Momentum Transfer in Fluids
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Good morning. Today, we are going to start with a new chapter and this will be the last major topic of this my part of the course and we are going to deal with something that you probably already know I am sure you already know that that is Bernoulli's equation. So, we are going to talk something about the Bernoulli equation, the fundamentals, its limitations and what kind of modifications are to be incorporated in the Bernoulli’s equation such that it can be applied over a wide range of conditions for a wide range of fluids and what are the implications of relaxing the constraints originally incorporated into Bernoulli's equation. So, I am going to start first with the just the writing down the Bernoulli's equation and then we will identify what those conditions what those conditions are. So, if you look at Bernoulli's equation all of us know that the sum of the pressure head, the velocity head and the gravitational head would be a constant. So, this equation therefore, relates the changes in pressure to velocity and change in the height along a streamline.

$$\frac{p}{\rho}+\frac{V^2}{2}+g\,Z=Const.$$

So, this equation is known to all of us, but we would like to see under which conditions this equation was derived and for reference I am following the textbook of Fox and McDonald for this chapter and you can get more detailed derivation of the equation and discussion on that. But I am going to give you the summary of the restrictions that are to be incorporated that are to be understood before we use Bernoulli's equation. The first one is it is going to be for a steady flow. So, there is no time dependent component in it, the velocity at any fixed location does not change with time, the same applies for pressure as well.

The second condition is there is this is the most I think the most vital restriction on the use of Bernoulli's equation that this equation is can be used only for frictionless fluids, only for an ideal fluid Bernoulli's equation is applicable. However, all of us know that for flow of any fluid through a real conduit there is going to be some sort of resistive force, some sort of frictional forces which opposes the flow. Therefore, the it is not just going to be the sum of these three heads the pressure, velocity and elevation, the losses due to friction should be somehow incorporated before one can use Bernoulli's equation for real situations. So, we will have to investigate that. The third one is that the flow is going to be along a streamline.

So, this adds additional constraint to the use of Bernoulli's equation and the last one is that this equation is valid for incompressible flow. So, if you have a fluid flowing through a pipeline in which the temperature keeps on changing and the change of temperature is such that the property is going to get affected for example, in this case the density is going to get affected then one cannot use Bernoulli’s equation. So, these are the four major restrictions on the use of Bernoulli's equation and if I can expand it further to make our self-aware of what we can do with the simple form of Bernoulli’s equation and what we cannot then it would be easier for us to suggest changes suggest modifications which could be incorporated in Bernoulli's equation. So, the first one is friction should have a negligible effect which of course, is not going to be true as we have discussed already and no separation of boundary layers on the walls. So, what exactly I mean by no separation of a no separation of a boundary layers at the walls let me draw what is going to happen when we have when we have a flow along a curved path.

So, let us say we have a situation in which the flow this is the surface and we have a flow along this. Now, over here the tendency of the fluid particle will be to move in a direction like this keeping this region devoid of any fluid flowing with the same velocity as was happening here. So, this kind of change in flow separation and formation of the boundary layer that we also know that if we have a solid plate and if we have flow coming towards it then there is going to be a thin region exaggerated for clarity here. In a thin region the flow is going to be 2 dimensional where the effect this is going to be viscous the effect of viscosity would be important. So, it is 2D flow inside and it is 1D and inviscid flow outside.

So, in this region the flow is going to be 1 dimensional and however, in the inside the boundary layer. So, this is the boundary layer the flow is going to be viscous there would be effect of viscosity inside this thin boundary layer. So, Bernoulli's equation cannot be used in this region because here the effect of viscosity is pronounced it is going to be highly it is going to be a viscous flow. Whereas the boundary layer I mean the Bernoulli's equation can be used for the region outside of the outside of the boundary layer. So, we must be very specific about the region where I am applying the Bernoulli’s equation there is not going to be any flow separation and there is not going to be any formation of boundary layer such that the flow inside is viscous.

