Momentum Transfer in Fluids
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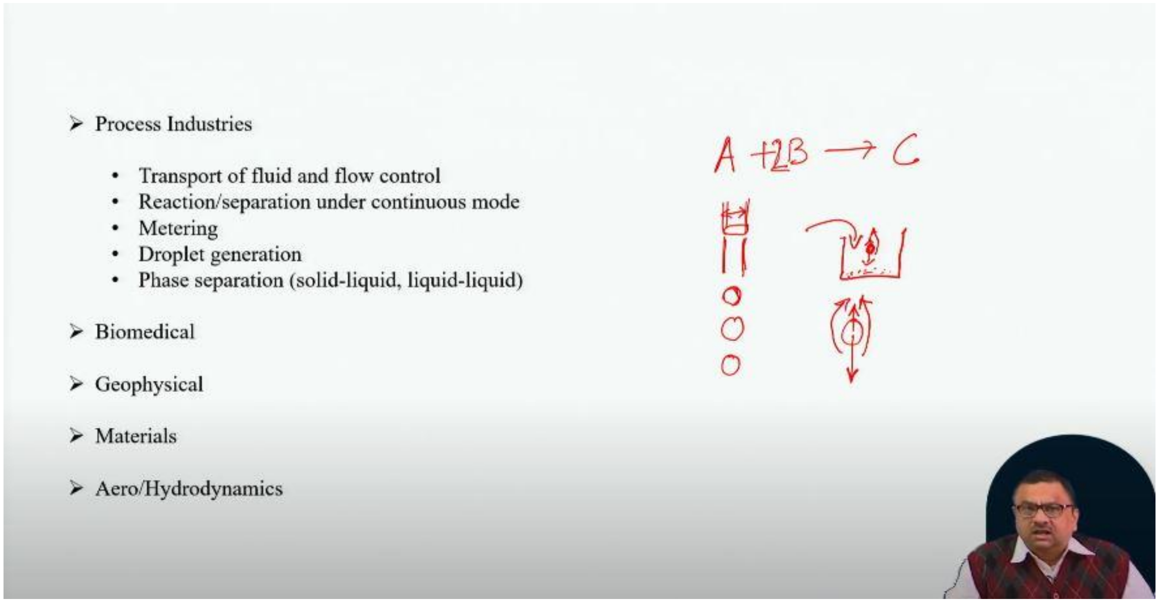
Lecture-02

I welcome you all to this course of Momentum Transfer in Fluids. I will be discussing some elementary frameworks first before getting into the details and theories of momentum transfer in fluids. What we have here is momentum transfer in fluids; this is required in many different applications in our day-to-day life, in the production of various ingredients, various chemicals, and various materials that we use in our everyday life. So, I have classified them into different categories, you can see process industries, biomedical, geophysical etc. So, in various categories momentum transfer in fluids is extremely important. In process industries, process industries means every day what we use that has to be processed, that has to be produced because we cannot rely completely on natural raw materials.



The natural raw material has to be processed and then only we can get the products that we use every day be it polymer, be it some other functional materials, for example, in eye surgery people are using certain materials that that that that that material has to be produced, or for coming to various other day to day needs, we have to produce those materials. Maybe the precursor is some raw material available in nature, but one has to play with those and come up with the best quality material that can serve the purpose and for the transport of fluid and flow control, which is absolutely important. For example, you want to produce some material, which is, say, product C, and you know that through a reaction, you will generate those. So, you can see that there is a reaction, let us say A + B, forming C.

