Momentum Transfer in Fluids
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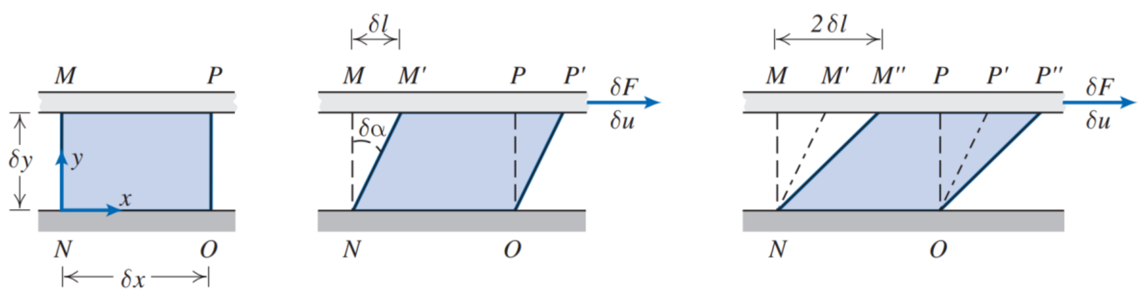
Week-01
Lecture-04

Welcome to this class of Momentum Transfer in Fluids. We were discussing about some elementary framework, and we already defined what is a stress tensor. Let us build on this further and try to define what is viscosity in this context. So, if we take a fluid element between two infinite plates, first of all, infinite plates appear highly theoretical. But the fact of the matter is this concept of infinity when it comes to this fluid flow, we will be mentioning this over and over.

You will find that when the gap between the two plates is on the order of millimeters, then 1 meter can be treated as infinity without much error. Because 3 orders of magnitude higher than something that can be assumed as infinity. So, it is not that it has to truly extend to infinity; it is a large magnitude compared to the size of the gap that is what we are referring. So, here I have two plates.



Here I have two plates: one is this one, and the other is this one. So, the lower plate is fixed, and the upper plate is moving. The gap between the two plates is given by δy and this is the x and y direction. So, I am focusing on this element, which is MNOP, which has a dimension δx in this direction, and δy is in this direction. So, now if the upper plate moves at constant velocity δu, under the influence of constant force δF_x.

So, I am putting a δF here, and let us say this is δf_x. So, that is the force given on the upper plate, and this is because of this the upper plate is moving at a velocity δu. So, now, after some time δt, we find that the fluid element got deformed instead of MNOP, now it is M'NOP'. So, that is the new shape of this liquid element and the angle that it has made is δα. The distance between M and M' is δl and that same distance would be here P to P'.

So, now, we can see here the distance

$\overline{MM'} = \delta l = \delta u \cdot \delta t$

δl would be equal to δu into δt because of velocity of this upper plate is δu, let us say meter per second, and over δt second, I have allowed this deformation to take place. So, this distance would be δu.δt. So, that is what is MM', and then we note here that if this angle δα is small, that means, if the deformation takes place for a very short time δt, then we have for small δα you have $\delta l = \delta y \cdot \delta\alpha$. So, what is our assumption here? Our assumption is $\delta y \cdot \delta\alpha$.

So, δl by δy, what is that? tan($\delta\alpha$) = $\delta l/\delta y$ and you are assuming here tan($\delta\alpha$) ≈ $\delta\alpha$ when $\delta\alpha$ is small. So, that is the assumption made here. So, once you do that, you can write then $\delta l = \delta u \cdot \delta t$ and $\delta l = \delta y \cdot \delta\alpha$. So, then you write $\delta y \cdot \delta\alpha = \delta u \cdot \delta t$ or in other words, you can write this from here from this you can write $\frac{\delta\alpha}{\delta t} = \frac{\delta u}{\delta y}$. So, $\frac{\delta\alpha}{\delta t}$ is the rate of angular deformation.

So, $\delta\alpha$ is the angular deformation when you divide it by δt that gives you the rate of angular deformation. So, now, what it says is when $\delta\alpha$ is small, the rate of angular deformation that is equal to the velocity gradient. What is the velocity gradient here? I have the velocity here as far as this fluid element is concerned. Velocity at this layer is 0, because this plate is held static, and the layer of fluid that is next to this fluid that is attached to this plate will also have the velocity is 0. This is a common assumption we always make in the fluid flow. That assumption is known as a no-slip boundary condition. There are cases when one has to use a slip boundary condition, but that we are keeping away from the purview of this course.

