Momentum Transfer in Fluids
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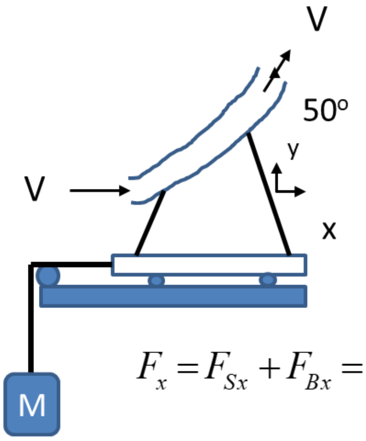
Lecture-37

Good morning. We will continue with our studies on integral equations of the basic the equations that we already know equation of continuity and equation of motion. And we have seen the need for the integral equations where instead of point velocity I can use some sort of an average velocity and do all the calculations and the derivations which would be valid for the entire control volume. If we think of the differential approach to fluid motion and if we think let us say we are talking about Navier Stokes equation the solution of which would give the velocity at every point in the flow field. But as I have mentioned earlier the we do not need in engineering applications we may not need to know the velocity at every point in the flow field rather we would be more than satisfied if we find out an overall behaviour of the system in terms of what is the force to be applied on a certain mass of fluid to make it move with a constant velocity or what is the force that a jet of a fluid jet of a liquid would exert on a plate which is placed in its path. The applications of these integral equations are therefore, many for example, starting with any fluid machine moving machinery for example, a turbine blade when the steam strikes the turbine blade it imparts a motion and this motion is then converted to electricity.

So, what is the force that is going to be exerted on the turbine blade? Now we have seen some examples of the forces acting on the fluid forces acting by the fluid and so on and the application that I was referring to just moments back refers to the movement on the movement initiated by the impact of a fluid stream on a surface which is curved. So, when on a curved surface the fluid impacts the fluid hits a curved surface then part of the fluid is going to be deflected away from the surface and this change in direction of the fluid requires that a force be applied on the fluid on the control volume which in turn would exert an equal and opposite force on the surface on which it struck. So, this kind of a curved surface commonly known as vane they would give us an idea of what is the force to be exerted by a moving fluid. So, in this in the in today's this class and the next class we will see how the motion of a of a fluid stream striking a surface can initiate motion in the in the in the on the to the surface or to the bounding surface on which the fluid is striking.

So, we would like to examine the behaviour of a vane when struck with a fluid. So, we will start the first one with a very simple conceptual problem. The problem that you see over here is it is a vane and which is which is situated on a frictionless platform and in this frictionless. So, this is the this is the vane and we have the it is resting on a platform and there is no friction. Now some fluid is going to strike the vane it will then change its direction and will exit from the top ok.



Vane on a frictionless platform with a block of mass, M, attached to it

V = 15 m/s, A = 0.05 m^2, θ = 50°
Find the mass, M, Needed to hold the cart stationary

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\, \vec{V}.d\vec{A}$$

This angle by which it turns is 50 degrees. So, at this point it creates an angle of 50 degree with the horizontal. In order to prevent the vane from moving in the plus x direction as a result of the fluid striking it a mass m is attached to the vane with a pulley. So, there is going to be a force due to the weight of this mass m that is going to act in the minus x direction. So, this there is a there is an externally applied force in the minus x direction and which will exactly counter balance the force exerted by the moving fluid when it hits the vane changes its direction and goes out of the other end of the vane.

So, this is the problem which we are going to we are going to analyse. Now, if we see the properties the I mean the variables are provided as the velocity of the of the this the jet that strikes its 15 meter per second the area is 0.05-meter square and the angle by which it changes its direction is 50 degrees. So, we need to calculate the mass m that is needed in order to keep the vane cart stationary. The one point that I would like to mention here which would we would have to stress again in the subsequent problems is the equation that you see the F s the force in the x direction is going to be some of the surface forces in the x direction the body forces in the x direction which would be the time rate of change of the intensive time rate of change of the extensive property there is momentum in this case since its momentum equation within the control volume.

And then the second term on the right-hand side is through the control surfaces through all the control surfaces that define the control volume how much of momentum per unit time is entering or leaving the control volume. However, this equation or any such equation that you see is for a stationary coordinate system. So, you know we can apply this equation without any modification since the mass the m that you see in the figure over here the mass m essentially makes essentially makes the surface makes this entire coordinate system with 0 velocity it makes it static. So, the coordinate system that you see x y in the figure it does not move at all had it been moving then I would not be able to use this equation in the in the way it is written in over here. The u that you see is the x component this u is the x component of the velocity.

