Momentum Transfer in Fluids
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Week-06
Lecture-30

Good morning. We will continue with our treatment of the integral equations and their applications. So, what we have seen in the past, in the last lectures, last two lectures are that how an integral equation can be used in order to define the change in the extensive property in a system. So, in this just to recap what we have covered, N, capital N is the is for the is the extensive property and since intensive property is extensive property per unit mass. So, eta would be equal to 1 if we are considering mass. Now, the conservation of this equation the dN dt system is equal to del del T of del del T of eta rho v and this the first term on the right-hand side refers to the transient effects.

Conservation of Mass N = Mass, $\eta = 1$

$$\left.\frac{\partial N}{\partial t}\right|_{SYSEM} = \frac{\partial}{\partial t}\int_{CV} \eta \rho d\forall + \int_{CS} \eta \rho \vec{V}.d\vec{A}$$

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V}.d\vec{A}$$

The second term is the net flux of mass through the control surfaces in the, in the control volume. So, when the system and the control volume coincide for that condition conservation of mass tells me that the left-hand side of this equation would be 0 and we are left with two terms on the right-hand side with eta being repressed by 1. So, for an incompressible fluid and for a constant size of the control volume, what we have is in the first term disappears and what we have is rho v dA is going to be equal to 0. So, when we integrate it when we express it, it is simply going to be rho 1 v 1 A 1 plus rho 2 v 2 A 2 etcetera would be equal to 0.

Incompressible Fluid $0 = \int_{CS} \rho \vec{V}.d\vec{A}$ The size of the CV is fixed

$$\int_{CS} \rho \vec{V}.d\vec{A} = \pm \left|\rho_n V_n A_n\right|$$

When uniform flow at section n is assumed

The magnitude of the mass flow rate rho v A n is to be considered whether it is going to be plus or minus, it is going to be decided by whether flow is out or in. As per our discussion of the previous class mass flow in is always negative mass flow out is positive that is because the area vector is always pointed outwards from a surface. Therefore, v dot A would be negative if v and A are oppositely directed which is going to be the case for flow in through a control surface. And similarly for the outflow both v and A are going to be directed in the same direction. So, it is going to be that the v dot A in that case is going to be positive.

Momentum Equation for Inertial CV, N = Momentum, η = Velocity

$$\left.\frac{\partial N}{\partial t}\right|_{SYSEM} = \frac{\partial}{\partial t}\int_{CV} \eta \rho d\forall + \int_{CS} \eta \rho \vec{V}.d\vec{A}$$

Then we have discussed about the momentum equation where we took the extensive property n to be the momentum of the system and since n the extensive property is momentum. So, momentum per unit mass would be the intensive property and we know that momentum per unit mass is simply going to be the velocity. So, once again we write del del T of n d del n del T of the system and when we consider about the time rate of change of momentum since my n over here is the momentum. So, the time rate of change of momentum is essentially the force, the force acting on the control volume. And we also understand that the force can be of two types one is the surface force the other is the body force.

$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V} \rho d\forall + \int_{CS} \vec{V} \rho \vec{V}.d\vec{A}$$

$$\vec{F}_B = \int_{CV} \vec{B} \rho d\forall \qquad \vec{F}_S = \int_{A} -p\, d\vec{A}$$

So, the surface and the body force are expressed as F s and F b and in this in this on the right-hand side the intensive property eta is going to be replaced by the velocity. So, this is going to be the complete momentum equation which is a statement of Newton's law. One point to note is that in the previous slide the all the velocities are sort of the same average velocity through coming through or going out of a control surface. Now, this F b the body force is the body force per unit volume multiplied by rho dv over the entire control volume and F s the surface force most likely example of surface force that we are going to consider is the pressure. So, it is minus p dA integrated over the control over the surface area surface area of the control surface.