So, you have a no-slip boundary condition, which means the fluid that is attached to this wall has a velocity 0, and the fluid that is attached to this wall has it has a velocity of δu. So, velocity transition takes place from velocity of 0 to velocity δu. So, you can see that there would be a velocity gradient. There would be a velocity gradient δ, which is a change of velocities from 0 to δu. So, what would be this gradient, $\frac{\delta u - o}{\delta y}$ over distance δy.

So, how the velocity is changing? Let us say this is let us say, 2 mm, and here the velocity is 0, and here it is let us say 4 mm/s. So, then what is $\frac{\delta u}{\delta y}$? If I assume that the velocity profile is linear, that means velocity is linearly changing from 0 velocity to 4 mm/s. So, then $\frac{\delta u - o}{\delta y} = \frac{4 - o}{2}$. So, that is so, numerator would be mm/s, and this would be mm. So, then this would be 2 s^{-1}.

So, this is the velocity gradient. We are talking about this, and this velocity gradient is equated with $\frac{\delta\alpha}{\delta t}$, which is the rate of angular deformation. So, now, the reason we are interested in this is because we would be linking τ_{yx}, which is the shear stress. Once again, the y is the first subscript, which is the area on which the force is acting, and the second subscript is in the direction in which the force is acting. That is what we have learned in our earlier lecture.

So, this is τ_{yx}. This is the area on which the shear stress is acting. So, the area has a direction and that direction is y. So, the first subscript is the direction of the area and the second subscript is the direction in which the force is acting, the shear stress is acting in this direction. So, $\tau_{yx} = \lim_{\delta A_y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y}$, that is what we have defined the shear stress as and when $\delta A_y \to 0$ this is $\frac{dF_x}{dA_y}$.

$$\text{Deformation rate} = \lim_{\delta t \to 0} \frac{\delta\alpha}{\delta t} = \frac{d\alpha}{dt} = \frac{du}{dy}$$

Now, there is something called Newton's law of viscosity. That is, what Newton's law of viscosity says is Newton's law of viscosity relates this shear stress with the deformation rate. So, the deformation rate, you can you have seen that $\frac{\delta\alpha}{\delta t} = \frac{\delta u}{\delta y}$, $\frac{\delta\alpha}{\delta t}$ is an angular deformation rate $\frac{\delta u}{\delta y}$ is the velocity gradient. When $\lim_{\delta t \to 0}$ then $\frac{\delta\alpha}{\delta t}$ will be written as $\frac{d\alpha}{dt}$, and $\frac{\delta u}{\delta y}$ would be $\frac{du}{dy}$. So, then Newton's law of viscosity relates this shear stress to this deformation rate. So, that is exactly what you would see here, and as Newton's law of viscosity, $\tau_{yx} \propto \frac{du}{dy}$, which happened to be also the velocity gradient.

$$\tau_{yx} = \mu \frac{du}{dy}$$

That means, how velocity changes from 0 velocity at the wall to δu at the moving surface at the moving plate. So, $\tau_{yx} \propto \frac{du}{dy}$, and then we add a proportionality constant, and then that proportionality constant is referred to as the viscosity. So, this is what we are trying to aim at, but there are a couple of other things that we may like to look into before we proceed from this slide. One thing is that Newton's law of viscosity states shear stress is directly proportional to the rate of deformation this is something which we understood. Note that u increases with y.

So, $\frac{du}{dy}$ is positive, u increases with y, what is the y direction? This is the y direction, right? So, in this y direction, the u increases u is 0 here, and u is the highest at the wall here. So, what we see is that u increases with y.

So, that means $\frac{du}{dy}$ is positive, $\delta\overrightarrow{F_x}$ is positive, $\delta\overrightarrow{A_y}$ will be positive if $\hat{n}$ is in positive y-direction. So, that means, this is the shear stress imposed by the plate on the fluid. Now, here, there is a catch: shear stress imposed by the plate on the fluid or shear stress imposed by the fluid on the plate. I mean, say, for example, I am taking one layer here. So, who is imposing shear stress on whom? Because of this lower layer, if I look at a lower layer and an upper layer, they are at the interface.

So, what I will see is that the upper layer will be imposing a shear stress in this direction on the lower layer, whereas the lower layer will be imposing a shear stress in the other direction on the upper layer. So, that is defined by the normal of that area. Is it in the positive y direction, or is it in the negative y direction? So, that defines the sign. So, δF_x is positive.

