Momentum Transfer in Fluids
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Welcome back to our discussion on the equations of change. And by equations of change what I mean is that when there is a flow in the liquid or in a fluid, then there is going to be, it has to satisfy, it has to obey certain natural laws. One of that law is equation of conservation of mass from where in the last class we have seen how we have obtained the equation of continuity. In this class we are going to correlate the change in momentum with the forces that are acting on a control volume which is open, which is defined by a set of control surfaces, but the control surfaces allow the entry of mass, entry and exit of mass through it. Now, whenever a mass enters through a control surface into a control volume, it will carry some momentum with it. Additionally on any such surface if there exists a change in the velocity, then this change in velocity would also create a shear stress.

And we already know that shear stress is nothing but momentum, getting molecular transport of momentum or diffusive transport of momentum in a direction perpendicular to the flow. So, these are the two main mechanisms by which momentum can come into a control volume. So, rate of momentum change, rate of momentum, time rate of momentum coming into a control volume or molecular transport of momentum, rate of molecular transport of momentum is essentially a force. So, then we have to consider the other forces which could act on the control volume.

And what could be the other forces? They are categorized into two different types. One is a body force where the force acts on every particle present inside the control volume equally. So, it works, it acts on the entire volume. And the examples of body force, the most common example of body force is the gravity. There could be electrostatic, magnetic and similar such forces which are broadly termed as body forces.

There are also forces which are, which act only at the periphery only on some of the control surfaces. They are called surface forces, the example being that of pressure. So, when you apply pressure, it is the surface which is going to experience a force due to the pressure existing at that point. So, all these forces through momentum, through body forces and surface forces if we club them together and if the algebraic sum of this is unbalanced then this would give rise to an acceleration or deceleration of the control volume, its velocity will change. So, that is nothing, but the statement of Newton's second law for an open system, that is the sum of all forces acting on the control volume will or may result in an acceleration.

So, mass times acceleration of the control volume should be equal to the algebraic sum of all these forces acting on it. So, this expression or this statement we are going to write for a control volume and we will identify as you would see in the figure that the forces that are going to be considered for such a control volume. So, we begin our journey with the equation, with the derivation conceptual derivation not step by step derivation. Step by step derivation is there in your textbook in Bird Stewart Lightfoot chapter 3. So, I will not derive, that derive that in here I will only point out the concepts involved in the derivation of that equation.

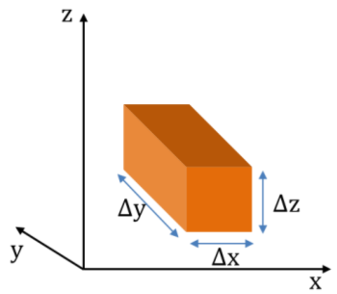
So, for that first I will recap of what we have done before. So, there are three types of derivatives which we have defined in the past previous class, one is the partial time derivative where the axis is fixed, the x y z is constant, the total time derivative where the axis has a velocity, the coordinate system has a velocity on a of its own where d x d t d y d t and d z d t they refer to the to the velocities of the coordinate system. And then we have the substantial time derivative or derivative following the motion where the coordinate system moves with the velocity of the surrounding fluid. And therefore, anything any variation any measurement that you take with respect to time is called the substantial time derivative. Based on this we have derived our equation of continuity which I will not discuss any further, let us move on to the conceptual derivation of the equation of motion.

$$\text{Partial time derivative: } \frac{\partial c}{\partial t} \text{ (x,y,z constant)}$$

$$\text{Total time derivative: } \frac{dc}{dt}, \frac{dc}{dt} = \frac{\partial c}{\partial t} + \frac{\partial c}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial c}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial c}{\partial z}\frac{\partial z}{\partial t}$$

$$\text{Substantial time derivative: } \frac{Dc}{Dt}, \frac{Dc}{Dt} = \frac{\partial c}{\partial t} + v_x\frac{\partial c}{\partial x} + v_y\frac{\partial c}{\partial y} + v_z\frac{\partial c}{\partial z}$$

