Momentum Transfer in Fluids
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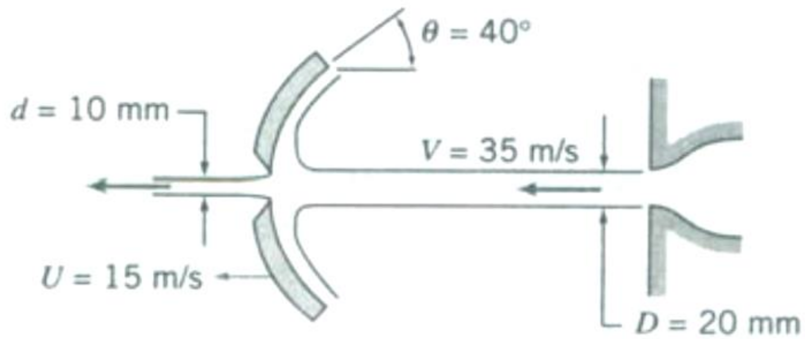
Week-08
Lecture-38

Good morning. This is going to be the final lecture on Integral Equations. So, so far, we have seen how the integral equation can be used for a stationary coordinate system and for a system where the coordinate system is coordinates are moving with a constant velocity. And the last class had two problems about what would happen if let us say a jet strikes a vane. The whole purpose of the vane is to change the direction of the fluid and therefore, extract some amount of momentum in the terms of power and to be used in various applications. Now, one such application as I have seen is in a turbine where the turbine blades change the direction of the steam which is hitting it in the form of a jet and had power which starts moving and therefore, the power is then converted into electrical energy.

So, the kinetic energy of the of the steam is going to be converted into the electrical energy. So, we also have seen that we can use the same equations as before provided we start using only relative velocities. So, the relative at the point of entry and the exit the relative velocities are to be used and when that is used the two the equation the previous equations for the integral approach can be can be utilized without any other modification. So, the last class on this topic would feature two problems and that in both the cases the fluid flow is either obstructed or changed its direction because of hitting a curved surface and what is going to be the power generated what is the force and so on.

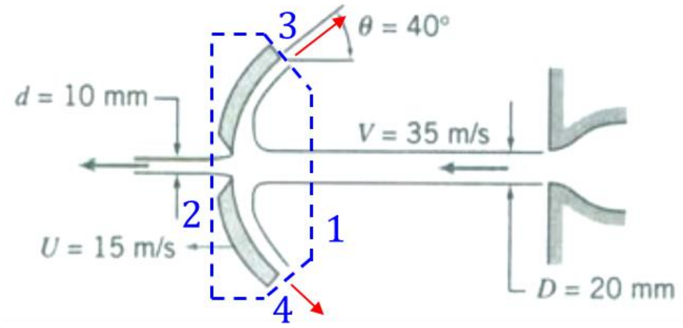


So, the first problem in this case deals with a circular dish what you see over here in the as an outside diameter of 0.2 meter and you have a water jet which is coming with a speed of 35 meter per second. So, this is the water jet which comes with a velocity and it hits the circular dish which has an opening right at the middle of it. So, with this opening some amount of air is some amount of fluid is going to leak out. The diameter of the initial jet is 20 millimeter where the diameter of the jet that leaves through the small hole over here is 10 millimeter.

The jet is striking it with a velocity 35 meter per second as a result of which as a result of this constriction and change of fluid direction in all along the circular path the entire thing is moving to the left with a velocity 15 meter per second. So, a circular dish with a small hole struck by a jet part of the jet goes all along this curved surface part of it goes through the small hole right at the middle and what you need to find out is the force required to maintain dish motion. What is the force that is necessary to maintain the motion of the dish? Here also you can clearly see that this is a case of a coordinate system which is moving with a which is moving with a constant velocity. So, this constant velocity in this case is 15 meter per second. So, now we must calculate the force.

One thing for sure is that we have we need to use the relative velocity. So, depending on how I draw my control volume if this is the point one point the velocity here is simply going to be the relative velocity the algebraic sum of these two. The velocity is also going to be the relative velocity in here. One more point is that if you look the last, but one line the remainder of the jet is deflected and flows along the dish and the main one the main stream is going to pass through the hole without any resistance. So, there is no resistance due to the sudden contraction of area there is no resistance due to friction.