So, δA_y is positive. So, $\hat{n}$ is in positive y direction that is shear stress imposed by the plate on the fluid. If you consider shear stress imposed by the fluid then it becomes negative. So, that you must keep in mind. Why I am putting it here is that when it comes to using this equation, or using this equation for that matter, you will find that in some books, it is referred as $-\mu\frac{du}{dy}$. So, it is which shear stress is imposed by whom on what matters that is one thing.

Second thing is this scheme of the upper plate moving and the lower plate fixed this provides a unique opportunity to measure viscosity. Suppose I have some arrangement, I have some rack and pinion arrangement. Rack and pinion means you have a gear like this, and you have another gear which is meshed with this gear you have another gear here, and suppose this gear rotates. So, as this gear rotates this would be moving in this direction. So, the amount of torque you are putting on this gear as the gear rotates and the rpm that you produce. So, you are rotating this gear at a certain rpm and amount of torque that you have to put.

So, the rpm will give you what is this δu, and the amount of torque you have to put that will give you what is this δF or δF_x. So, if you have some type of device where you know the gap between the two plates, let us say it is you hold it at 1 millimeter 1.5 millimeters, or 0.5 millimeter that you know the gap between two plates you have some arrangement by which you can control the rpm and you can control the δu and also you can measure the amount of force that has to be put there. So, we can get the value of τ right away, and also, τ is what, you know, the area.

Suppose you made a device you have a lower plate and an upper plate, the upper plate is moving by some arrangement here, and then you have those measurements. So, then you measure the δF from this torque, you have the area because the plates are your choice. So, you know the area of the plate, and then that will give you the shear stress, and the other one is the $\frac{du}{dy}$, the velocity gradient you know what is δu and you know what is the gap between the plate. So, you know $\delta u,$ the velocity gradient $\frac{du}{dy}$. So, by a simple arrangement, you can measure the velocity gradient, you can measure the shear stress, and if you divide one by the other, you get an idea of the viscosity.

So, essentially, this concept of one plate moving and the other plate fixed in an extended way is used to measure the viscosity. Viscosity is measured viscosity of a fluid you pick up any fluid and measure the viscosity then you can measure the viscosity by this arrangement. Of course, if one plate has to move what was found is that if the upper plate instead of moving in a particular direction, if the lower plate is fixed and the upper plate rotates. So, then it becomes easier to manage. So, typically, these concepts are used to measure viscosity in a device, which is referred to as a viscometer.

So, you will find there are many different ways the viscosity can be measured. For example, you can have a fluid filled in a beaker and drop a ball from the top, and see how much time it takes for the ball to travel. One can measure viscosity, one can measure take a capillary viscometer or you can have how much time a fluid takes to flow through a capillary under gravity, there are many different ways, but this is typically when it comes to measuring viscosity one relies on something called a viscometer, and which operates on a very similar principle. Two plates, one plate, the bottom plate is fixed, and the upper plate is moving at a constant velocity instead of moving it, probably it is rotating at a constant velocity, and you find out the torque given and the other geometric parameters, and from there, you can measure the viscosity. So, this viscometer is an analytical device. A lot of times in the labs, you will see this viscometer device where the one can do the viscosity measurements. That is one thing.

