Momentum Transfer in Fluids

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Week-08

Lecture-36

The fourth lecture on the Basic Equations in Integral Form would deal with a number of problems and each one is different from the others in terms of application and in terms of the methodology which is used to solve them. So, far we are dealing with a system which is fixed in space and therefore, the coordinate the origin is fixed. I have mentioned that all velocities to be used in this equation are relative to the control volume. So, we will see not in this lecture, but in the next lecture a situation in which the control volume itself is moving with some velocity. But for now, this lecture we are only going to deal with situations like what we have done before that the forces due to the body force and due to the surface force would be equal at steady state if there are no transient term will be equal to the net efflux of momentum through the control surfaces to the control volume. So, we move on to the next slide which tells us once again the same equation that is the x component and we have discussed how to decide about the sign when it is going to be positive for the case of flow out and negative for flow in.

Scalar Component

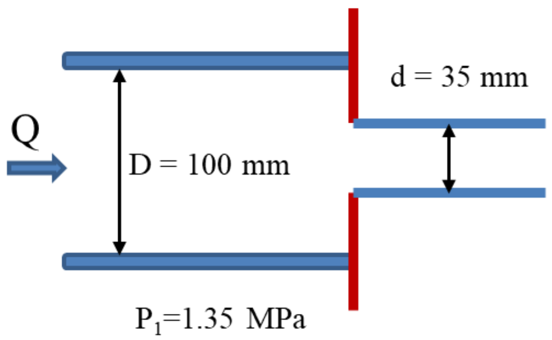
$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho \, d\forall + \int_{CS} u\,\rho\,\vec{V}.d\vec{A}$$

1. To determine the sign of $\rho\,\vec{V}.d\vec{A} = \pm\left|\rho\,V\,dA\cos\alpha\right|$

2. To determine the sign of each velocity component

$$u\,\rho\,\vec{V}.d\vec{A} = u\left\{\pm\left|\rho\,V\,dA\cos\alpha\right|\right\}$$

And then each one of those mass flow rates with appropriate sign are to be multiplied with the component of velocity in this case the x component of velocity. So, this is the guiding principle which we will use in solving problems in this class as well. So, the first one is about a flat plate orifice what you see in the figure. So, what you have here is there is some flow which is coming through a larger pipe with D equals 100 millimetre and then the red one is the orifice plate which is circular with a hole of diameter 35 millimetre at the centre.



So, the orifice plate is placed like this and where from a large diameter a pipe the flow is directed through the orifice to the outside. One and two are essentially this essentially the inflow and outflow points for this. The pressure in the pipe is 1.35 mega Pascal and once again note that this is the gauge pressure not the absolute pressure. The other end too is open to atmosphere.

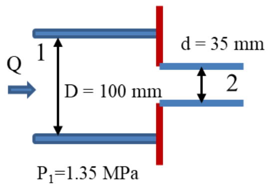
So, whenever you see a jet let us say by a by some fire the by a by a firearm by someone a fireman then there is a larger diameter pipe and a nozzle is connected to the other end through which the liquid water is going to come out with a high velocity. So, that is the that is the common arrangement. We need to we can neglect friction on the pipe wall and we must find what is the force needed to hold the orifice plate. The once again the red is the orifice plate the purpose of which is to reduce the flow area to increase the velocity such that the liquid which is coming out of two is delivered to a point to a far to a point far from the person who is holding the pipe. So, this is again I am going to write the x component and in the x component I understand that I have the flow rate mentioned I have the flow rate q that is known the diameter is known.

So, I should be able to find out what is the velocity because q the volumetric flow rate is simply v 1 times a 1 and the continuity tells me that v 1 a 1 must be equal to v 2 a 2 since there is no other escape or entry of the fluid which is passing through one must pass through two with a reduced area. So, I can calculate v 1 as q by a 1 and which is 6.3 meter per second whereas, v 2 is about 52 meter per second. So, there is considerable increase in the flow velocity because of the constriction in the area between 1 and 2. So, start with this and R x is going to be the force p 1 g is the surface force due to the due to the pressure over here and the pressure related force is acting in the x direction.