Scalar Component

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u \rho \, d\forall + \int_{CS} u \, \rho \, \vec{V}.d\vec{A}$$

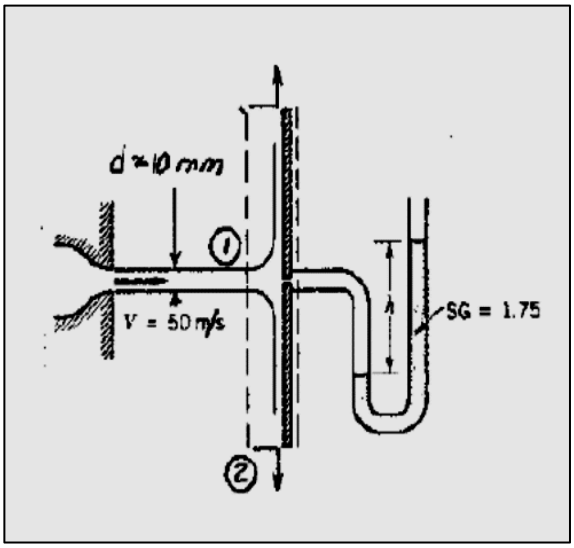
The scale that is a vector equation. So, I can write the scalar component here what you see is the scalar component that is the x direction has been written. So, my v the velocity I am going to use the x component of the velocity which is small u and the rest remains the same. So, when we apply for steady state then for the steady state this part is going to be 0. The del del t the right-hand side the first term on the right-hand side would be 0.

And once again to evaluate the sign of the sign of the momentum inflow or outflow through the control surface what we have to do is we are first going to figure out what is the sign of the mass flow rate that comes in coming in or going out plus or minus depending on in in is minus out is out is plus and then the mass flow rate is to be multiplied by each velocity component. So, the mass flow rate is essentially rho v dA and the x component of momentum associated with that mass of fluid is simply u times the mass flow rate. So, once again any mass which comes in through a control surface can have multiple velocity components u, v and w velocity in the x direction y direction and z direction. So, the contribution of the x component of the momentum would simply be the total mass flow rate multiplied by the component x component of velocity. So, this is what the term will look like the second term on the right-hand side will look like.

1. To determine the sign of $\rho \vec{V}.d\vec{A} = \pm\left|\rho V \; dA\cos\alpha\right|$
2. To determine the sign of each velocity component

$$u \; \rho \vec{V}.d\vec{A} = u\left\{\pm\left|\rho V \; dA\cos\alpha\right|\right\}$$

And we must do it for each of the control surfaces through which momentum is coming into the control volume. So, with this background we have proceeded to solve a few problems and I will continue solving problems of different types in this class as well. So, as to make yourself more familiar with how to solve problems of momentum transport using the integral approach. So, the first one is an interesting one where you have a nozzle over here and the nozzle directs a jet of water jet of fluid towards a plate which is held vertical to the direction of the velocity of the liquid. Now obviously, when it strikes when the water strikes it cannot proceed further and it is going to move out of the system in the y direction, but there is a small hole at the centre and the small hole at the centre relates to a manometer.



And therefore, when the water strikes this hole, it cannot get in here, but it is going to cause a deflection in the manometer. So, the liquid over here is carrying some kinetic energy that kinetic energy at this point of stagnation is going to get converted into completely into pressure and that pressure is going to be reflected by a difference in the head in the of the manometric liquid. So, this is a well-known principle which is used in flow measuring devices. For example, if you would like to figure out what is the point velocity in a moving flow moving fluid. So, what is done in this case is that something called a pitot tube is employed.

So, let us say you have a system in which a fluid is moving and obviously, we understand that for viscous situations the velocity is going to be different at different points. So, if the velocity is going to be different and different points and you would like to know what is the experimentally, what is the velocity at these points what you do is you employ something a tube sort of apparatus which is filled with a manometric liquid that is far heavier than the liquid which is flowing through this. So, the kinetic energy of the moving fluid at this point is going to get converted to pressure head and that pressure head is going to force the liquid the manometric liquid down through this arm and up through this arm. So, the difference in height of the manometric liquid multiplied by rho and g would give you what is the pressure at this point. So, the instruments and you can move this up and down you can move this up and down and to bring the mouth of this nozzle at different points and figure out and find out what is going to be the deflection in such cases and the deflection will change depending on where this point is located right at the centre the velocity is maximum.