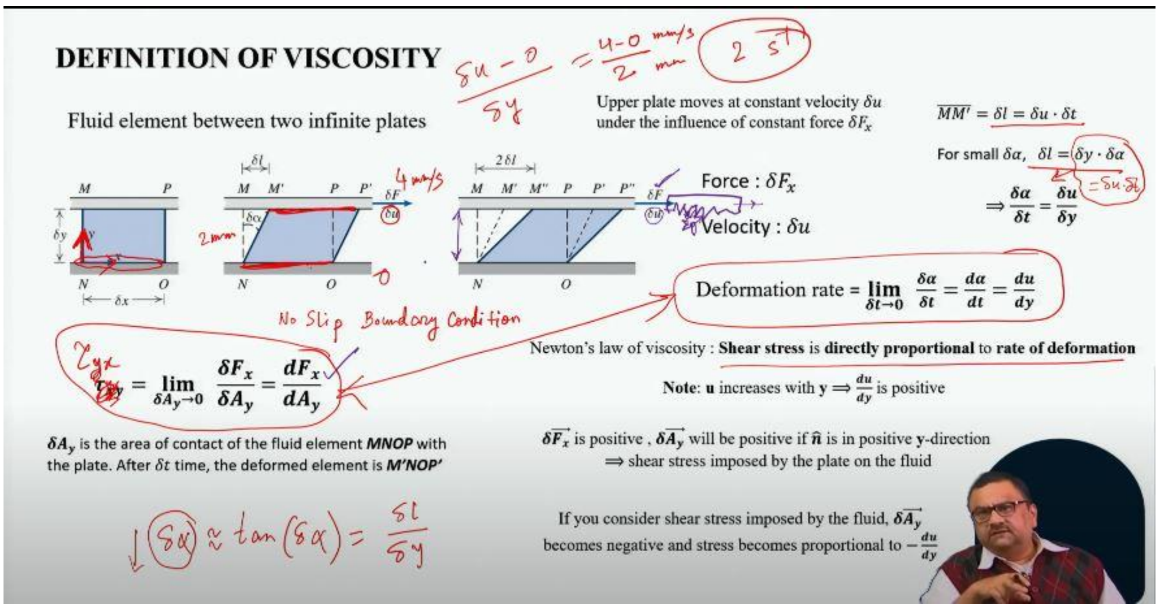
The second thing is you must understand the viscosity of common substances. For example, viscosity of water, typically the viscosity is in SI system, the viscosity unit is Pascal second. The CGS system if you look generally work with poise. $Poise = \frac{gram}{cm.s}$. You can see how they are related; typically, water has a viscosity of 1 centipoise, close to that, of course; I mean, it is not exactly 1. It depends on temperature and other factors, but it is 1 centipoise which is

1 Centipoise = 10^{-2} Poise

In Pascal seconds, if you convert it comes to m.Pa.s. So, you can convert these units. You should be familiar with this because water viscosity typically needs to know. Air viscosity, on the contrary, is at least a few orders of magnitude lower than this. So, air would be easy to flow. Viscosity is a characteristic parameter that tells you how easy the fluid is, you will be able to flow of fluid. For example, honey is more viscous than water.

So, these are the common definitions and common considerations one has on viscosity. Typically, you may like to note here that this is referred to as Newton's law of viscosity, and further down, the fluids that obey this law of viscosity, those fluids are referred to as Newtonian liquids. For example, water is a Newtonian liquid that means, this is a liquid which obeys Newton's law of viscosity, but there are a whole bunch of fluids that do not obey Newton's law of viscosity, and the fact is those are the most important fluids in our day-to-day life. When it comes to a polymer suspension, it comes to a sludge, there are many different materials. In fact, coming to think of it, many since we are more interested in the application point of view, we must appreciate the fact that most of the materials that we work with in our day-to-day life and with the functional materials that are helping to make our life easier, those materials are at the interface between solid and liquid frankly speaking to paste, gel, gum, they are in between liquid and solid.

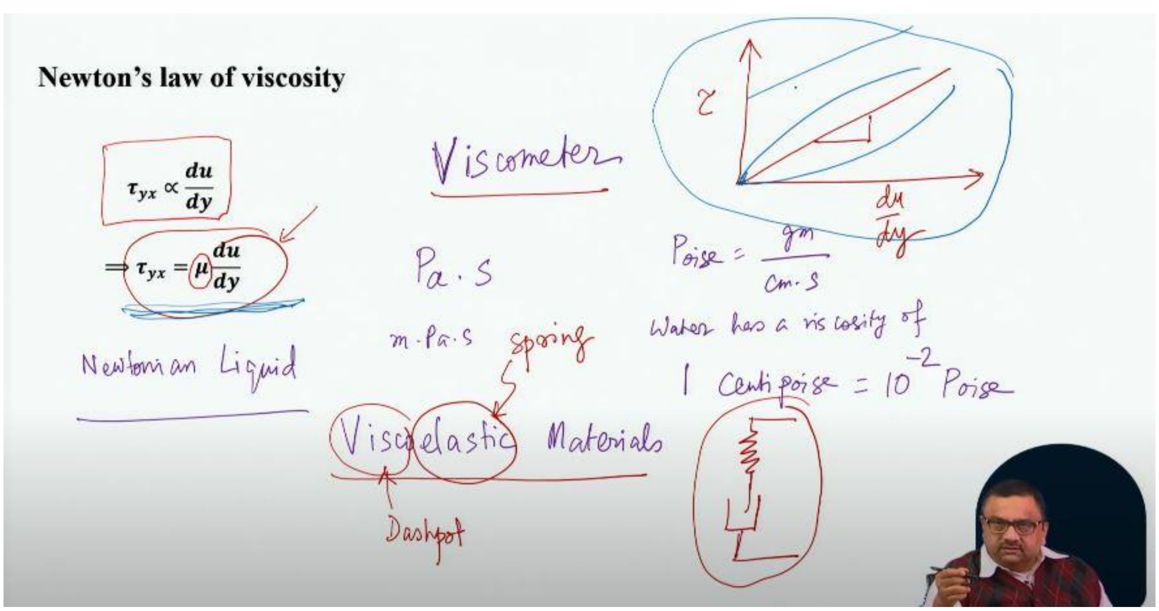
So, coming to, I mean though the Newtonian liquid is a theoretical thing, I mean for water, it is fine, but most of the fluids, so to say, are at the fluid-solid interface. So, generally, there is a name attached to this type of fluid. These are referred to as viscoelastic materials. That means it has some amount of viscosity and some amount of elasticity, and elasticity is a property of a solid, and viscosity is a property of a fluid. So, we say that this has both characteristics present there.