So, as if this jet does not see feel the presence of this curved path and goes unimpeded at least this 10-millimetre centre part is going to go through this without any resistance without any change in velocity without any friction so to say. So, this is an idealized condition and this idealized condition we are going to analyse in the subsequent part. So, this is what it looks like that blue dotted lines are the is the control volume and here the control volume is includes the plate. So, the control volume includes the plate inside this and there are 4 points of entry and exit this is 1, this one is 2 and it goes like this so 3 and 4 though we understand that there is going to be symmetry between point 3 and point 4. So, the assumption is steady horizontal uniform flow.



The uniform flow assumption is something that you have to make in order to use the equations that we are using for integral approaches. No change in jet speed so the jet speed whatever speed with which it was coming with the same speed it goes out. So, here also the velocity is equal to 35 meter per second like this and this can idealize situation this can only happen no friction no resistance. So, I first start with continuity equation relative velocity entry point. So, that is why this exit point it is going to be plus because fluid is moving out of the system a side refers to 3 and 4 this is the relative velocity.

$$(V - U)\left(-\frac{\pi D^2}{4} + \frac{\pi d^2}{4} + A_{Side}\right) = 0 \text{ (Conti. eqn)}$$

$$A_S = \frac{\pi}{4}(D^2 - d^2)$$

So, this is the algebraic sum of all this would be equal to 0 and this follows directly from equation of continuity. Therefore, you would be able to figure out what is the surface area, what is the side area through which it leaves it simply pi by 4 d square minus small d square. Now, we write momentum equation. So, what is going to be the momentum equation? Part x the force is simply going to be equal to rho V A 1. So, I am talking about this part rho V is the relative velocity.

$$R_x = u_1\left\{-\rho(V-U)\frac{\pi D^2}{4}\right\} + u_2\left\{+\rho(V-U)\frac{\pi d^2}{4}\right\}$$

$$+u_3\{+\rho(V-U)A_{3,4}\}$$

$$u_1 = V - U \qquad u_2 = V - U \qquad u_3 = -(V-U)\cos 40$$

So, it becomes V minus u area is pi d square capital D square by 4 that is the area at this point. Now, I come to point 2 the velocity remains unchanged as I have decided that there is no change in jet speed. So, if there is since there is no change in the jet velocity. So, if it is V minus u over here it is V minus u over here and over 3 and 4 as well the area is small d it is leaving. So, that is why it is positive u 2 is going to be the x component of velocity that I must decide later.

Now, the area 3 4 the side area this is essentially A s the A s which we have calculated which we have measure which we have expressed over here. So, this is the side area this one the velocity will again will remain unchanged as it is a frictionless flow frictionless interaction between the solid and the liquid. So, the velocity will still be V minus u the way we have done it over here and u 3 is going to be the x component of velocity at this location. So, the velocities as you see in your continuity equation will be the same at the inlet at outlet through the small hole at outlet through the side area. Only thing that is going to be going to change are u 1, u 2 and u 3.

I hope this is clear to all of you is how we are going to write the momentum equation when the control volume is moving with a with a constant velocity. So, what is u 1? The u 1 over here is simply the relative velocity that is the x component of the velocity at 1 V minus u, u 2 is also going to be equal to V minus u. So, this is sorry this is u 2 u 1 is the velocity over here and we have u 3, u 3 is going to be equal to minus because the x component of the velocity is going to be in this direction which is minus x. So, that is why I have the minus over here times cos 40 the velocity the relative velocity once again times cos 40 the same way we have done it for the for the when moving when problem. So, u 1 is the relative velocity, u 2 is the relative velocity once again, u 1 is equal to u 2, u 3 is the x component of the velocity over here the velocity is V minus u x component of that is cos 40 minus sign because this is in the minus x direction.