So, when you see the x component of the velocity over here and the x component of the velocity here you can see a change. The magnitude of the velocity will remain unchanged. So, if you have v over here with a certain cross section and if the cross section does not change and if the fluid is incompressible with it will go out of the vane with the same velocity capital V. However, the x component of the velocity will change and we will be able to use this equation because this is a system in which the coordinate system is fixed in space due to the weight of due to the mass m which keeps the vane stationary. So, this point is to be very clearly understood that the equation is for a stationary coordinate system can be used only when it does not have a velocity of its own had it been there then in we have to think about using relative velocities.

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\,\rho\vec{V}.d\vec{A}$$

So, we will come to that later. So, again starting with the x component of the equation we are not writing the y component of the equation there would be a y component of the equation I will discuss about it at the time. So, what we have is that you have a body force which is equal to minus m g its acting in the minus x direction and then you have this one refers to the mass flow which is into the control volume at this point at the entry point what is the mass flow rate to the control volume since it is in. So, we have this negative sign and over here at point 2 at the exit of the exit from the vane this is again the mass flow rate and we understand since this is incompressible rho 1 v 1 a 1 must be equal to rho 2 v 2 a 2 we also understand that that a 1 is equal to a 2 rho 1 is equal to rho 2 since its incompressible. So, you have v 1 is equal to v 2.

$$-Mg = u_1\left\{-\left|\rho_1 V_1 A_1\right|\right\} + u_2\left\{+\left|\rho_2 V_2 A_2\right|\right\}$$

$$u_1 = V \qquad u_2 = V\cos\theta$$

$$M = \rho V^2 A\left(1-\cos\theta\right)/g$$

However, this u 1 and u 2 are going to be different and this is plus since the mass is leaving the control volume. So, we can clearly see that the u 1 the velocity the there is no y component of velocity there is only the x component of velocity. So, its equal to v whereas, in u 2 the location to the x component of velocity that is in this direction is simply going to be v cos theta where v this theta is equal to 50 degrees. So, we my mass flow rate remains constant, but the momentum flow rate becomes different at point 1 and point 2. This difference in the momentum flow rate is going to be counter balanced by the weight m by the weight mg which I have attached to the vane which is pulling it to the left with the velocity pulling it to the left with the force equal to mg and that is why I have this mg.

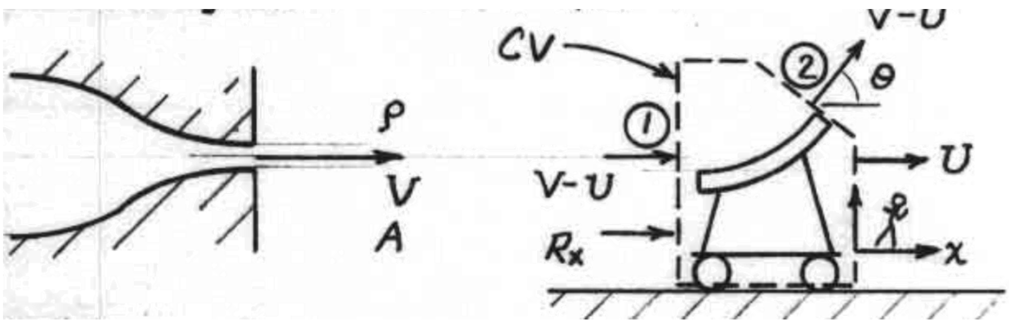
So, the net flux of momentum is counter balanced by a body force which is equal to minus mg for the coordinate system that we have chosen here. The velocity at point 1 is the x component of velocity and at point 2 its again the x component of velocity and when you put them into the equation and simplify what you would get is m the mass needed to have the system coordinate system static is simply going to be this. You can put the values of rho v a theta etcetera and figure out what is the numerical value of m in this case. One point here I would like to mention is that I did not do I did not calculate the y component of the force. The y component I could I could similarly write what is the y component of the equation of motion over here.

In this case the x will be substituted by y and this u is simply going to be equal to small v where small v is the component of velocity y component of velocity at point 1 and then point 2. So, when you think about the y component of velocity at 1 at this point it is simply going to be equal to 0 there is no y component of velocity. Over here the three is going to be an y component of velocity at this point the y component of velocity we can we can we can find out since we know the angle. So, the difference in there is going to be a difference. So, this is being 0 that means, the first term is going to be 0 and this this is going to be v sin theta times rho 2 v 2 a 2.