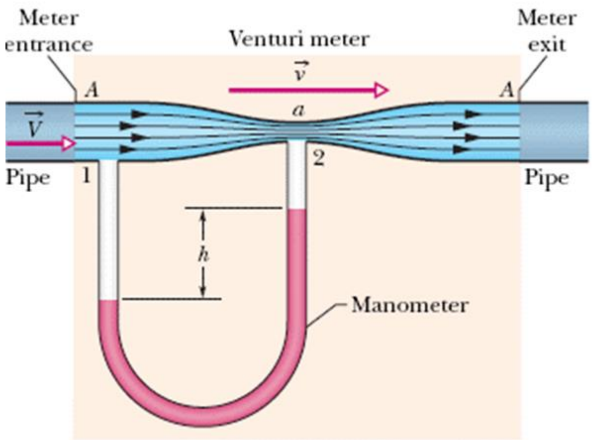
Second is what I mean by sudden expansion and sudden contraction. So, if you have a diverging passage and sudden expansion. So, the flow cannot be the Bernoulli's equation cannot be used. So, if you have a flow which suddenly expands like this the flow region expands like this so there is going to be additional pressure changes at the exit and which cannot be modelled using Bernoulli's equation. The next one is so where can it be used? In anywhere where the effect of viscosity and the friction and the related pressure drop is not too large where would that be? So, if you have a well-rounded entrance if you have flow coming in from a large area to a small area where the entrance as you can see is well rounded it is very smooth in that case Bernoulli's equation can be valid.

If you have a bend in the path where the bend is rather gentle so that there are the fluid streamlines are not forced to make a sudden change in direction then it can be the Bernoulli's equation can be used. If the Bernoulli's equation is used between two points which are close to each other the length the separation is not going to be too large then the pressure drop in that small distance would be can be neglected and therefore, the frictional effects for a short length of the pipe can be neglected and Bernoulli's equation can be used. The other is if you have a machine for example, in most of these situations you have a fluid moving machinery present in the pipeline. So, you have a pipe that comes in the drawers let us say water from a reservoir then you have a pump over here and from the delivery side of the pump this goes to some other reservoir. So, whenever there is a source additional work is being done on the flowing fluid that is not that cannot be handled with the form of Bernoulli's equation with the idealized form of Bernoulli's equation that I have just shown.

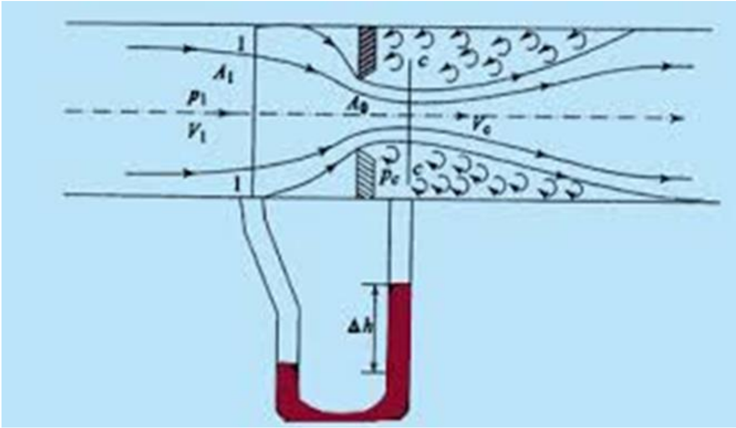
So, there cannot be any fluid moving machinery which adds work or extracts work out of the flowing fluid for example, a turbine. So, Bernoulli's equation in that form is unsuitable for use for that specific use. And compressibility of gases can have to be considered specially when the Mach number is about 0.3 and above then one should consider the compressibility and the change in rho for such situations. And the temperature change may cause a significant change in rho and therefore, Bernoulli's equation cannot be used when there is a property variation and one example of property variation is initiated by a change in temperature.

So, all these 7 points are to be kept in mind before one can use Bernoulli's equation confidently, but as you can see that most of these things are going to happen in any working real conditions. So, what is how do I how do I handle such situations? What is the point in having an equation having a relation that must that has so many restrictions on it, but it is a very useful relation between the pressure head, the velocity head and the gravity head. So, can something be done about it and that is why to handle that a modification to this has been added, but before I get into this I if you look at the number 3 that diverging passage and sudden expansion cannot be modelled and it is a reasonable model for well-rounded entrance. So, I am going to show you 2 or 3 figures about usual flow or a very common flow measuring instruments. The first one that I show you show you here is the Venturi meter.

So, what is a Venturi meter? A Venturi meter essentially measures what is the what is the what is the velocity. So, you have you have a flow which is coming in through the pipe, you would like to find out what is the flow rate and this is the flow out of this and in between you add a Venturi meter. So, from here to here is a Venturi meter. So, the flow compresses the streamlines come close to each other and then slowly they regain their original separation the original flow. So, there is going to be pressure difference between these two which can be measured using a manometer.



