Momentum Transfer in Fluids
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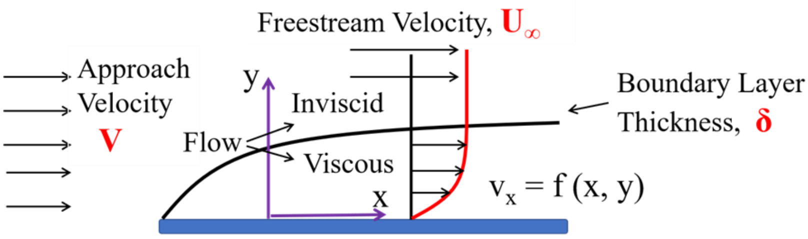
Week-06
Lecture-29

So, we are going to start this class which is a continuation of our previous class on the application of integral equations. So, the basic equations in integral form we have discussed extensively in the previous class. So, in this we will see specific applications of those equations into problems which are slightly more involved than the one that I have presented before which is simply a mass balance equation. So, for to discuss that I am going to go back to a concept which I briefly touched upon in one of my previous classes which is the concept of boundary layer. Now, I explained to you that at the junction of the liquid and a flow at the junction of a solid and a flowing liquid there forms a thin layer in which the viscous forces are going to be predominant. And outside of that thin layer the viscous forces are going to be negligible and therefore, the flow inside that thin layer adjacent to the solid surface is 2 dimensional for flow over a flat plate, but outside of that thin boundary layer the flow is going to be 1 dimensional.



Flow over a flat plate

$v_x = f(x, y)$ Viscous 2D flow inside BL

$v_x = U_\infty$ Inviscid flow outside BL

$v_x = 0.99U_\infty$ at $y = \delta$

δ - boundary layer thickness

So, if you look at the figure over here there is a plate in there is a solid plate and a fluid is approaching this plate with a constant velocity let us call it as approach velocity and that is v the capital V. And at the junction, at the interface between the solid the blue one and the liquid the flow is affected by the presence of viscosity. So, due to no slip condition the fluid layer close to the surface will have a velocity equal to 0 and it will try to slow down the velocity of the subsequent upper layers less than that of the approach velocity v. So, there is going to be a velocity distribution as shown over here this velocity distribution is going is something which you see over here starting with a value equal to 0 and then asymptotically merging with the free stream velocity that means, velocity outside of the boundary layer which I call as u infinity.

So, u infinity is the free stream velocity essentially this stream is moving free of any viscous forces present in the system. Whereas the approach velocity is where the fluid has not encountered the solid plate as yet. For the special case of flow over a flat plate this v and u infinity are equal, but for a curved plate and in presence of pressure gradients this v and u infinity can be different. So, I have kept it as u infinity, but we understand I stress that a stress upon the point that u infinity is equal to v for flow over a 0-pressure gradient flow over a flat plate. Now, this inside the inside this thin layer the velocity is a function both of x and y where the x is this the direction in the direction of flow and y is a direction perpendicular that of the flow.

Now, this velocity there are viscous 2-dimensional flow inside the boundary layer and inviscid flow outside the boundary layer. So, v x is going to be almost equal to u. Now, to define a practical thickness of the boundary layer what we say is that when v x the x component of velocity reaches 99 percent of the free stream velocity that point the thickness is called the boundary layer thickness. So, you could see that the boundary layer thickness is defined by this black line which is also called the edge of the boundary layer. So, at any point the boundary layer thickness is essentially the distance of this layer from the solid plate.

So, boundary layer is a function of x. Now, at higher values of x the boundary layer would be more, but if you look at the growth of the boundary layer it is the growth is very rapid at the initial part of it and then the rate of growth of boundary layer somewhat decreases and it is still growing over here, but the rate of growth of the boundary layer is going to be quite less as compared to what happens in at the beginning. And since the boundary layer grows the boundary layer cannot be a streamline. A stream one of the basic fund properties of streamline requirements of a streamline is that there can be no flow across a streamline. Now, if there is no flow across the edge of the boundary layer the black line that you see in the figure the black line over here then if there is no flow across this then the boundary layer cannot grow.

So, which means that there is flow across the edge of the boundary layer and hence the edge of the boundary layer is not a streamline. The relative concepts would be quite handy later on in the in the in subsequent classes when we talk more about the friction which is generated by flow on a solid plate. So, let us think about flow in a pipe. So, the boundary layer is going to form from the pipe walls till up to the centre of, the centre of the pipe. Now, in the growing part of the flow the velocity the axial velocity the velocity in the z direction will be a function of z, but once the flow is fully developed that means, the boundary layers from the side of the walls come and meet at certain point beyond that point the velocity is not going to be a function of z that condition it is known as the fully developed flow.