So, there is going to be a force there is going to be a force which is R y which is going to act on this, but the entire force which is acting downward is going to be balanced by the normal reaction force and therefore, it is not going to impact any motion any perceived motion of the control volume. So, we could do the y component balance as well figure out what is f f B y or what is R y, but we understand that it is simply going to be the normal reaction force and it will be balance it is going to balance everything. So, this is in a in a nutshell what this problem the vane problem is all about. Now, I was I told you that the equation that we have seen previously or we have used so far is for a control volume moving with constant velocity is for a control inertial control volume that is a control volume which does not have any velocity, but what if the control volume moves with a constant velocity. So, the from the inertial control volume now I have moving control volume and they will have to be brought into their modifications are to be added to the entire equations.

So, that my moving control volume can be converted to an inertial reference frame capital X Y Z. So, instead of small x small y small z coordinate system I now must define a new coordinate system where it is inertial and the way I can make a control volume moving with a constant velocity I can make it inertial if I start using relative velocities. So, if I use relative velocities then to an observer the control volume will remain inertial and the previous equations can still be used, but in all cases the velocities are to be taken as the relative velocity. So, the trick to use this because the same equation for the for the moving contact moving control volume moving with a constant velocity control volume is to use everything is expressed in terms of relative velocity. So, all velocities are measured relative to the control volume and all-time varied derivatives are also measured relative to the control volume.

So, the relative velocity the relative part is going to be very important it must be kept in mind and I am going to demonstrate how to do that using an example. So, the example that you see here is from a jet from a nozzle a jet of a fluid is hitting this control this vane which is on a frictionless rail on a smooth surface. So, there are no frictional forces no opposing frictional forces no forces opposing the motion and because of which the vane starts to move to the right with a velocity equal to capital U. So, v is the velocity absolute velocity of the jet and U is the velocity of the control volume, but to note that this is a constant velocity with which the control volume is moving. So, that is important and what we need to find out is expressions for the forces R x and R y exerted by the vane.



Second is what is the power produced by the vane because as I told you that one of the purposes of this exercise is to find out what kind of a power that this vane which deflects the fluid stream hitting it how much power it can produce. In what is going to be the relation between U and V, V is the constant velocity of the jet, U is the constant velocity initiated by the jet striking the vane. So, what it what should be the relation between U and V to maximize this power. So, that is that is the problem which we are going to solve and clarify our in our mind what could be the velocities to be used for this case. And as I have said that there are no surface forces and no frictional forces and the jet speed does not change.

Even though the vane starts moving to the right moving away from the nozzle, let us say the nozzle is also moving and hitting the vane and therefore, the jet speed V capital V does not change it is not a function of x. So, wherever the vane is it is struck by a constant jet velocity of capital V. So, with this we go on to solve the problem. First, I write the equation the integral equation, but note that I have written here is U x y z. So, this x y z is the ins is the change that I had to make to make the control volume inertial such that the control volume remains steady that is what I must provide.

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho\,\overrightarrow{V_{xyz}}\cdot d\vec{A}$$

$$R_x = \int_{CS} u_{xyz}\,\rho\,\overrightarrow{V_{xyz}}\cdot\overrightarrow{dA}$$

And this is the accumulation at steady state this part is going to be 0 and this is the net addition of momentum to the control volume and we know what does it signify. So, the first term since it is not there and there are no surface forces. So, the R x is simply going to be the net efflux of momentum into the control volume U x y z V x y z this x y z refers to the changed control volume the velocity with which they to consider the velocity with which the control volume is moving as a result of the jet. So, I expand this is rho V x y z A 1 that is the point 1 and since it is entering the momentum is coming in to the or the mass is coming into the control volume. So, that is why it is negative and it is positive over here and we note that A the entry points the area and this area are these two areas are the same.

$$R_x = \int_{CS} u_{xyz}\,\rho\,\overrightarrow{V_{xyz}}\cdot\overrightarrow{dA}$$

$$= u_{xyz_1}\{-|\rho V_{xyz}A_1|\} + u_{xyz_2}\{+|\rho V_{xyz}A_1|\}$$

$$V_{xyz_1} = V - u \qquad V_{xyz_2} = V - u$$

$$u_{xyz_1} = V - u \qquad u_{xyz_2} = (V - u)\cos\theta$$

So, I have simply used A 1 it is an incompressible fluid. So, rho remains unchanged as well. Now, we have to see what is U x y z and what is V x y z. The V x y z at 1 is simply the is simply the relative velocity. So, it is V minus U, the V x y z is also the relative velocity V minus U these two V x y z must be equal in order to satisfy equation of continuity.