This pressure difference is directly proportional to the flow rate, but since the bend this is this is very small gradual bend that you encounter that we encounter in Venturi meter and then it expands slowly from the throat it expands slowly and regains its original position. So, the pressure drop across a Venturi meter is going to be a lot smaller than what you would get in the case of an orifice meter. So, what is an orifice meter? Again, you have the flow and the flow out of this and you have a constriction placed in the path of the fluid. So, it could be just a circular disc with a small hole at the middle. So, this region shows the small hole at the middle for an orifice meter.



So, the streamlines are going to come they are going to come very close to each other and suddenly it is going to expand and since it is expanding suddenly there is going to be some there is going to be recirculation. So, what you see over here this is the low-pressure region there is going to be lot of recirculation and a significant pressure drop is going to take place as compared to that of the Venturi meter and therefore, orifice meter is going to be relative to the Venturi meter it is a crude it would give you a crude measurement of the flow rate. So, there the recirculation and the formation of the low pressure also have additional problems associated with them. If this fluid the incoming fluid contains suspended particles it is likely that they are going to deposit over here. So, orifice meter has a number of problems, but still in many situations the orifice meter is preferred over the Venturi meter that you see over here because as you can see the Venturi meter needs precise machining.

So, it must be machined properly with a very gentle slope very gentle slope from here to the throat of the Venturi meter and then expanding to its original size. So, this requires precise machining it is more expensive it takes up more space. However, it is more accurate and you almost do not lose I mean your loss of pressure is going to be considerably smaller than that of the orifice meter. So, these are examples of sudden contraction and sudden expansion sudden contraction over here and sudden expansion that would give rise to that would give rise to this kind of known applicability of the known applicability of the Bernoulli's equation for situations like this. Next, we move on to what can what can we do then if Bernoulli's equation is so useful, but at the same time versatile, but at the same time if it cannot be used so what are the modifications that one can suggest.

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+g\,Z_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+g\,Z_2\right)+h_{LT}$$

So, the modifications that are proposed is that the pressure on the upstream side the sum of the pressure the velocity head and the gravitational head at the upstream side must be equal to the sum of the pressure head the velocity head and the gravity head at the downstream side. This is where this is where the Bernoulli's equation ends. However, we know that there is going to be some difference between the two heads because there could be losses present in the system. The losses could be due to friction, the losses could be due to the presence of a sharp change in the direction of the fluid. The losses can be by the introduction of a flow measuring in measuring instrument like a venturi meter or an orifice meter in the flow path or it could also be due to the presence of valves which are used to control the fluid flow.

So, we have all seen valves which are controlling the fluid flow the work flow of water at the tap at our houses. So, that is essentially a valve. Now, there are different types of valves one is the crude off on valve some are where you can control the flow rate and some where you can control the flow rate very precisely. So, the tap this for the valve that we have at our home is something called globe valve. A valve which is only opens and shuts is known as the gate valve and something where you would like to add let us say a reactant or a catalyst or something in precise small quantities at a rate which you can controlled very carefully there are other types of valves which are used for which are which are suitable for that purpose.

Each of these valves will create additional pressure losses during the flow. So, valves like metering devices, the flow measuring devices and the bends which you have to provide in a pipeline like if you look at a pipe look at a pipeline that is pipe that supplies water to your house to your home which essentially it is a straight pipe and then it changes its direction multiple times there could be branches of the main line we should then go to through again some number of bends and valves and it reaches the point of use. So, this is also going to create some losses. So, the losses due to the metering devices, the losses due to the valves, the losses due to the presence of bends in the path, the losses when you have sudden contraction or sudden expansion all these are clubbed together in this term which is the head loss the total head loss. So, L stands for loss, H head.

So, H L is head loss and T signifies the total head loss from all such sources taken together. If I could evaluate what is H L T then and put them on the downstream side of the Bernoulli's equation then essentially, I have an equation I have a relation which can be applied in real situations because it takes care of friction takes care of shape the area changes and everything else. So, this is the practical form of Bernoulli's equation which is mostly used for calculating the pressure drop between two points in real situations. So, what are the major features I have described most of them, but to be to be to complete the discussion the velocities that is one more point the velocities that you have that you have seen before in the Bernoulli's equation the velocities that you have seen here these velocities are along a streamline. So, this velocity is along a streamline.

So, it is a point velocity it is not an average velocity across the flow area, but we understand as from our engineering input that it is difficult to get an estimate of the point velocity, we would rather be more comfortable using an average velocity an area average velocity. So, the bar that you see over v 1 and v 2 in here essentially that tells me that these are area average velocities. So, that is something additional term which we have additional simplification to the Bernoulli's equation that we have incorporated for greater ease of operation. Now, to do that I have introduced something which is known as alpha 1 and alpha 2 these are kinetic energy coefficient, I will describe them in the next slide. So, by incorporating the kinetic energy coefficient alpha I can replace the point velocity by an average velocity.