So, the depression is going to be the difference is going to be the maximum and near somewhere over here the velocity is small. So, the kinetic energy to be converted to pressure head is going to be small and therefore, the deflection is going to be small. So, instruments which operate on this these principles are known as a pitter tube. So, what we have to figure out in this problem is the in this case the fluid is air and the manometric liquid has a specific gravity equal to 1.75. We need to find the deflection and the force exerted by the jet of air on the disk. So, the first thing that we do is we identify the two points marked as 1 and 2 in the figure and then write the Bernoulli's equation between these two points and we assume that there is no friction. Since the air jet is moving through air possibly. So, therefore, there is little frictional effects that we need to consider and we can simply write the sum of the pressure head and the velocity head at these two points are going to be the same. So, once you come to point 2 right over here the velocity disappears and therefore, you’re the rho v 1 square rho times v 1 square by 2 is simply going to be rho manometric times g and h.

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} = \frac{p_2}{\rho} + \frac{v_2^2}{2}$$

$$\boxed{\frac{\rho v_1^2}{2} = \rho_m g h}$$ PITOT TUBE

$$h = 0.089 \text{ m}$$

So, the as I mentioned the instrument which is based on this simple principle is known as pitter tube which is used to measure the velocity of the flow field at different points and you could by varying by measuring by noting down the value of h over here the noting down the value of h you should be able to figure out what is the value of the velocity at that point. So, this is the principle of pitter tube and when you plug in the values you would see the value of h the deflection in the manometer over here is going to be 0.09 meters. Next comes the question of what is the force exerted on the plate by the jet of air. So, of course, this is the motion is in the x direction.

So, I would I am going to write only the x component of the equation x component of the integral momentum integral equation and we understand also that this the it is a steady state problem. So, this part would disappear it is a problem in which there are no body forces since it is in the x direction only and no gravity force or anything else acts in a or it acts only in a direction perpendicular to the motion. So, it does not affect the motion at all in the x direction. Therefore, my R x is simply going to be u where u is the x component of velocity times rho v dA. So, I am going to individually figure out what is going to be the two control surfaces.

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\cancel{V} + \int_{CS} u\,\rho\,\vec{V}.\,d\vec{A}$$

$$R_x = \int_{CS} u\,\rho\,\vec{V}.\,d\vec{A}$$

$$R_x = u_1\{-\rho vA\} + u_2\{\rho vA\}$$

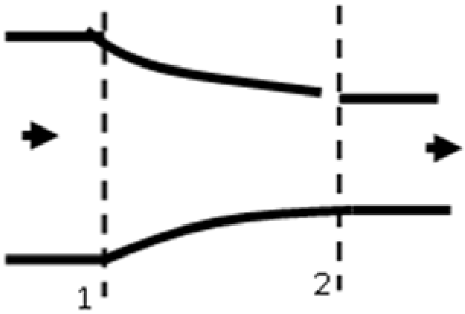
$$R_x = \{-\rho v^2A\} = -0.24\ \mathrm{N}$$

$$\text{Force by the jet} = +0.24\ \mathrm{N}$$

One is the u 1 which is coming in to the control volume and that is why it is negative rho v A and the other one is u 2 which is going to be rho v A once again the mass flow rate remains constant. And we also understand that u 2 is simply going to be 0 there is no x component of velocity beyond this point. So, my R x is simply going to be minus rho u 1 is essentially equal to v where v is the velocity 50 meter per second. So, once you put the value of v in here multiply it with rho and with A you would get the x direction the force exerted is going to be equal going to be equal to minus 0.24 N. So, this is the force on the control volume. So, force by the jet the control volume is the jet. So, force on the control volume is 0.24 N and force by the jet on the plate on this orifice plate with the small hole in here is going to be the reverse of that.

So, it is plus 0.24 N which is intuitive because if I have a jet which is going to strike this the plate is going to experience a force towards this direction because of the motion of the jet. So, anything that we have calculated is essentially the force on the control volume and the force by the control volume on the plate in this case is going to be negative of that. So, this is a clear example with potential application in flow measuring devices where a pitted tube is employed to evaluate what is the velocity, what is the point velocity at every point in the flow field. So, that is a very common instrument. We move on to the next problem now.