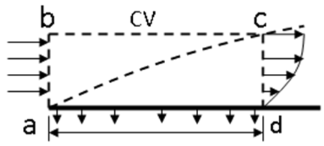
Most of the problems which we have considered in this course is for fully developed flow where the flow velocity the axial velocity is not a function of axial position. So, the few things that we have understood in this is that there is something called a boundary layer which denotes the interaction the viscous interaction between the flowing fluid and the solid. The edge of the boundary layer is defined as where the velocity has reached 99 percent of the free stream velocity, but it if you look carefully over here the rate of the rate of change of the boundary layer is so small that it is experimentally quite difficult to pinpoint the, pinpoint where it exactly reaches 99 percent of the free stream velocity. So, there are other ways not a differential way differential method where you want an exact point where the velocity is 99 percent of the free stream velocity, but something else to define the thickness or the effect of the thickness of the boundary layer. So, though there are integral ways integral methods integral boundary layer thicknesses which may come which is not part of this course.

So, we are going to solve our next problem based on the boundary layer concept where there is going to be suction or injection through the bottom through the solid surface. So, how am I going to use the integral approach use the integral approach for conservation of mass for such a system that is the topic of the next problem. So, what you see here is the growth of a boundary layer. So, ABCD the AC surface the AC surface essentially is the edge of the boundary layer which keeps on growing. So, a steady flow of water past a porous plate the plate the solid plate in question over here is porous with a constant suction velocity.

Consider the steady flow of water past a porous plate with a constant suction velocity of 0.2 mm/s (i.e., V = - 0.2j mm/s). A thin boundary layer grows over the flat plate and the velocity profile at section cd is

$$\frac{u}{U_\infty} = \frac{3}{2}\left[\frac{y}{\delta}\right] - 2\left(\frac{y}{\delta}\right)^{1.5}$$

where U_∞ is the velocity of approach at section ab and is equal to 3 m/s. Find the mass flow rate across section bc. Given: width of the plate = 1.5m, length, ad = 2m. δ at CD = 1.5 mm



So, liquid is sucked out of the sucked out of the flowing flow flowing liquid at a rate of minus 0.2 j millimetre per second and a thin boundary layer grows over the flat plate and the velocity profile at section CD is approximately expressed as 3 by 2 y by delta where delta is the film boundary layer thickness at location D and y by minus 2 y by delta to the power 1.5. U infinity is the velocity of approach. So, this is where the U infinity is acting on at section AB and it is equal to 3 meter per second.

You have to find out the mass flow rate across section BC. So, how much mass is either coming in or going out of the control out of this control volume through the surface BC. So, I first draw my control volume as ABCD and then try to make a balance based on our integral approach. So, the problem is then flow over a porous surface and the porous from the porous surface liquid or the fluid is being sucked out and you have to figure out what is the flow rate across surface BC. You can see that across surface AB the flow is coming in with a constant velocity, but across CD the velocity varies from 0 at D to some value at C and the distribution is provided in the problem.

So, in order to find out the mass flow rate across CD you need to integrate this velocity expression over the area and the area being CD. So, the and through A D it is constant just equal to minus 0.2 J. So, these factors are to be kept in mind while solving this specific problem. So, what I am what we should do then is we are going to write the conservation equation and the conservation equation simply tells me the del del T of rho dv plus the efflux of mass through the control surface and we know that it is a steady state problem.

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V}.d\vec{A}$$

So, for so, if it is steady and incompressible flow then the first term first term on the right-hand side this term would be 0. So, what we what we end up with is the control surface integration over the control surface rho v dA to be equal to rho v dA to be equal to 0 and v is minus v 0 J. So, it is directed downwards and all along AD and therefore, this over the control surface the mass flow rate is simply going to be whatever be the mass flow in or out through AB that means, through this surface the m dot BC the one that we need to calculate m dot BC is mass flow in or out I do not know yet mass flow in or out through B C whatever is the mass flow rate through CD and mass flow rate through DA. So, all 4 surfaces the mass flow rate in or out the algebraic sum of all this must be equal to 0 that is our starting point and then we are going to put the different values in there to to calculate what is m dot BC. So, what we do next is that we understand that through surface AB the velocity with which the fluid enters through AB is capital U.