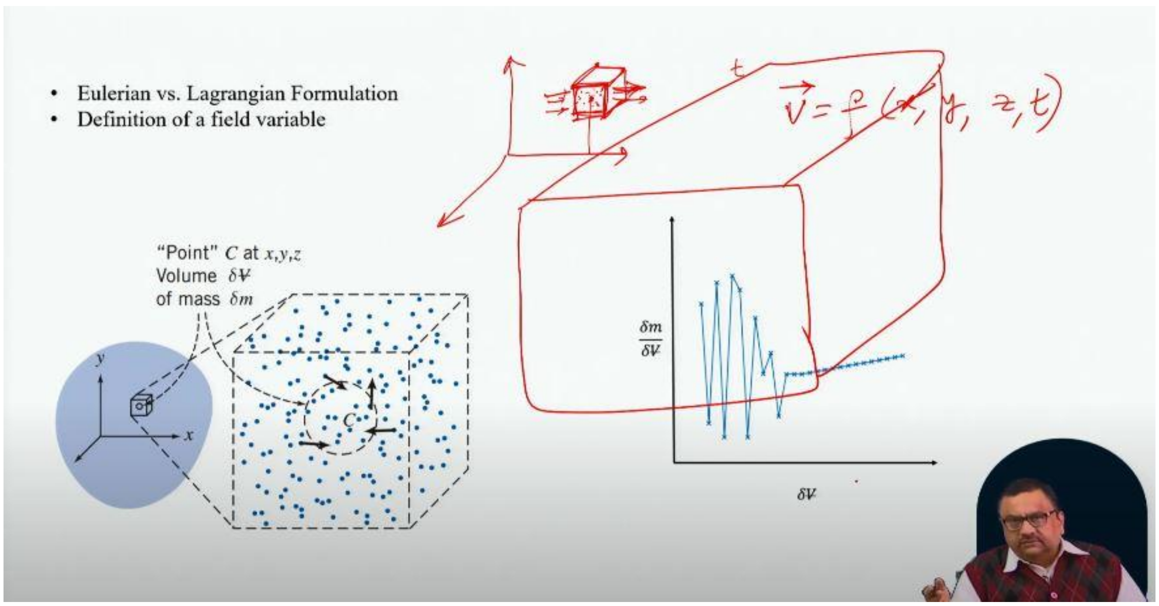
So, we have to provide A and B in a place so that they can react to form C. When it comes to working on a small scale in a test tube that can be done, but when you need in ton scale every hour few tons of material have to be produced, then you have to rely on some continuous process, and there, that is in fact, the second point reaction and separation under continuous mode. So, one has to work with A and B that have to be supplied in a common place, and then the C that is produced and some unreacted A and B will also be present. The customer will not accept C with A and B present; they want pure C or at least 99% pure C, and the remaining 1 percent can be unreacted A and B. So, there has to be a separation. So, not only a reaction, there has to be a separation, and this entire process has to be done in a continuous mode; otherwise, you cannot rely on a batch process when you are working on a very large scale.

So, the transport of A and B is extremely important, and the control of A and B is important because there is a certain stoichiometric ratio by which these A and B have to react. So, let us say, I have the stoichiometry says that A + 2B is forming C. So, the amount of A that you are providing, and the amount of B that you are providing, there is a certain ratio, and that ratio has to be maintained. So, for that purpose, flow control is absolutely essential, and it has to be very precisely controlled; otherwise, you will be generating extraneous products that will destroy the product purity, that will make the product unacceptable to the customer. So, these issues are extremely important, and at the same time, you need metering not only just the flow or just the reaction or separation. One has to meter and keep a log in real-time of how much of A is going in and how much of B is going in.

So, these are some of the common areas where every day momentum transfer in fluids concepts are used. The other area is here, I mentioned is droplet generation. For example, one wants to make a pharmaceutical product a powder. So, the powder would be the first one to be produced. One has to say, from a liquid phase, you have to evaporate the water, and at the same time, you produce the powder material, which will form a tablet later. So, there, this, say I have an aqueous phase in which I have some dissolved solids, and I want that aqueous phase to be evaporated, the water to be evaporated, and the remnant that solid will remain as will be there as particle.