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho \, d\forall + \int_{CS} u\rho \vec{V}.d\vec{A}$$

$$Q = V_1 A_1 = V_2 A_2; \quad V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = 6.37 m/s$$

$$V_2 = \frac{Q}{A_2} = \frac{4Q}{\pi D_2^2} = 52.0 m/s$$



So, it is p 1 g a 1 there is a pressure over here which is acting in the reverse direction, but this pressure is equal to the atmospheric pressure. So, if this is the atmospheric pressure if 2 is open to atmosphere then the gauge pressure at that point is equal to 0 because gauge pressure is atmospheric pressure the pressure minus the atmospheric pressure. So, over here the gauge pressure is 0 and there is no contribution of pressure force the surface force at point 2 which need to be which needs to be considered on the left side left hand side of this equation. So, since p 2 g is 0 therefore, there is no mention of that term over here and then I have the E 1 the velocity at location 1 with the mass flow rate coming in that is why it is negative u 2 the velocity out at this point the mass flow rate with the positive sign since the flow is going out and then my E 1 is v 1 u 2 is v 2 the values of which I have already calculated in the in the previous slide. So, once you plug that values in this R x are going to be 8.32 kilo Newton and the point to note here is that the gauge pressure is to be used for this all problems of this type. So, this is a fairly straight forward problem the only point to note is the use of gauge pressure is highlighted in this specific problem. The next one is something which you which you see very commonly in let us say supply of water in your in at your home you have some water. So, a pipeline comes in then it bends and it bends and the water is directed to some other point. So, under such situation the flow since the flow path is changed by the presence of a bend like this there is going to be a force which is felt by the wall the outer wall where the flow direction is going to be forcefully changed to a new direction.

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho \, d\mathcal{V} + \int_{CS} u\rho \vec{V}.d\vec{A}$$

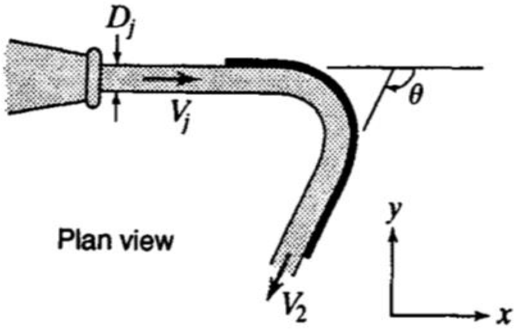
$$R_x + p_{1g}A_1 = u_1\{-\rho Q\} + u_2\{+\rho Q\} = (V_2 - V_1)\rho Q$$

$$u_1 = V_1 \qquad u_2 = V_2$$

$$R_x = -p_{1g}A_1 + (V_2 - V_1)\rho Q$$

$$R_x = -8.32kN$$ Gage pressure is to be used

So, whenever you change the flow path you have to apply you have to apply a force to change its direction. So, that is why this this this this black proportion is going to experience a force. Now, what is this? This is this is a blade is put in the path of the water jet. So, this is a water jet with some velocity a blade is put in its path to change the velocity of the jet to change the direction of the jet in this path. Assume that friction is negligible so, there is no question of adding friction we will add friction in when we deal when we are going to deal with the application of Bernoulli's equation for real fluids.



So, real fluids they have they have viscosities which are not 0. An ideal fluid is the one which has 0 viscosity. If an ideal fluid flows through a conduit because of its 0 viscosity there is not going to be any friction experienced by the fluid as it flows through a solid conduit, but that would not be the case because all real fluids have some viscosity. It may be small, but it is always going to be there. So, a frictional pressure drop is to be added to the Bernoulli's equation to balance the two sides.

So, sum of pressure head and gravity head and velocity head we used to write it as at location 1 is going to be equal to the sum of pressure head, velocity head and the kinetic head at location 2, but these two are not going to be equal for a real fluid. Some amount of pressure drop due to viscous forces will take place when the fluid flows from 1 to 2. So, that additional pressure drop due to viscosity is to be added to the right-hand side of Bernoulli’s equation in the form of head loss because of viscosity. But for this specific problem we are not dealing with we are dealing with an ideal fluid in which case we can assume that the friction the effect of friction is negligible. So, there is no friction to talk about and theta the turning angle is 115.

So, this angle by which the fluid is made to turn is 115 degree and that the water jet has a velocity of 25 meter per second, the velocity of the water jet is 25 meter per second and the diameter d J over here is 40 millimetres. Find the component of force acting on the blade, the blade is shown by the black portion over here in the direction of the jet. So, the direction of the jet is in the x direction. So, what is the force due to the jet in the x direction on the blade that is what we must calculate. And secondly, the force component normal to the jet.