So, the total mass which enters through this has to go out over here and while expressing the total mass I am simply using the relative velocity. So, these two are the ones inside the mod sign the ones that represent the total amount of mass coming into the control volume will remain the same. Next terms what is U x y z 1? If you think about U x y z 1 that is also the relative velocity. So, it becomes V minus U, but with the conceptual part comes in here what is going to be U x y z 2, what is going to be the x component of velocity at location 2. So, the x component of velocity at location 2 would simply be whatever be the velocity V x y z 2 times cos theta.

So, it is V minus U times cos theta. So, if you once again I explain this because this will come this will going to be a recurring theme in many of the problems involved with moving control volume. Your mass flow rates at two points are the same velocities are to you to be used are relative velocities to make the coordinate system inertial. Only if the coordinate system is inertial, I will be able to use the integral equation. So, the to make it inertial the relative velocity at 1 and similarly at 2 would simply be equal to V minus U.

However, the component x component of the velocity at location 1 is simply going to be equals to V minus U the same as V x y z 1 whereas, at location 2 it is going to be the cosine component of the velocity as you could clearly see from here. So, this concept is to be kept in mind all the time that you have to use relative velocity all the always. So, we get into the next part. So, when once you put the expression for U x y z 1 and U x y z 2 you get the force is simply going to be rho times V minus U square cos theta minus 1 and therefore, the force exerted that is the force exerted on the control volume on the fluid mass and therefore, the fluid mass is going to exert an equal and opposite force on the vane which is denoted by this k x. So, this is the force this is the this is the force on the vane and which is equal to the minus of the force exerted on the fluid so you will simply get this expression for the force generate force on the vane.

$$R_x = \rho(V-u)^2(\cos\theta - 1)$$

$$k_x = -R_x = \rho(V-u)^2(1-\cos\theta)$$

Power produced by the vane:

$$k_x \times \text{velocity} = \rho(V-u)^2 U\, A(1-\cos\theta)$$

Maximum power produced by the vane:

$$\frac{dW_{Out}}{dx} = 0 \quad \Rightarrow \quad \boxed{u = \frac{V}{3}}$$

And what is the power produced by the vane? We understand that power is simply force times velocity ok. So, Newton times meter per second so force times velocity so Newton times meter per second is work meter per second is work done per unit time and work done per unit time is the power. So, k x into velocity and therefore, this multiplied by the velocity is going to be the power produced by the vane. And what is the maximum power produced by the maximum power produced by the vane? The maximum power produced by the vane would be we really have to figure out what is d w out by d x you make it equal to 0 and when you make it equal to 0 what you are going to get is that the velocity with which the v and u are related as u equals v by 3 oks. So, the relation between the velocity of the vane striking this striking velocity of the jet striking the vane and the and the velocity of the control volume that the vane they must be related to be u equals v by 3.

And this is simply d w out it is some simply the derivative of the work done with respect to the variable. So, this x could be equal to u ok. So, this would give you and this has given you an idea of how to tackle problems of a system in which it is moving control volume in which it is moving control volume. So, if I recap it once again, I would go back to this the initial statement that I have made over here once again the equation that you see here is valid for a stationary coordinate system inertial coordinate system. So, to an observer the control volume must be inertial.

So, in this in this in this problem the control volume was moving with a velocity equal to u to the right. So, for me to have to make this stationary I must artificially impart a velocity equal to minus u on this. So, as that u on this side and minus u on this side makes this stationary. How can this be made? This can only be made if I use relative velocity at the point of exit entry and at the point of exit. So, then to an outside observer it would have it would be correct to say that my inertial my reference frame the is the coordinate system is inertial is static in space.

So, the velocity with which it enters is v minus u the velocity with which it leaves is also equal to v minus u when we are talking about mass conservation. Since mass in is equal to mass out. So, v minus u will remain the same at both entry and exit. When you are talking about the x component of the mass which is coming in. So, the x component is simply going to be the mass flow rate at the in multiplied by the velocity and the velocity component as the figure suggests is simply going to be equals to v minus u, but when you talk about the x component of the velocity at the top it is going to be v minus u times cos v minus u times cos theta ok.

So, this essentially is the problem where you the this is probably the most important slide where it clearly shows what are going to be v x y z u x y z cap v x y z 2 and u x y z 2. So, once you follow this rule that you use that use only the relative velocity and nothing else. Component velocity is because of the angle could be different, but the velocities the velocities capital Vs are going to be identical at the inlet and at the outlet, but even though they are identical we must use the relative velocity to satisfy the condition of inertial reference frame inertial coordinate system. So, I hope this is clear to you and we will have more problems in the coming class as well as in your tutorials.

So, we on an assignment. So, we should hopefully clarify any doubts that you may have. Thank you.