So, with this I would put the values in there and you get the complete expression for the R x. Now, you have you know all these V u capital D small d and etcetera. So, plug in the numbers and when once you put the numbers the R x the force that is going to be minus 167. So, the force must be applied to the right. Similarly, R y if you go back to the figure over here R y is simply going to be the weight of this and the normal force must take care of the force in the y component.

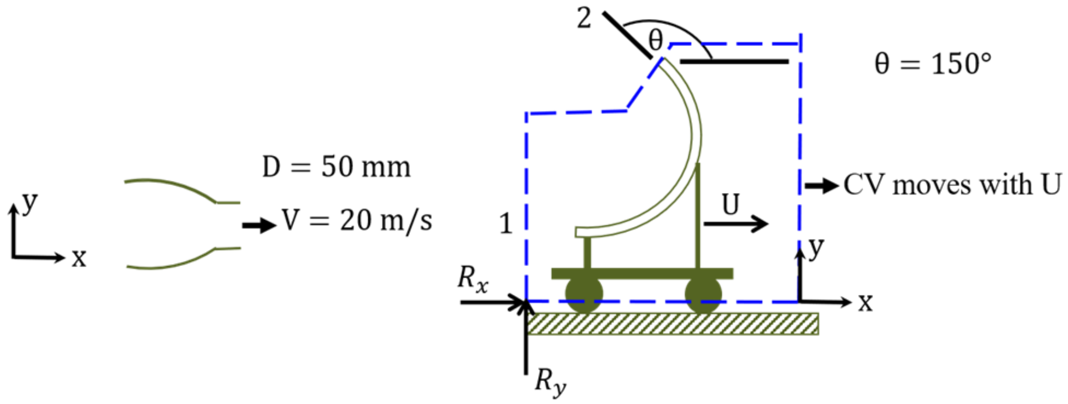
$$R_x = -\rho(V-U)^2\frac{\pi D^2}{4} + +\rho(V-U)^2\frac{\pi d^2}{4} - \rho(V-U)^2\frac{\pi}{4}(D^2-d^2)\cos 40$$

$$R_x = -167\ \text{N}$$

So, the y component of the force is simply going to be mg. So, there is no net flux in the in the y direction and that that is essentially the entire problem. So, once again the crux of the problem is to use relative velocity. Identify from equation of continuity what is going to be the magnitude of the velocity at the point of entry and at the point of exit. In this problem it was made simpler because it is mentioned that there is no change in the velocity due to friction.

$$R_y = Mg,\ \text{Since there is no net flux in the y-direction}$$

So, everywhere it is simply V minus u, but there is a change in the y in the in the x component of velocity at location 3 and 4. So, it is V minus u cos 40 whereas, in location 1 and 2 you have only the x component of velocity and you could figure it out. So, that is one more problem about moving control volume. The last one in this case is also a moving control volume problem similar to the one that we have just discussed. So, water from a jet it is striking a moving it is moving to the right with a velocity equal to u as you can see over here.



The jet diameter is 50 millimetre and its velocity is 20 meter per second. Find the force needed to maintain a constant speed of the vane equal to 5 meter per second. So, essentially this u is having been told to be equal to 5 meter per second and the angle of deflection of the water jet that you see here is 150 degree and the control volume moves with u to the right in the plus x direction and the R x and R y the directions are shown over here. Once again, the surface on which it acts it is a frictionless surface. So, there are no frictional forces opposing the motion.

So, whatever be the change in the whatever momentum net momentum is being added per unit time to the control volume because of the jet and its changing direction will result in the force in a constant force. Therefore, a constant velocity and it starts to move it with a constant velocity to the right and we must calculate what is the force needed to maintain a constant speed of the vane equal to 5 meter per second. So, once again this is the this is the control volume that has been drawn over here and we start with the same equation the x component of the equation of motion. This u x y z is the velocity at for the inertial control volume. It is the velocity which an observer standing outside would see and therefore, this has to be the relative velocity which would make the coordinate system inertial does not move it will appear to be static.