$$0 = \int_{CS} \rho \vec{V}.d\vec{A} = \int_{ab} \rho \vec{V}.d\vec{A} + \dot{m}_{bc} + \int_{cd} \rho \vec{V}.d\vec{A} + \int_{da} \rho \vec{V}.d\vec{A}$$

$$0 = -\rho U_{\infty} W \delta + \dot{m}_{bc} + \int_{0}^{\delta} \rho U_{\infty}\left[\frac{3}{2}\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{1.5}\right] W dy + \rho v_0 W L$$

Now, it enters the control volume since it is entering the control volume the it is going to be negative. So, the mass flow rate is going to be negative rho u infinity W times delta. So, W is the width of the plate the value of which is provided in the problem and delta is simply the equal to the film thickness at CD because I have drawn a rectangle as my control volume. So, whatever be the value of delta at CD is essentially AB. So, that is why I have put delta in here then m dot BC is the unknown.

Next is what is going to be m dot C D. Now, in order to obtain the m dot CD, I need to integrate the velocity expression which has been provided if you look at this expression which have been provided. So, the velocity is simply u infinity multiplied by this function as my as my velocity. So, it is rho u infinity the functional form which has been provided multiplied by the width W of the plate which is constant times dy. So, this integration provides me from 0 to delta provides me with how much mass is crossing the surface CD and once again since it is going out of the control volume.

So, therefore, it is positive. The third last one is rho v naught where v naught is the suction velocity at this point which has been provided in the problem multiplied by W times L. So, the area through which the mass goes out of the control volume is whatever be the width multiplied by L where L is the length of the plate. So, this essentially is my governing essentially my governing equation applied for this special case where I have flow in through AB, flow out through AD, flow out through CD and the only thing that we have to ensure is that while evaluating the flow out through CD, I am using the expression for velocity that is provided for the section C D which is this and I integrate this over dy to obtain whatever be the whatever be the total flow out of the control volume. I guess conceptually it is clear to all of you what you need to do is then integrate this and figure out what is going to be the what is going to be the numerical value of each of these terms.

$$\dot{m}_{bc} = \rho W \left[1.05 U_{\infty} \delta - v_0 L \right]$$

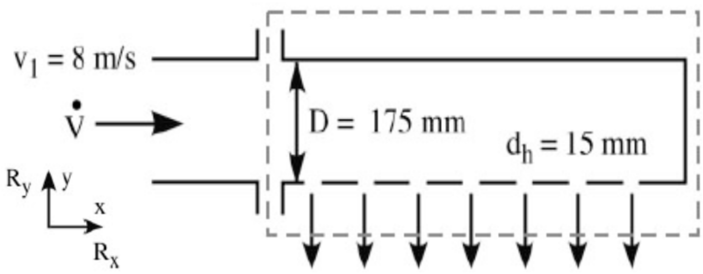
$$\dot{m}_{bc} = 6.48 kg / m^3$$

Since, $\dot{m} > 0$, so flow is out of CV

So, I will not do this the next step which you can do and you would see that the m dot B c simply turns out to be rho times W where W is the width 1.05 u infinity delta minus v naught L where v naught is the suction velocity. So, when you put all these values in this expression you are going you will see that m dot B c is going to be 6.48 kg per kg per it should not be meter cube, this should be kg per second. Let me let me let me say it here that this is mistake this should be this meter cube should be essentially kg per second.

So, I will correct that at the before I before I do this, but this one is simply going to be kg per second ok, meter cube to be replaced by kg per second. And so, essentially this then gives us a nice example of the use of concept the boundary layers and whatever how do you calculate the flow rate when the velocity at one surface is a function of location. The next one is the previous one was about use of mass equation, the conservation equation, the continuity equation. This one is going to be about the use of both continuity equation and the momentum equation. So, what we do here is we take the example of a spray pipe in a garden.

Water exits a pipe from a series of 109 holes drilled into the side as shown in the figure, along with the coordinate systems. The pressure at the inlet section is 35 kPa. Calculate the forces required to hold the spray pipe in place. The pipe and the water ($\rho = 10^3$ kg/m^3) it contains weighs 5 kg.



So, water enters through this water enters through this point with a velocity equal to 8 meter per second. It enters a spray with holes a number of holes 109 holes which are drilled into the sides. The dimension the diameter of each hole is 15 millimetres whereas, the dimension of the supply pipe and the pipe along which the holes have been made are both equal to 175. This is the x direction and this is the y direction. What you need to find out is the force required to hold the spray pipe in place.