Of course, you could have crystallized, that is another thing, I have the second thing phase separation solid liquid separation, but the other way is to atomize that aqueous solution into very fine droplets and as these droplets travel you blow air from the reverse direction and probably apply hot air. So that the evaporation gets completed, and at the end, you are left with powders. So, this droplet generation process is extremely important. That means you are having a liquid string just like you have. Let us say, all of you must have seen the dripping faucet problem. There is a faucet from which a liquid string is coming out, and then you see that liquid string within a short distance it will start forming droplets. Because, say liquid string, there is certain instability that sets in, and it cannot retain its cylindrical shape.

So, it breaks down into droplets. So, you can discretize and generate droplets this way. Now, the diameter of this droplet depends on the diameter here, diameter of this nozzle. So, if you reduce the diameter you expect the diameter of the droplet will also get reduced. So, this generation of droplets and making a powder out of it or for other is I mean droplets can be used for other purposes as well.



So that, momentum transfer in the fluid becomes extremely important. And phase separation, a lot of times, we have solid and liquid, and we want to separate the solid from the liquid. So, generally, it goes by gravity, but many times, you do not have time to wait for gravity. If the particle size is small there, it will take a long time for that particle to settle in the bottom. So, you can think of, I mean, the only way if we are relying on gravity, what we can do is to increase the residence time. That means I have, let us say, tank, fluid plus solid getting in there, and then I expect the solid to precipitate at the bottom.

Now, these particle size, let us say I have a particle which is settling. So, this is the particle. This particle is undergoing what all forces? First of all, there is a gravity force acting on this particle because of which the particle wants to come down. There is a buoyancy force on this particle that would be acting upward, and as the particle travels, you might have you remember what you studied as the particle travels, you would expect that the particle will start from 0 velocity and then the velocity continues to increase. So, as the velocity increases at one point, the drag force that is applicable on this particle would be drag force plus the buoyancy force that would equate to the force of gravity here, and at that point, the particle will achieve a terminal velocity.

Now, of course, if the particle size is very small or the particle is light, so you need a very long time for the particle to come down, to the floor of this tank. So, the only way out is you have to increase the residence time because you are working in a continuous mode, but you have to ensure that the tank is sufficiently big, and you have to provide a lot of time for the particle to settle. Now, it may so happen that you cannot afford to do that there is such a big tank. So, in that case, one may use instead of gravity one may use centrifugal force, a process which is going which goes by the name centrifugation where you have you apply centrifugal force instead of gravity force that is basically a body force by which the solid particle would be drawn to the bottom of a vessel. So, these are some of the aspects which we will be discussing in this momentum transfer operation.

Apart from process industries biomedical applications, there are several biomedical applications that relies on momentum transfer in fluids, very simple thing is flow of blood in the veins and arteries, and there are many other aspects of momentum transfer. For example, you have a tissue through which continuously there is a flow of fluid, and simultaneously flow and diffusion that because, blood has to reach every part of the body and one it has to it has to capture all the carbon dioxide and delivered oxygen. So, that cells get their necessary nutrients and to survive. So, this is a very unique process and one has to ensure if the process gets hampered because of some problems. So, that has to be reinstated artificially.

So, for these reasons, there are many different biomedical applications where this momentum transfer comes into play. Then the next point is geophysical. Geophysical is for example, I can think of underground reservoirs or the flow of wind, say underground reservoirs when it comes to that we still rely on hydrocarbons a lot because the petrochemical products that we use for our everyday, the polymers that we generate from petroleum which is obtained from underground reservoirs, I mean from the mother earth, and there the flow of fluid because essentially when it comes to crude oil that remains trapped or when it comes to natural gas that remains trapped inside a porous structure, inside the pore space and one has to inject another fluid mostly water which is the most abundantly available resource in nature. So, one has to inject water to displace that crude oil from the reservoir. So, that is essentially a momentum transfer operation, and a lot of tuning has to take place at what pressure you should inject that fluid, what viscosity of the fluid should be there.

So, that you can have a better sweeping out of the crude oil. So, many different aspects would come and that definitely comes under the momentum transfer operation in fluids.