So, for that the first statement that I have already explained is that the rate of momentum accumulation must be equal to the rate of momentum in minus rate of momentum out plus sum of all forces acting on the system. If this is at steady state, then the left hand side of this equation would be 0. So, the all the forces and the rate of momentum in and out would be perfectly balanced at steady state and therefore, the control volume will have no acceleration. So, let us first draw the figure for such a situations for such a control volume where you can see once again that the control volume is defined by the space del x del y del z and we are going to write the x component of the momentum equation. This is important because the equation that we are going to get in the through this exercise is a vector equation.



Rate of momentum ACCUMULATION = Rate of momentum IN – Rate of momentum OUT $+\sum$ Forces acting on the system

x-component of the MM Eqn.: Rate of momentum IN – Rate of momentum OUT

So, we can start with the x component of the equation the y and the z components once we have the x component of the equation of motion, the other components can be written simply by changing x to y in at the right places. So, if I can derive one component, I can derive all the other components. So, that is why I am going to derive only the x component of the momentum equation, y and z components can be written in a similar fashion. So, let us first identify the rate of momentum in and the rate of momentum out. So, if you look at the expression that I have written over here, once again del y del z it is the area of the x face.

$$\Delta y \Delta z(\rho v_x v_x|_x - \rho v_x v_x|_{x+\Delta x}) + \Delta x \Delta z(\rho v_y v_x|_y - \rho v_y v_x|_{y+\Delta y}) + \Delta x \Delta y(\rho v_z v_x|_z - \rho v_z v_x|_{z+\Delta z})$$

So, it is this is the x face and the x face has an area del y times del z. Now, rho v x, this part rho v x is multiplied by del y del z is nothing, but the rate of mass that the mass which is coming in to the control volume. So, once you have this as the mass coming into the control volume with the velocity, essentially you are talking about rate of momentum coming in to the control volume. Whereas the fluid leaves at x plus delta x. So, all these v x is evaluated at x plus delta x would give you the time rate of momentum that is entering or leaving the control volume because of motion of the fluid around it.

So, since it involves the motion, since it involves the actual motion of the fluid particles. So, this is a convective transport of momentum. So, the circled one is the in term and the next one is the out term for the x component. Now, we are going to talk about, this is this is the now we are going to find out what is the y what is the x component of momentum being transported through the y face. So, for that I first write the area of the y face and the area of the y face is delta x delta z rho v y it is the.

So, rho v y multiplied by del x del z, this is the amount of mass which is entering through the y face. I think that is clear I will go through it once again rho times v y. So, it is the area rho times v y this is the y component of velocity. So, only the at the y face that there is the y face the amount of mass which comes in would simply be equal to rho v y. This multiplied by the area would give me the amount of mass which enters through the y face per unit time. Now, in order to obtain the x momentum of the mass associated with this, the x momentum associated with this I need to multiply that with v x.

So, this is to be clearly understood once again let us say I have this is my y face and this is v y. So, this amount of fluid is entering through the y face, but this is the velocity is 3 dimensional. So, even though this is my v y, this packet also has an x component associated with it. So, I figure out what is the mass of the fluid crossing the y face that is the amount of mass I am adding through the y face, but this amount of mass carries with it because of its nonzero v x some amount of x momentum associated with it. And since I am writing the x component of equation of motion.

So, I multiply the amount of mass that comes in through the y face with the corresponding value of v x at that point in order to obtain what is the x component of momentum being added through flow through the y face. So, once again the bracketed one that you first see here is the mass that comes in through the y face multiply that with v x and what you get is the amount of x momentum being added to it. Because you remember that we are writing the x component of the momentum equation. So, if I am writing the x component of the momentum equation then this is necessary that I identify the amount of mass being entering through each of the control surfaces and then multiply that mass with the corresponding x component of velocity in order to obtain the x momentum being added through the y face. So, the first term over here is the x momentum being added through the x face.