So, then the question is how we carry. I mean, Newton's law of viscosity applies to viscous material, but it is not applicable to elastic material. But when it comes to these applications, we have to stretch Newton's law of viscosity to some of these materials. So, how to do that? If you want to handle a classical treatment of viscoelastic materials, there is a well developed subject, and you must agree that there would be when it comes to the viscous part, visco part that is synonymous to something called a dashpot. Whereas the elastic part is synonymous with something called a spring. So, you will find that to characterize viscoelastic material, what researchers have done is they have put a spring and then they put a dashpot, and then they said that whatever characteristics we see with the spring with its spring constant with dashpot with its characteristic constants and the overall behavior.

Now, it could be just one spring and one dashpot in a series, or it could be some other network of springs and dashpot that would truly represent a viscoelastic material. But the fact is many different materials are more towards the viscous part, more towards the liquid part instead of an elastic part. So, there has been a major thrust towards extending this Newton's law of viscosity towards these materials by making some modifications of this particular equation. So, that we can capture these effects. Now, when it comes to these viscoelastic materials first thing that will happen is that τ_{yx} is not proportional to $\frac{du}{dy}$.

So, what can happen, you have used a viscometer you have measured, you have obtained we talked about some method by which we can measure the δF_x we can measure the δu we can measure $\frac{du}{dy}$, and the shear stress and by taking the ratio we can find out the viscosity. So, what you see is that if someone plots the viscosity, first of all, if someone plots the shear stress τ as a function of $\frac{du}{dy}$, the velocity gradient, and if Newton's law is valid, there is no intercept to it. So, I would expect that the shear stress will go like this and the slope of this line will give me the viscosity. So, that is what we would expect. If it is not following this curve.



So, we will follow. We may have a curve like, let me put some other color here. We can have a curve like this. We can have a curve not negative. Sorry, we can have a curve like this. We can have a curve like this, we can have a curve like this, we can have even in they may not have to start from 0 we can have a curve like this. So, it will not follow a straight line like this. So, these are the various curves that one can get. So, immediately what professionals try to do is that they said that if we can extend somehow these equation to these modified forms, then at least we can go somewhere.

So, that is one approach we have. The other point is that shear stress is a function of $\frac{du}{dy}$ that is the type we said, but the moment you have a dashpot and spring in this combination, you may have a class of fluid which is not just with the deformation gradient shear stress is changing, shear stress would be a function of time as well. That means I have imposed a velocity gradient now, and after half an hour, it starts moving. So, there are classes of fluids whose shear stress would be time-dependent. So, that is also another class of fluid. So, what we will do is, as we continue this exercise, we will try to characterize these as an extension of Newton's law of viscosity and see how well we can do that, and how well we can characterize this particular aspect.

So, that is something which we will be addressing in the next lecture, and what we will do is further. I mean, our approach would be that we will start with an open mind. Suppose I am given a fluid, and I do not know whether the fluid is Newtonian, whether fluid will be of this line, that line, or some other line. So, we have a very open mind. So, I will start with that same viscometer, that same apparatus that we discussed, and we will start measuring viscosity with an open mind, but the only thing we do is measure viscosity at different deformation rates. That means, at different velocity gradients, and see how viscosity varies and from there we try to relate which class this fluid belongs to and try to put theories accordingly.

So, that is the approach that we are going to follow in the next lecture. So, that is all as far as this particular module lecture class is concerned. Thank you very much for your attention.