So, the force the jet is in the x direction. So, normal to the jet that means, it is going to be in the y direction. So, this is the coordinate system towards the right is the positive plus x over here is plus y. So, we need to figure out what is the force in the x direction and in the y direction and the magnitude and direction of the resultant force. So, what we are going to do in part a and part b are the are the components the x component and y component of the force exerted by the jet on the blade.

And in part c we are going to find out the resultant force exerted on the blade. The so, we will start first with this x component and I am not writing everything over here. The x component has no body force, it the x component also has no force due to pressure. Because this pressure over here since it is a jet coming through air. So, the pressure over here is atmospheric it is going to be go it is going to go out to atmosphere.

$$R_x = u_1\{-|\rho v_1 A_1|\} + u_2\{+|\rho v_1 A_1|\}$$

$$u_1 = 25 \text{ m/s}$$

$$u_2 = 25\cos(115) \text{ m/s}$$

$$R_x = -1117 \text{ N}$$

So, the pressure over here is also atmospheric. So, the so, the gauge pressure at location 1 and gauge pressure at location 2 are both 0. So, if they if they are and there is there is there is there is no friction in here. So, if they are both 0 that means, the contribution due to surface force principally due to pressure on the momentum transport net momentum transfer is going to be 0. So, I am only I only end up with an r x on the left-hand side.

When I come to the right-hand side this is the mass flow of the jet of air and with a minus sign since mass is coming in multiplied by E 1 which is the x component of velocity at this point. Over here the mass flow rate with a positive sign and u 2 is going to be the velocity component x component of velocity v 2 at location 2. So, these so, u 1 is straightforward u 2 will be obtained from the geometry of the bend that is the value of theta which is provided in the problem. So, u is 25 meter per second and u 2 is going to be 25 times cos 115 this value is 115 degrees. So, that is going to be the value of u 2.

So, once you plug in the values in putting the values of rho a etcetera the value of the diameter is also provided the diameter of the jet you are going to find out that r x is going to be minus 1117 Newton. So, the only difference as compared to the previous problems is that we must take the x component of velocity at location 2 based on the angle of deflection of the jet by the blade. Now, we come to the right through the y component. In the y component once again, there is not going to be any force due to pressure and what we have then is whatever be the y component of velocity at this point the y component velocity at this point and the mass flow rate mass flow rate of the liquid and the associated momentum which is coming through this. Now, over here you can clearly see that v 1 is going to be equal to 0.

$$R_y = v_1\{-|\rho v_1 A_1|\} + v_2\{+|\rho v_2 A_2|\}$$

$$v_1 = 0 \quad v_2 = -V_2 \sin(115)\ m/s$$

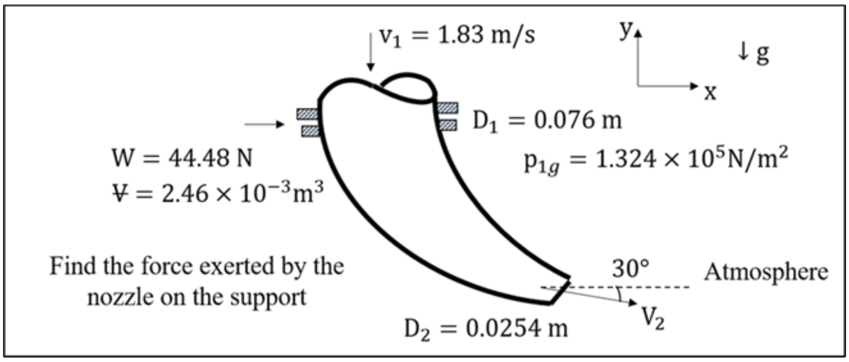
$$R_y = -712\ N$$

Force on the blade +1117 N and +712 N

So, there is no y component of velocity in here and the v 2 is simply going to be minus of v 2 sin 115 it is going to be directed in the in this direction. So, you are going to find out that r y is going to be minus 712 Newton. So, these are force on the control volume. The control volume is defined as the that of the jet. So, if the force on the control volume is minus 1117 in the x direction and minus 712 in the y direction.

So, the force on the blade on the confining surface is going to be negative of that. So, it is going to be plus 1117 and plus 712 Newton. So, that is about this problem where something is deflected from its original path. The last problem before we end this class is about also about a reducer, but you can see that it is a complicated nozzle that has been shown over here. The nozzle is kept in its place by a support over here.