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho\,\overrightarrow{V_{xyz}}.d\vec{A}$$
$$F_y = F_{Sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v_{xyz}\,\rho\, d\forall + \int_{CS} v_{xyz}\,\rho\,\overrightarrow{V_{xyz}}.d\vec{A}$$

So, with this the first thing is we understand that there is no force no body force in the x direction. So, this term would disappear it is a steady state system. So, obviously, this term the first term on the right-hand side would also not be present. Then we talk about the y component of the equation and the y component of the equation for the first thing is that there is no change in the no time variation. So, del del t in this case would be 0 and then we are going to proceed with that the no pressure with the assumptions that there are no pressure or friction force frictional forces.

So, F s x that we have will be only part is going to be equal to R x, F s y only going to be equal to R y and F B x is 0 we are not considering F B y. If you wish you could also include F B y as the weight of this, but you we understand that the weight is going to be exactly counter balanced by the normal reaction force from the bottom surface from the surface over which it moves. There is steady uniform flow at all sections at the entry and the exit. So, the it is a frictionless fluid. So, the velocity is not going to be a function of function of any special coordinates, it is uniform everywhere and with this assumption the first equation can be written as R x E 1 where E 1 is the velocity over here, U 2 is the velocity at this point, this is the entry the mass which enters through 1 per unit time and look once again that I have used V minus U.

So, the relative velocity is used over here and I also understand that as per the statement of the problem the area at 1 and area at 2 the area for flow at 1 and the area for flow at 2 are equal. So, I have used only A in here. So, the only thing that remains in this problem is to figure out what is going to be U 1 and U 2. So, the mass flow rate in with a negative sign mass flow rate out therefore, positive to be multiplied with the components of the velocity at location 1 and location 2 considering it is relative velocity. So, the first one is straightforward the at the entry point there is there is the velocity x component of velocity is simply going to be V minus U at the y at the at the location 2 it is simply going to be V minus U times cos theta which is the velocity in this direction.

$$R_x = u_1\{-|\rho(V-U)A|\} + u_2\{|\rho(V-U)A|\}$$
$$u_1 = V - U \qquad u_2 = (V-U)\cos\theta$$

So, once again this is a straightforward equation straightforward use of the equation if you use relative velocity. So, the idea is that anyone standing outside would have to that person the control volume the coordinate system should appear to be static. Now, in order to do that you artificially give it a velocity equal to minus U, U is the velocity with which it is moving. If I add a minus U and if I am standing outside then this coordinate system would appear as static to me, but the moment I give it a velocity equal to minus U that means, I am using relative velocity for any entry and exit of the fluid. So, that is why at location 1 it is V minus U and location 2 it is again V minus U times cos theta.

$$R_x = \rho(V - U)^2 A(\cos\theta - 1) \qquad \rightarrow R_x = -822\ \text{N}$$

So, once we incorporate all of that what I get is R x the force in the x direction is rho V minus U square times A times cos theta minus 1. When you plug in the values in there this R x is going to be equal to minus 822 Newton. So, this is this is this is the x component of the force. Once again, I would tell you to solve problems like this portion is entirely from Fox and McDonald.

So, this portion is from Fox. So, the book textbook of textbook as I have mentioned in the first one. So, this is the name of the book. So, in the entire portion over here is from Fox and McDonald's book. There are many example problems, many exercise problems and if you keep on practicing with them, I am sure you will not have any problems, but concept I am trying to clear clarify your concepts for the situation where the control system control coordinate system is moving with the constant velocity. The next job that will the last thing that we have to do is find out what is R y.

$$R_y = v_1\{-|\rho(V - U)A|\} + v_2\{|\rho(V - U)A|\}$$

$$v_1 = 0 \qquad v_2 = (V - U)\sin\theta$$

$$R_y = \rho(V - U)^2 A \sin\theta$$

$$\rightarrow R_y = 220\ \text{N}$$

So, if you think about if you look at the figure once again the y component of velocity at location 1, the y component of velocity at location 1 that means, over here it is not there. There is there are no y component of velocity at this point, but there is a y component of velocity because fluid is going out and therefore, there is going to be a component of the y component in the up positive y direction. So, that is why while figuring out what is the R y you may have to make sure that V 1 may be 0, but V 2 will not be equal to 0. However, the mass in and mass out with the proper signs will remain unchanged because of equation of continuity. So, that is the that is the change over here.