Similarly, for material synthesis, not just synthesis material engineering when it comes to that. For example, I can think of, let us say, chemical vapor deposition: one wants to deposit a thin layer, or one wants to grow an oxide layer, let us say, or grow a carbon layer on a substrate, a very thin layer that is supposed to act as a functional material. So, there we have to apply a vacuum, we have to apply so that the pressure has to be very low.

So, we can increase the mean free path of the vapor that goes through the tube, and then we sublime some material, which means from solid, we take it to the vapor phase, the material that we want to deposit on a substrate. So, we take the vapor out using sublimation by heating, and then create a highly vacuum-inert atmosphere, and ensure that the substrate is held on its path where the temperature is controlled such that the vapor is about to get deposited, the vapor is going to form a solid thin layer on the substrate. So, this entire dynamic works based on momentum transfer. Here it is it operates in a high vacuum environment, and the vapor phase has to travel through the tube and heat the surface of the substrate. So, one can start having growth of a layer of very small dimension, and that is what is required for certain functionality of that material. So, in every aspect you will find that there would be a momentum transfer one has to calculate, at what pressure I should have this vapor I should send.

So when it reaches, it will be at this pressure, just the right pressure, right temperature. So, it can, there would be a deposition. Similarly, for aero or hydrodynamics, when it comes to airfoil or hydrofoil and then there is a flow around that airfoil or hydrofoil. So, there is an immense amount of momentum transfer operations one has to keep track of. We will discuss some of them in due course here in these lectures. First of all, when it comes to this momentum transfer operation, I must point out one immediate point, which is that there are two types of formulations possible that. Is one is known as the Eulerian formulation, and the other is the Lagrangian formulation.

What is Lagrangian formulation? For example, we just now mentioned about a particle getting settled. So, we said the forces that are acting on that particle is one is the gravity force acting downward, and another is the buoyancy force acting upward. Generally, they call it buoyancy corrected gravitational pull. So, you know which one and the same thing if you subtract, and then what you get is the buoyancy-corrected gravitational pull. But on the other side moment, the moment the particle starts moving and accelerating, there is a drag force acting on that particle because the particle is moving downward. So, there would be a drag force upward.

So, one has to balance between the drag force and these gravity and buoyancy forces, and one will see that at one point in time, as the velocity increases, the drag force will increase, and at one point in time, you will find that the acceleration becomes 0. We said that the particle has reached terminal velocity, which means that from now on, the particle will move at a constant velocity. So, this is a typical Lagrangian formulation that means you are working on a particle, and then that particle is moving because of certain forces acting on that particle. Now, you think of fluid comprising 100,000 millions of such particles, and then you keep track of the movement of each and every particle, and also the collision of the particles with themselves, and collision with the wall. So, let us say we want to find out the pressure or temperature; we can calculate that based on these interactions of particles within themselves and the interaction of the particles with the wall.

So, this type of formulation is referred as Lagrangian formulation. Here, you will simply note the position of that particle, and take a derivative of it. We get velocity. Take the derivative of velocity with respect to time, and you get acceleration, the typical physics that we have studied so far. So, these are all these all come under Lagrangian formulation. Whereas there is another method, which goes by the name Eulerian formulation, where we talk about the velocity field. We do not segregate particles. We do not talk about this particle moving at that velocity or the other particle moving at some other velocity. Here, we are more concerned with particles in mass and how they behave.

That means, in the space, let us say these are my x, y, and z coordinates, and I pick up a differential volume. So, within this differential volume, I will have a certain number of particles, hundreds or thousands of particles, they are a certain velocity. So, what I will say is at a particular time at this location x, y, and z, some coordinates you have. So, at the center, it is x, y, and z. So, on this side, dx/2, other side is dx/2.

So, this is a differential element. So, this is dx, dx, and dx dy and dz. So, these are the dimensions. So, if we try to find out the average velocity of all the particles present inside the box, and then we call that average velocity as the velocity at that point. So, that is not the velocity of a part single particle the way we talked about in the Lagrangian framework.