So, alpha times v 1 square is taking care of the converting the point velocity into an average velocity. So, the significance of that I will discuss in detail in the next slide. H L as I said is the major head is the total head loss. So, the total head loss is divided into two categories one is called a major loss and one is a minor loss. So, what are these? Major loss is essentially the frictional loss that you would get for flow in a straight pipe.

So, major losses are purely due to friction for flow through any for flow of a real fluid through any channel your frictional force is going to give you the major loss. What is a minor loss? If you change the flow direction, if you add a metering device, if you if there is a sudden expansion or sudden contraction those the pressure losses associated with those are collectively called as the minor loss. Now, the major loss and minor loss does not mean that major loss is going to be always more than minor loss. There could be situations in which the minor loss could be more than major loss, but once again major loss comes from the flow in a straight pipe frictional loss, minor loss comes minor losses come from the presence of a metering device, sudden change of a direction, expansion, or contraction in the flow area and so on. And so, there the way to calculate the major and minor losses would be the topic for my next lecture.

Now, one more thing to note here is that the dimension of h l t the loss is energy per unit mass. So, it is consistent with the other terms present in the Bernoulli's equation. All terms in Bernoulli's equation in the way that I have written is energy per unit mass. And if the flow is frictionless, there is no friction. If there is no friction, then alpha 1 and alpha 2 would be equal and of course, there are not going to be any head losses since the friction is absent.

So, this equation is used to calculate the pressure difference between any two points provided we know what is h l t. So, h l t is something that we would like to figure out the way to calculate the major and the minor losses, the sum of these two would give me the total loss. But let us concentrate on slightly more about what is alpha. Now, this kinetic energy coefficient alpha, here you see that I have the point velocity, velocity along a streamline and here I have the average velocity. So, alpha is the parameter which I introduce to make these two identical.

$$\int_A \frac{V^2}{2}\rho V\,dA = \alpha\int_A \frac{\overline{V}^2}{2}\rho V\,dA = \alpha\frac{\dot{m}\overline{V}^2}{2}$$

$$\alpha = \frac{\int_A \rho V^3\,dA}{\dot{m}\overline{V}^2}$$

For laminar flow in a pipe $\alpha = 2.0$

For turbulent flow, large Reynold's number, $\alpha \approx 1.0$

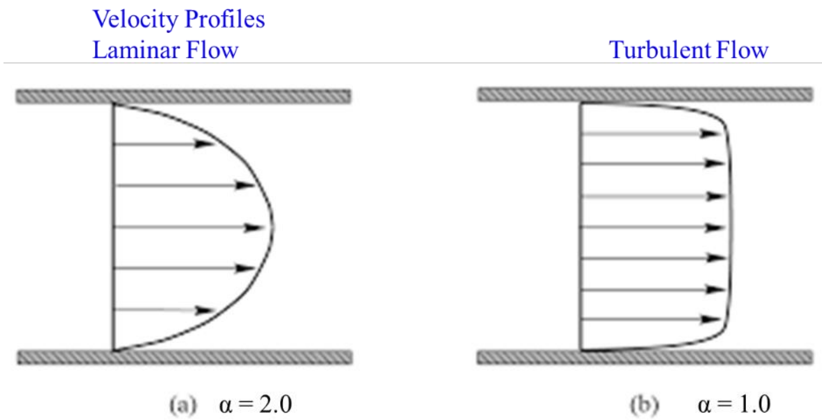
So, the definition of alpha is that the you are going to it is going to be the same when I use average velocity and over here, I use the point velocity. Now, if you look over here rho times v times d A is essentially the mass flow rate, there should be a bar over here. So, the rho times average velocity times area is m dot. So, this is m dot. So, therefore, this expression and v bar since it is already area averaged.

So, therefore, it can simply be taken out of the integration sign and what you have is m dot v bar square by 2. So, this is the defining equation for alpha and alpha is therefore, area integration over rho times v cube v square and v rho times v cube divided by m v square ok. This is the kinetic energy coefficient. Now, we would like to figure out what is the value of the kinetic energy coefficient. If I have a laminar flow in a pipe, then we know what is the expression what is the expression for velocity.