So, V 1 as I have mentioned is going to be equal to 0 and V 2 is simply going to be equals to V minus U times sin theta. Let me go back to this and bring the figure over here at this location there is no y component of velocity. So, V 1 is going to be equal to 0 at this location the y component of velocity is going to be V minus U since everything has to be in expressed in terms of relative velocity times sin theta and this is the velocity is this. So, it will have a negative x component as you have seen in the previous slide and a positive y component the positive y component. So, that is why it is positive V minus U sin theta.

So, once you plug this in over there. So, what you have then is r y to be equals to rho times V minus U whole square times a sin theta this is the r y 1 and this is the r x 1 because it has cos theta minus 1. So, it turned out to be negative and we have seen why this r y is going to be positive and once you plug in the values for rho V etcetera you are going to get r y is equal to 220 newtons. So, the bottom line from here is there is a force of 822 Newton to the left and 220 Newton upward must be applied to the vane to maintain its motion at 5 meter per second.

So, once again you have taken the control volume and the control volume encompasses the vane. So, the force on the vane force on the vane it is equal to minus 822 N and since it is minus 822 the force to the left to the left is to be applied and 220 N is positive 220 Newton.

So, an upward force must be applied to the vane to maintain its motion at U equals 5 meter per second. So, if we have to I think it is clear to all of you. So, to recap this specific chapter in the next 2 minutes I am going to talk about what we have covered in here. For the first time after dealing with Navier Stokes equation we have we have thought that it is better to express everything in integral form rather than in differential form. In differential form is advantageous if you want to know the condition at a specific point.

So, if you want to know what is exactly happening at the liquid solid interface, what is happening at the centre line, what is happening midway between the centre line and the solid surface. So, the differential approach would give you a continuous profile for the velocity if you could solve it analytically a continuous profile in the entire flow domain. So, in order to examine the physics of the flow it is very useful. So, you get the information at every point in the flow field, but in many engineering applications you do not need to know that into those intricate details of what is happening in the flow field. Rather you would be more satisfied if an average idea about the system can be can be provided.

Now for this we have selected situations in which there is no frictional forces. If there are no frictional forces then the flow at any cross section is going to be uniform. If there are forces if there are frictional forces then we would substitute the velocity with the average velocity which can be evaluated easily comparatively easily from experiments. So, this gives us the overall integral idea not the exact point information what we get in differential approach, but overall velocity. With this we have written the equation for the system and for the control volume.

And for the special case when the system and the control volume coincide, we obtained the equation the equation that you see the equation essentially tells us about what is the relation between all the forces surface forces. One of the examples of surface forces is the pressure force the example of body force is gravity and then you have the time rate of change of the extensive property in this case the momentum inside the control volume which is the first term on the right-hand side and the net efflux of the extensive property momentum again through the control surfaces. And we have now figured out what is going to be the total amount of total force acting on the control volume acting by the control volume and so on. And finally, lastly, we have looked at what if this entire thing the motion of the fluid is going to create a motion of the control volume itself like what you get in a jet vane assembly. So, since this equation is derived for inertial control volume to use the equation where the control volume or the coordinate system is moving with the constant velocity I need to use relative velocity.

So, as to bring back the condition satisfy the condition of an inertial control volume. So, everywhere then v the velocity is going to be replaced velocity is going to be replaced by the relative velocity and once the relative velocity we replace it with the relative velocity then it is straight forward. So, small u and small u 1 and u 2 that you see over here are the component velocities whereas, v minus u in the mass flow rate are the average velocity are the relative velocities. So, with this we have solved problems of moving control volume where the coordinate system is moving keeping that in mind that all the time we must use relative velocity. So, that is that brings the end of this chapter and from next class onwards we will see a Bernoulli's equation, its development, restrictions, potential use and calculations using

Bernoulli's equation to figure out what is going to be the total pressure drop in a pipeline, what kind of a pump power is to be provided in order to make the fluid flow in the pipe.

So, all those calculations etcetera will be the last part of this course. Thank you.