So, the nozzle from the top of the nozzle there is going to be some velocity which is entering the nozzle, it is changing its direction and then it goes out in the angle over angle over here is 30 degrees. So, v 1 value is known, v 2 value is unknown, this is vertically entering vertically downward entering the control volume, this is going out with some angle. These are the x and y directions, this is the coordinate system and of course, g is acting in the minus y direction. The weight of the dry nozzle is 44.48 Newton. So, the weight has been converted. So, it is essentially mg where m is the mass of the nozzle multiplied by g which is a force which should have units of Newton. So, the weight of the dry nozzle is 44.48 Newton and when it is in operation there is going to be some amount of some amount of volume of the liquid present in present in here that volume which will be occupied by the liquid is 2.46 into 10 to the power minus 3-meter cube.



The diameter at location 1 is 0.076 meter, the pressure at the gauge pressure the g denotes the gauge pressure. So, the pressure at 1 is 1.3 times 10 to the power 5 Newton per meter square where whereas, this portion is open to atmosphere. So, that is the description of the problem and the diameter at the inlet and inlet and the outlet are also provided. Now, you we must figure out what is the force exerted by the nozzle on the support.

So, there would be some control if the control volume is that of the liquid any force, we calculate out of the momentum equation is going to be force on the liquid. The force on the liquid so therefore, it will act a force equal and opposite on the nozzle. Now, the nozzle to keep the nozzle in place a force equal and opposite to that on the nozzle is to be supplied by the support. But in this case, we figure we need to figure out what is the force exerted by the nozzle on the support by the control volume on the nozzle and the nozzle is going to apply the same force on the support that is what we need to figure out. So, we have to be careful about we have to we have to carefully read the statement of the problem in order to figure out what is required, what is needed, is it the force on the control volume by the control volume or to keep the control volume in its original position an external agency has to apply some force.

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho \, d\forall + \int_{CS} u\,\rho\,\vec{V}.d\vec{A}$$

So, what is required and that would depend on how you have chosen the control volume and do it correspondingly because keep in mind that any force that you get out of this equation is force on the control volume ok. So, we are going to use this equation once again to figure out all these terms. So, this is the same figure and I am going to start with the force in the x direction. Now, if you look at the force in the x direction there is no body force ok, there is no body force in the x direction in the in this x direction. So, whatever be the body force is going to act in a direction of minus y, no body force in the x direction.

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho \, d\forall + \int_{CS} u\,\rho\,\vec{V}.d\vec{A}$$

$$R_x = \underbrace{u_1}_{=0}\left\{-\left|\rho V_1 A_1\right|\right\} + u_2\left\{+\left|\rho V_2 A_2\right|\right\}$$

$$\boxed{R_x = \rho V_2^2 A_2 \cos\theta = 117.3N}$$

$$R_y - p_{1g}A_1 - W - \rho g \forall = v_1\left\{-\left|\rho V_1 A_1\right|\right\} + v_2\left\{+\left|\rho V_2 A_2\right|\right\}$$

$$v_1 = -V_1 \qquad v_2 = -V_2 \sin\theta$$

$$\boxed{R_y = 616N}$$

And then what about the pressure force? The pressure over here is more than the atmospheric pressure is 1.3 times 10 to the power 5 Newton per meter square. So, this gauge pressure this is more than atmospheric and the gauge there is some value of the gauge pressure that is given. But the force due to this pressure is going to act in the minus y direction, there is no component of this pressure force in the x direction. So, there cannot be any force at location 1, any surface force, any pressure force at location 1 on the control volume that we have chosen.

Once we come over here the it is at an angle with the angle with the horizontal axis. So, if the pressure at this point had the pressure at this point more than the atmosphere, then there would have been a component of pressure force in the plus or in the minus y direction minus x direction. So, if there is if the pressure here is let us say 1.2 times the atmospheric pressure. So, then the gauge pressure is going to be equal to 0.2 atmosphere. So, 0.2 atmosphere multiplied by the area would be the would be the would be would be the component of that with the proper angle would be the force in the x direction. But here it is mentioned that it is atmosphere it is open to atmosphere. So, there cannot be any contribution of pressure force due to pressure at location 2. Thus, the left-hand side will only contain r x, it is a steady state situation. So, the right-hand side the first term on the right-hand side would also disappear.