So, the spray pipe is connected to the main body of the pipe at this location. So, what is the force needed to hold the spray pipe in place? This spray pipe and the water contained in it at any given point of time is 5 kg. The mass of the pipe and the water contains it weighs 5 kg. So, how am I going to find out what is the force to be to be exerted by an external agency on the pipe to keep it in place? Once again, I stress upon that all the forces that we calculate out of the equation F s x plus F b x etcetera that is force on the control volume. So, if we use that equation to find out what is the what are the forces then essentially, we are finding out what are the forces on the control volume.

So, this control volume will exert an equal and opposite force to wherever it is connected to. So, the pipe is connected to the main pipe. So, the spray the action of spray in a different direction will result in a force on the pipe. So, therefore, in order to keep the pipe in place keep the main pipe in place keep this pipe in place another force equal and opposite to the force exerted by the pipe by the spray pipe on the main pipe is to be applied. Let us first see how it is calculated and then it would become clear to all of you.

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho \, d\forall + \int_{CS} u\rho \vec{V}.d\vec{A}$$

$$F_y = F_{Sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho \, d\forall + \int_{CS} v\rho \vec{V}.d\vec{A}$$

So, the first thing that we do is write the equation the x component of the conservation x component of the momentum equation y component of the momentum equation and see what are going to be there. Now, before we do that, we need to figure out what is the velocity of the velocity of the water that leaves the pipe through these holes. So, we know what is the area of the inner in area in at this point where the diameter has been provided. So, you can calculate what is the diameter what is the area of the surface of the main pipe. Then area out through each one of the holes which has a diameter of 15 millimetre is this one.

$$A_{OUT/hole} = \frac{\pi}{4}(0.015)^2 = 1.766 \times 10^{-4} m^2$$

$$A_{OUT}(Total) = 109 \times 1.766 \times 10^{-4} m^2$$
$$= 1.925 \times 10^{-2} m^2$$

So, the total area out for all of these holes is going to be this much. So, the velocity in is 8 meter per second at this point the velocity in is known the area is known. So, you would be able to find out what is the what is the q the volumetric flow rate of water coming in through the main pipe into the spray pipe. And you would so, this one the area out you are trying to figure out what is going to be the what is going to be the velocity of the water which goes out through each pipe and you calculate that to be equal to 9 about 10 meter per second. So, you get the water in at a velocity of 8 meter per second and you are getting out at 10 meter per second which lets it travel to a further distance and to wet the surfaces to act as a proper spray pipe.

$$A_{IN} = \frac{\pi}{4}(0.175)^2 = 2.404 \times 10^{-2} m^2$$

$$V_{IN} = 8\ m/s$$

$$Q = 1.92 \times 10^{-1} m^3/s$$

$$\therefore 109 \times V_{OUT} \times 1.766 \times 10^{-4} = 1.9 \times 10^{-1}$$

$$\boxed{V_{OUT} = 9.98\ m/s}$$

So, now, I have the inflow rate in velocity I have the out velocity. So, I should be able to calculate what is the what are the net momentum added through the control surfaces. So, what we do is the first we write the x component of the equation. So, x component of the equation simply tells me F s x plus F b x the transient term and the momentum efflux term. Now, looking at this you understand that since gravity is the only body force present in here there is no component of gravity in the x direction.

$$F_{Sx} + \underbrace{F_{Bx}}_{=0} = \underbrace{\frac{\partial}{\partial t}\int_{CV} \eta\rho\, d\forall}_{=0} + \int_{CS} \eta\rho\, \vec{V}\cdot \vec{dA}$$

So, F b x is 0 and we are dealing with a steady state situation. So, therefore, this term is also going to be equal to 0. Whatever the force on the control volume is essentially due to the force through the control surfaces. So, I write R x which is the surface force through the in the x direction and there is some force due to pressure which is acting over here.

$$R_x + P_{1g}A_{IN} = -\rho Q u_1 + \underbrace{\rho Q u_2}_{=0} = -\rho Q u_1$$

$$R_x = -1000 \times 1.92 \times 10^{-1} \times 8 - 35 \times 10^3 \times 2.404 \times 10^{-2}$$

$$R_x = -2377\ N$$

So, the pressure at this point is provided. So, p 1 g A i n is the force in the plus x direction acting on the dotted control volume. So, that is the total force one is the reaction force and the other is the force due to pressure acting on the control volume. What do I have on the right-hand side for this one? One is the momentum in with the inflow. Since the mass is coming into the control volume it is negative. So, rho q the volumetric flow rate multiplied by E 1 the same flow is going out through all these holes.