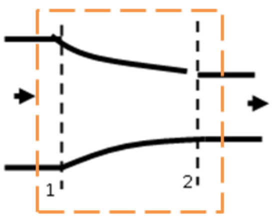
So, this is the flow of this represents a flow of gasoline of known specific gravity 0.72 through a reducer. So, sometimes in a pipeline in a gas pipeline or in a crude oil pipeline the area is reduced. So, that whatever be the velocity at this point the velocity increases over here since the flow area is smaller. Now, the velocity at the inlet and its diameter is given as 0.4 meter is 3 meter per second and the at the outlet at this point it is 12 meter per second. The pressure at the inlet is 58.7-gauge pressure kilo Pascal and at the outlet is 109 kilo Pascal, but note here that this pressure is the absolute pressure ok. So, the pressure mentioned over here is 58.7, but gauge. So, the absolute pressure is going to be 58.7 plus the atmospheric pressure. So, that is the pressure over here and at the outlet it is going to be 109 kilo Pascal. So, but that is the absolute pressure. So, we when we when we figure out what is the pressure difference, we must be aware of whether we are dealing with absolute pressure or dealing with gauge pressure.



And I have told you that all calculations in this we are going to take the pressure always as the gauge pressure because if you just have a surface, you have atmospheric pressure acting on both sides and there is no net pressure due to no net force due to pressure on this control surface. Only if the pressure at this side is going to be more than atmosphere by an amount equal to the gauge pressure this surface is going to feel a force due to pressure it towards in this direction. So, it is always the gauge pressure which is to be used in any such calculations. And this entire system also has a mass and it also has some liquid present in it with its own weight. So, all these are going to have a force in the y direction.

The force in the y direction is only principally going to be due to the weight of this weight of this reducer and the gasoline contained in the reducer which is going to be balanced by a normal force exerted by the support on which these this reducer is placed. So, I do not need to compute what is F y, I already know what is F y. It is going to be m times g where m is the mass of the reducer and mass of the gasoline contained in the reducer at any given point of time. The flow is in the x direction. So, any momentum associated force due to flow I need to consider only the x direction, I do not have any y component of velocity and I do not need to consider that.

So, I am going to write only the x component of the equation, x component of the momentum equation and evaluate the terms to see what is the force exerted by this moving gasoline on the reducer that is essentially the problem. So, no need to calculate R y, I need to calculate only R x. So, let us take the control volume, but here the control volume is marked by this yellow dotted ones and R x and R y are the forces needed on the control volume to put it in its place. This plane is essentially it should be it should be place alright. So, I start once again with the equation momentum equation only in the x direction and it is clear from this you would you can see that there is no force no body force in the x direction, it is a steady state process.



$$F_x = F_{Sx} + \cancel{F_{Bx}} = \cancel{\frac{\partial}{\partial t}}\int_{CV} u\rho \, d\mathcal{V} + \int_{CS} u\rho \, \vec{V}.d\vec{A}$$

$$F_y = F_{Sy} + F_{By} = \cancel{\frac{\partial}{\partial t}}\int_{CV} v\rho \, d\mathcal{V} + \int_{CS} v\rho \, \vec{V}.d\vec{A}$$

So, this term and this term the second term on the right-hand side and the first term on the left-hand side would disappear. So, what we are left with is that the force is simply going to be the net efflux of momentum through the control surfaces. So, I am going to figure out what is the momentum in and what is the momentum out through these control surfaces and that would give me F s x which is the force needed to keep the reducer in its place. So, F y, I can similarly write that, but this part is going to be 0 since its steady state. So, let us start writing the x component of this equation.

$$R_x + p_1gA_1 - p_2gA_2 = u_1\{-|\rho v_1A_1|\} + u_2\{+|\rho v_2A_2|\}$$
$$R_x = -p_1gA_1 + p_2gA_2 + (u_2 - u_1)\rho v_1A_1$$

So, one is going to be one is going to be R x and the F s x is whatever be the pressure once again the gauge pressure at 1 multiplied by multiplied by the area minus whatever be the gauge pressure at 2. Let me correct this should not be any g in here whatever be the pressure over here multiplied by it multiplied by the area and on the left-hand side this is the mass flow rate in through location A. An equation of continuity tells me that whatever be the mass flow rate through A must be equal to the mass flow rate going out through 2. So, it is rho v 1 A 1, but since it is in so, that is why it is a minus sign multiplied by the x component of velocity at location 1 which is over here. The one that goes out is rho v 2 A 2 once again we understand that rho v 2 A 2 is equal to rho v 1 A 1 because of equation of continuity because of conservation of mass.

