valve	3
Lift check valve: globe lift	600
angle lift	55
Foot valve with strainer: poppet disk	420
hinged disk	75
Standard elbow: 90°	30
45°	16
Return bend, close pattern	50
Standard tee: flow through run	20
flow through branch	60

[a]Based on $h_{l_m} = f(L_e/D)(\bar{V}^2/2)$.

So, you already know what is the value of L E. So, if you know the if you know what is the what is the what is the device that you have in the pipe in the pipeline, what is the bend that you are provided in the pipeline then you can you know what is the equivalent length and you can find out F from Moody diagram and use it else you have to know what is the contraction coefficient that is the only thing that is different. And if you have sudden flow into this so, flow in here what is the minor loss coefficient if the it is square edged it is slightly less than that and if you carefully round the edges you would see that the value of k could be even lower. So, based on h l m k v square by 2 where v is the mean velocity in the pipe. So, if you have flow entering from a large reservoir over here through a square edge and through a rounded edge what would be the value of the pressure drop the what would be the value of the head loss that you can calculate by putting in the right value of k and the value of v the velocity in here. So, this gives you the way to calculate all the minor losses which are present in the path.

Entrance Type		Minor Loss Coefficient, K^a
Reentrant		0.78
Square-edged		0.5
Rounded	 	r/D: 0.02, 0.06, ≥ 0.15 K: 0.28, 0.15, 0.04

So, to recap once again just to give you an idea Moody diagram for turbulent flow, laminar flow 64 by R e the loss the loss is F L by D v bar square by 2 for F you calculate using Moody diagram either for laminar or for turbulent. The h l m the minor loss is k the contraction or the expansion coefficient defined in this formula and this curve gives you the value of the contraction coefficient k and where the velocity is always the velocity in the smaller area. It can also be calculated by incorporating an equivalent length and finding out F as we have done before. The value of the equivalent I have explained the concept of equivalent length over here equivalent length for different valves different elbows and all these are provided. So, you pick the right value of the F and then put it in their right value of the sorry the length put it in there and then do the calculation.

And similarly for the entry of fluid from a large reservoir through different types of entry points something which is projected out something which is square something which is well rounded well machined and the value of a k the minor loss coefficient you would see progressively decreases as I make this more and more smooth and this is very gradual if I can make it very gradual the value of k would be as small as 0.04, 0.02 and so on. So, in this lecture I have shown you that the Bernoulli's equation can be can be used if I can find out what is h l t the total loss the total loss is a sum of major loss and minor loss and I have explained how each one of these losses can be evaluated. So, we will see the example application of one of these the discussions that we had so far in our next lecture. Thank you.