So, q remains the same, but it is going out of the control volume. So, that is why it is going to be positive. Once again, this E 1 and E 2 are all the velocity x component of the velocities. Now, the x component of velocity at outlet is 0. There is no x component over here you only have the y component.

So, therefore, u 2 is 0 and this equation is simply minus rho times q rho q E 1. So, u 2 is 0 you plug in the values and you get the value of the R x the force in the x direction to be equal to minus 2377 Newton. So, the things that we understand are there is a force on the control volume, there is a force due to pressure acting on the control volume in the plus x direction, the flow out in is minus flow out is plus and the flow in as per this equation is to be multiplied with the x component of velocity at location 1 and the x component of velocity at location 2 which as per the figure as per the statement of the problem we understand that there is no x component of velocity in here and therefore, it is simply going to be equal to 0 and the entire right hand side is minus rho times q 1. With the addition of the values, you get this. One more very important point to note here is that whenever you use the pressure you always use the gauge pressure not the absolute pressure because a control surface if my hand is the control palm is the control surface I have atmospheric pressure over here atmospheric pressure over here.

So, the net pressure net force due to pressure is going to be 0. Only if one side is slightly higher pressure then the gauge pressure which is absolute pressure minus the atmospheric pressure that gauge pressure is the additional force additional pressure which is going to force my hand to this direction or my palm will feel a force due to the presence of a non-zero-gauge pressure. So, in all calculations of this type you will always use gauge pressure and never the absolute pressure and the relation between the absolute and the gauge pressure is total absolute pressure minus the atmospheric pressure is the gauge pressure. So, that is why the 35 that we use over here for the p 1 g, g stands for gauge pressure is 35 and on the other side I do not have this one is open to atmosphere. So, therefore, I do not have any pressure force if any pressure force acting on the control volume through the control surface because the gauge pressure here is 0.

So, that is another important point that you should keep in mind. Then go we go to the y component and the flow in the y direction is again going to be r y that is the force p 2 g pressure gauge at location 2 and I have already explained that p 2-gauge pressure over here is going to be 0. Since you do not have any this is open to atmosphere. So, the gauge pressure at location 2 is going to be 0, but there is an additional one the body force which is rho g v the body force present in by the because of the water and its water and the wet water and the wet of the entire pipe spray pipe is going to going to give rise to a downward force in the y direction in the minus y direction. So, therefore, minus rho g v is the body force which is present only for the y component and not for the x component.

For y-direction: $$R_y + P_{2g}A_{OUT} - \rho g\forall = -\rho Q v_1 + \rho Q v_{2y}$$

$$R_y = \rho Q v_2 + \rho g \forall$$

$$R_y = -1867 \text{ N}$$

And then you have rho q v 1 whatever be the mass y component of mass sorry y component of momentum coming in through this point and y component that is going out of this component. Now, of this we realize that v 1 the y component of velocity at location 1 is 0. We at the control surface 1 we only have the x component and there is no y component. So, therefore, v 1 is going to be 0 there is no y component over here, but v 2 is going to be going to be there. So, that is why v 2 is over here and I bring the rho g v on the right-hand side.

So, my r y, force on the control volume is in the y direction is going to be equal to this. Now, I put in the values when I get the value of r value of r y to be equal to minus 1867 Newton. Now, if you look at the previous one r x is minus something and then this one is r x r y is minus 1867 Newton. This is the force on the control volume force on the dotted line. So, therefore, this control volume exerts a force of minus r x and minus r y on this pipe.

So, the pipe will experience a force of minus r x and minus r y because of the flow out of the control volume through the pipe surfaces. So, since once again since the force that we calculate are force on the control volume, this control volume will exert an equal and opposite force on the pipe. So, therefore, in order to keep the pipe in place a force equal to that, but opposite must be applied. So, essentially therefore, the final force on the pipe must be equal to r y and r x.

One more time r x and r y are force on the control volume. So, therefore, force by the control volume on the joining pipe is minus r x and minus r y. So, to nullify that an external agency has to apply a force of r x and r y on this, which essentially means that the force on the control volume is going to be r x as we as in the previous slide and r y minus 1867 Newton, which is understandable because if this is the this is the spray pipe because of the spray it tries to push the pipe in this direction. So, in order to hold this, I someone external agency has to apply a force in the in this direction. So, these are the concepts which are we will keep on keep on expanding in our subsequent classes. So, that is that is all for this class. Thank you.