So, what I should have then is r x equal to the net efflux of momentum through the control surfaces that is what I have written over here. So, this u 1 is the x component of velocity at this point, u 2 is the x component of velocity at this point, this term under the mod sign that is the flow rate with a minus at 1 since the flow is in the mass flow rate with a plus since the flow is out. Now, if you look carefully, you would see that u 1 the x component of velocity at location 1 would be 0. There is an x component of velocity at location 2. So, the this is going to be equal to 0 and u 2 is simply going to be equal to v 2 cos theta, u 2 over here is simply going to be equal to v 2 cos 30.

So, you plug in the values of rho v 2 a 2 and cos and the value of theta and this is the expression this is the number for force in the x direction that you would get. And the since the velocity and the flow rate the flow rate and the diameter are known and in fact, the diameter the velocity over here is known. So, the velocity and the area is known. So, v 1 a 1 must be equal to v 2 a 2. So, here v 1 is known sorry here v 1 is known and the area which is pi d square by 4 that is known over here the area is known since pi d 2 square by 4 is the area.

So, you should be able to calculate what is the unknown v 2 at this point that is what you are going to use here and v 2 is simply obtained from 1.83 multiplied by the area is equal to area at 2 multiplied by the velocity. So, this r x is going to be equal to 117.3 Newton. Now, comes r y. So, what is r y? What are the what are the terms that we need to consider at r y? The first one the I mean the second one is whatever be the pressure force whatever be the force due to pressure at location 1. Now, we can see that the force due to pressure is going to be in the minus y direction. The force due to pressure is directed downward. So, according to the coordinate system that we have chosen this force is going to be equal to minus and the also once again the pressure is going to be the gauge pressure multiplied by A 1 where A 1 the diameter is given.

So, you should be able to obtain what is A 1. W which is again acting downwards the weight of the dry nozzle. So, W is acting in the minus y direction. So, that is why it is negative and this v is the volume of the reducer entire volume of the reducer it is filled with a liquid whose density is given rho the numerical value is given in the problem and g. So, this is the weight of the fluid weight of the liquid contained in the reducer and the force due to that is acting in the minus y direction. So, that explains the entire left-hand side of the entire left-hand side of the equation.

Now, come to the right-hand side. This is the mass flow rate at 1 minus sign denoting that it is mass flow rate in this is multiplied by v 1 which is the velocity over here and I could see that v 1 is simply going to be these 1.83 meter per second and this is the mass flow rate out. The mass flow rate out is equal to the mass flow rate in, but opposite in sign since it is out multiplied by v 2. So, this v 1 is going to be equal to minus v 1. The minus v 1 because it is in the minus y direction and v 2 over here is going to be equal to minus v 2 sin theta since the y component of v 2 is going to be in the minus y direction v 2 sin theta with a minus sign in it.

So, I have correctly identified the pressure force as one component, the weight of the nozzle, the weight of the fluid contained in the nozzle, the mass flow rate through A, the mass flow rate through 2, the minus and the plus sign, the velocity the y component of the velocity at 1, y component of velocity at 2 with the proper with the proper signs as per the coordinate system that we have chosen over here. So, with this we can calculate what is the value of r y after you put in the values. I did not I did not show you all the calculations which are straightforward just plugging in the values and you would get the you would get the values of r x and r y. So, this r x and r y are essentially the force on the fluid. So, the fluid is going to exert an equal and opposite force on the nozzle.

So, the if it is an equal and opposite force on the nozzle. So, the force on the nozzle would be minus 117 and minus 616. The nozzle is going to exert a force on these which would simply be equal to plus 117 and plus 616 because that is the force by the nozzle on the support. So, to summarize what we have done is we have we have seen two problems, one is due to the first one is due to a bend in its path. If you change the flow direction what is the force to be applied and this one is very interesting where a curved nozzle assembly with the nozzle being supported by at the at near location 1. The curved one changes the fluid path and therefore, there are contributions due to the velocity the momentum that comes in the y momentum that comes in the y and x momentum that goes out the pressure force in the y direction minus y direction in this case.

No pressure force at this point since it is atmosphere and then we have solved the x and the y components and we understand that what is the force it is on the control volume which is the fluid, what is the force on the nozzle that is minus of that and what is the force by the nozzle on the support that would be minus of that itself. So, whatever value of R x and R y that you calculate is going to be the force by the nozzle exerted by the nozzle on the support. So, that is all for this class. We will continue with problems in which something strikes a vane and it creates a motion how to tackle problems of that type that would be the topic of the next last two classes on this momentum transport using integral approaches. Thank you.