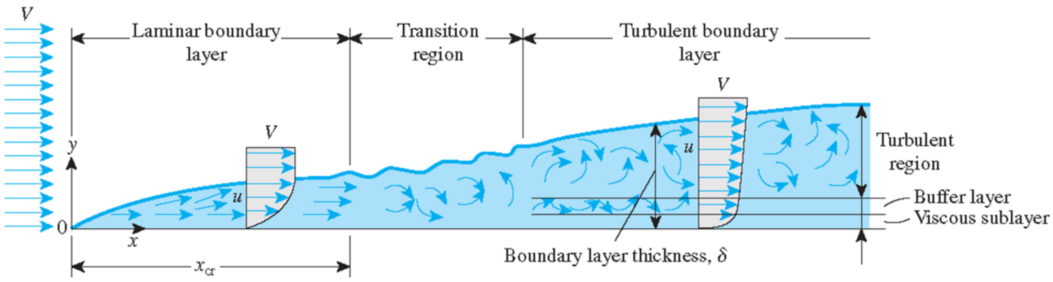
The velocity distribution in laminar flow we know that it is going to be parabolic. So, if it is parabolic and if that form is going to be substituted in here, then I would be able to evaluate I would be able to perform the integration and find out what is the value of alpha that value of alpha comes out to be around to be around 2. So, for laminar flow while using Bernoulli's equation the value of alpha is generally taken to be equal to 2 oks. This comes directly from the definition of alpha and the substitution of the parabolic distribution of velocity and figuring and performing this integration. However, for the case of turbulent flow that is for large Reynolds number situations, the value of alpha is equal to 1 and why it would be so would be clear from here.

Now, here in laminar flow I can see the parabolic distribution the value of alpha can be evaluated analytically to be equal to 2. Over here if you see the turbulent flow there is going to be a sharp variation of velocity near the wall and then the velocity is going to be constant which is known as the turbulent core. The velocity will not be a function of r the distance from the wall and near here where the viscous forces predominate it is going to be a sharp change from this velocity to a velocity equal to 0 due to the no slip condition at the liquid solid interface. So, the turbulent velocity profile looks something like this. Now, if you think about alpha, alpha gives you an idea of how much the velocity departs how much the velocity deviates from an average velocity.

Here most of the flow is moving with a constant velocity, it is as if this is a frictionless flow. Therefore, it tells us that the value of alpha is going to be very close to 1. So, for turbulent flow alpha can be safely taken as 1 whereas, for laminar flow it is going to be equal to 2 and the velocity profile in turbulent flow looks something like this. So, that is going to be laminar for velocity profile. So, it is going to be something like this then that is going to be a transition region and then a turbulent velocity turbulent boundary layer turbulent velocity profile.



So, these are the different ways by different representations of the velocity in a turbulent boundary layer. For the viscous sub layer for the layer in which the velocity is turbulent, it is the velocity profile is given by the one-seventh power law which is quite common. I am going to discuss more about turbulent boundary layer velocity profile later on, but suffice to say that this is the form the one-seventh power law this is mostly used to connect the velocity inside the boundary layer and the free stream velocity. So, for turbulent flow it is y by delta to the power one-seventh, where y is less than delta the film thicknesses the thickness of the boundary layer ok. And we will talk about this more and when it is when the point is outside of the boundary layer the velocity is almost equal to the free stream velocity as we know from the definition of us from boundary layer.



Turbulent Boundary Layer

- All BL variables [*U(y), δ, δ*, θ*] are determined empirically.
- One common empirical approximation for the time-averaged velocity profile is the **one-seventh-power law**

$$\frac{U}{U_e} = \left(\frac{y}{\delta}\right)^{1/7} \qquad y \le \delta$$

$$\frac{U}{U_e} \cong 1 \qquad y > \delta$$

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + g Z_1\right) = \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + g Z_2\right) + h_{LT}$$

$$h_{LT} = h_L + h_{LM}$$

h_L = Major loss due to frictional effects in fully developed flow

h_{LM} = Minor losses due to fittings, entrance, area changes

So, one-seventh power law just keep in mind that one-seventh power law is going to be used mostly for the turbulent boundary layer. Whereas for the case of laminar flow it is a parabolic distribution that is mostly encountered. The this is one the last slide on this which again tells me about this where the total head loss is a sum of the major loss h L and h L m, where h L is the major loss for fully developed flow due to friction only and h L m is due to the fittings entrance sudden changes of area and so on. So, what we are going to do in the next class is to figure out how do we evaluate h L and how do we evaluate h L m such that we can put them back into the Bernoulli's equation and figure out what are the pressure changes or velocity changes due to change in the other three parameters. So, the summary of this is that the Bernoulli's equation can be used provided we incorporate a loss term to the right and the loss term has two components a major and a minor loss which we are going to evaluate in the next class. Thank you.