But here the mass flow rate out of the control volume must be multiplied with the corresponding value of velocity at location 2. So, u 1 this is 1 and this is 2 u 1 and u 2 are the velocities of the inlet and the outlet. So, my r x is simply going to be equal to the pressure the all these with the we can we can calculate what is going to be the total amount of force and as these two are equal. So, I can use any one of them and it is simply going to be u 2 minus u 1. So, once you do that your r x is simply going to be equal to minus 4.68 kilo Newton. So, that is the force on the control volume and this is the force on the control volume force exerted by the control volume. So, the force is to be applied to the left. So, therefore, this equation this problem gives us the total the gives us the total story about the about whatever be the force needed force on the on the reducer by the moving fluid. Now, let us think about the r y. So, r y being the force on the reducer and then we are going to calculate that.

$\rightarrow R_x = -4.68$ kN

R_x being the force on the reducer as per the CV chosen

Force to be applied left

So, as I have mentioned this is the r y component and this r y component it also has some body forces. What are the body forces? The body forces that we deal with are m g the mass of the reducer and this v is the volume of the reducer. So, volume of the reducer multiplied by rho of gasoline and g. So, this is the total mass of the reservoir including the gasoline which is contained in it. Now, I come to the right-hand side I have v 1 and v 2.

R_y being the force on the reducer

$$R_y - Mg - \rho g V\!\!\!\!- = \cancel{v_1}\{-|\rho v_1A_1|\} + \cancel{v_2}\{+|\rho v_2A_2|\}$$
$$R_y = Mg + \rho g V\!\!\!\!- \qquad \rightarrow R_y = 1.66\ \text{kN}$$

This is the mass flow in negative mass flow out positive and we realize that both v 1 and v 2 are going to be 0 because we have flow only in the x direction no flow in the y direction. So, the moment y momentum contribution through the control surfaces to the control volume is 0.

As I have as I have while discussing the problem at the very beginning, I mentioned that the r y is simply going to be equal to the weight due to the weight of the control volume. So, what we get is r y is m g plus rho g v and this r y when once you put in the values it will come to be 1.66 N. And once again I emphasize that the control volume if I go back to the slide over here the control volume that that is the dotted line is essentially encompasses the reducer. So, the force on the control volume is essentially the force on the reducer. So, if that is the case if that is the case the control volume is defined including the reservoir therefore, r x and r y they refer to the force on the control volume which in this case is the reducer. So, this specific problem that the problems that deal with the problems that we have discussed in today's class one is about pitot tube where the pressure the velocity head is converted to pressure head and that pressure head can be measured by the deflection of a manometer connected to the pitot tube. So, with that with that arrangement we would be able to obtain what is the flow what is the flow velocity at every point in the in the in the flow field.

So, the problem that we dealt with was an orifice that means, a plate with a very small hole at the middle and a jet of air striking this with a manometer connected on the outside of the hole on the other side of the hole. So, whatever be the kinetic energy of the jet gets converted into pressure head and that is the that is the manometric the deflection in the manometer. This jet is going to apply a force on the plate. So, it would try to move in the direction of the jet. So, to keep it in place an equal and opposite force must be applied to the, to the plate.

So, how do I calculate this this how do I calculate this force is by the application of the momentum equation. And once again in is negative out is positive. So, we calculated the force on the control volume and therefore, we could figure out what is the force to be applied on the plate to keep it in place. And the second one is also an industrially relevant process in which there is a reducer which is used to increase the velocity of the incoming fluid. So, whatever be the velocity here is going to be less than the velocity at the outlet.

So, this difference in velocity would create a difference in momentum being added to the or net efflux of momentum to the control volume. And the pressure on two sides is also unequal and we are going to use we have used gauge pressure which you must always use. So, the f the x component will contain no body force since it is horizontal body force the surface force due to pressure because of the pressure imbalance on the two sides. And on the right-hand side you have some momentum coming in because of the velocity because of the lesser velocity some momentum going out because of the higher velocity. The net sum of this the algebraic sum after incorporating the conceptual position of in to be negative out to be positive you can figure out what is the total amount of momentum that is being added to the control volume.

And that would enable you to figure out what is R x and similarly for R y as you can see in this R y is simply going to be the mass of the entire system the weight of the entire system. So, this would give you the value of the force that is exerted on the reducer by the flow even without the flow it even the flow has no contribution on the y component of the force. So, that is all for this class we will solve slightly different type of problems in the next class. Thank you.