It is not the velocity of a single particle; it is the average velocity of all particles within that differential element centered around the coordinate x, y, z. So, that we will call Eulerian velocity. This is a velocity fiel, here we have this as the velocity field. Velocity field is a function of it is function of x, y, z, and t. So this is known as a velocity field. It is not the velocity of a single particle. It is the velocity of several particles within that differential element. So, now, if we operate with this velocity field, we ignore all these individual moments of particles and start putting theories using these types of variables.

What are the implications? So, that is something which is shown here in this plot. Here the fact is that you must understand there is something called a probe volume. So, you have a probe volume; say, for example, you are measuring velocity. So, you can measure velocity over a certain dimension. For example, what instrument you are using, it depends on that.

For example, you are measuring density. So, measuring density, there is a probe volume. You can measure density over 1 nanometer of a dimension if you have your own method, or you can measure it over 1 millimeter of a dimension. So, that means you are measuring density at a point, but that density is that point and surrounding area, which is, let us say, 1 millimeter by 1 millimeter by 1 millimeter, whatever is the average density that is what you are capturing. Maybe you are coming up with some other instrument where the density of the entire room, I mean, your coordinates, are in on the order of meters or maybe tens of meters, and the way your probe volume works is that the average density of the entire room you are measuring. When you are particularly going for density, let us say the density of the atmosphere you are measuring.

So, there you are measuring average density. Your probe volume is such that when you talk about density at a point, that is, the density of a point and the surrounding area, the average density of that surrounding area happens to be a few meters by a few meters. So, there is something called a probe volume, and then within this probe volume you have now, there would be a certain number of particles, a certain number of molecules, let us say. So, you are trying to keep track of the number of molecules within that probe volume. So, here you will see that when you are working with a smaller probe volume the number of molecules would be less. When you are working with a larger probe volume, definitely the number of molecules under your purview, number of molecules over which you are doing the measurement would be much larger.

What is the implication? I have a very small differential element here, and let us say the number of molecules that are entering and the number of molecules that are leaving. So, that is a certain fraction. I have another probe volume which is this big. So, here I would say that, in this case, the number of molecules entering and a number of molecules leaving as against the number of molecules sitting inside the probe volume, you will find that the number of molecules entering and leaving is a very large percentage of a total number of molecules present in the system. Whereas, when you are working with a larger probe volume already so many molecules are sitting there.

So, number of molecules entering and number of molecules leaving that would be a smaller fraction of molecules that are present there. In other words, I would say that when you shrink the size of the probe volume, here I have drawn the density. I am calling it $\delta m/\delta v$. Here δm is the mass, and δv is the probe volume, and δm is the mass of particles within or mass of molecules within this probe volume. So, $\delta m/\delta v$, if we are trying to plot against δv, we will see that when δv is large, I have a consistent value because the number of molecules entering and the number of molecules leaving are small compared to the number of molecules sitting in the system. So, the number of molecules sitting in this volume contributes to your density.

So, the density will remain more or less uniform. I mean, you are shrinking the probe volume and seeing the same density over and over, but when the probe volume is too small, then you will find that there would be fluctuations because, at one particular instance, you will find that the number of molecules entering and a number of molecules leaving there is a big difference. So, the density shoots up. In another way, at other times, there is a difference in the reverse direction.

So, you find the density goes down. So, there will be a lot of fluctuations. So, when you talk about the Eulerian framework, it is very important to keep a very clear idea of what is your probe volume. What I will do is I will continue this exercise, and I will see how these probe volume will feature in our Eulerian framework because when we define velocity field it is important that we understand what are the inherent assumptions based on which we are defining this velocity field. Let me tell you one thing: in this course, we will be following the Eulerian framework all along. Lagrangian formulation, at some point we will show some comparisons, but generally as a core discipline momentum transfer is handled by Eulerian framework, and that is what we will be doing in this course.

So, that is all as far as this module is concerned I will be continuing this lecture in this exercise of Eulerian formulation in my next lecture. Thank you very much for your attention.