This term is the x momentum being added because of flow through the y face and similarly the third one here you see this is the mass being added through the z face multiplied by the x component of velocity. So, this third term is the x component of momentum being added by flow through the z face. So, I hope I am clear that how do we evaluate the momentum being added through flow through each of the surfaces and these three are the corresponding out terms where everything is evaluated at the at the y plus del y or z plus del x faces. So, this is essentially then my convective transport of momentum. So, this three combined together would give me the convective transport of momentum.

$$\Delta y \Delta z(\tau_{xx}|_x - \tau_{xx}|_{x+\Delta x}) + \Delta x \Delta z\left(\tau_{yx}\big|_y - \tau_{yx}\big|_{y+\Delta y}\right) + \Delta x \Delta y(\tau_{zx}|_z - \tau_{zx}|_{z+\Delta z})$$

Next comes the other parts which is the conductive or molecular transport of momentum. So, for molecular transport of momentum the shear stress will have to be multiplied with the corresponding area. Now, when we talk about shear stress, we understand that shear stress is a tensor, it has 9 components. So, the 9 components are then tau xx, tau yx, tau zx and tau xy, tau yy, tau zy and tau xz, tau yz and tau zz. So, these are the 9 components of the shear stress tensor.

Of these 9 you can clearly see that these three are different, these three are normal stresses what commonly we encounter we feel as the pressure of this. So, these are normal stresses and sometimes they are expressed not as tau, but sigma xx, sigma xy and sigma zz. So, the concept is that this is the x momentum. So, this is the x momentum being transported in the x direction, x momentum being transported in the y direction and x momentum being transported in the z direction. So, when we transport these in the when we talk about when since we are writing the x component of equation of motion.

So, these three are the shear stress components that we need to consider. So, where does tau xx act on? It is in the x direction. So, if it is in the x direction then it must be acting on an area which is the x face. What is the area of the x face? It is del y times del z. So, del y time del z tau xx at x minus tau xx at x plus delta x gives me the x component of momentum being transported in the x direction itself and this being stress force per unit area.

So, I can express this force per unit this force per unit area into force by multiplying it with the corresponding area of the x face which is del y del z. In a similar fashion when we talk about the x component being transported in the y direction then I must multiply tau yx with the area of the y face and we understand that the area of the y face is del x del z. And for the x component of momentum being carried in the z direction the this is del x del y is the area of the area of the z face. So, when we talk about the 3, when we talk about the shear stresses which relate to the transport of x momentum then these are the 3 that we need to consider tau xx, tau yx, tau zx to be multiplied with the corresponding areas as I have as I have shown you here. So, together all these are essentially my conductive transport of momentum.

So, conductive transport of momentum. So, I have taken care of the convective transport and the conductive transport. So, what is left is the pressure component in the x direction. So, the pressure is p at x to be multiplied by del y del z and the other pressure at the other end is px plus del x which again is applicable which again is being applied on an area del y del z. So, this essentially gives me what is the force due to pressure in the x direction or the total force the x component of the total force due to pressure is essentially the local pressure multiplied by the area which is perpendicular to the x direction.

Pressure Force x-component: $\Delta y \Delta z(p|_x - p|_{x+\Delta x})$

So, this essentially is the pressure force the force due to pressure and one can write what is the body force. So, body force is simply the volume, the control volume multiplied by rho. So, this gives me the mass of the control volume multiplied with the gx or the x component of the body force that is acting on the control volume. So, this g could very well be the acceleration due to gravity if we if we think that it is the gravity which is the only force that is present in this system. So, now, I have all the necessary terms that take into account the total amount of momentum being added to the control volume, the rate of momentum being added to the control volume, the pressure forces that is a surface force and the body force.

$$\text{Body Force x-component: } \rho g_x \Delta x \Delta y \Delta z$$

As a result of which the momentum accumulated inside the control volume may change. So, what is that? What is the time rate of that? So, the time rate of that would simply be the volume multiplied by rho giving you the mass multiplied by the velocity in the x direction gives you the gives you the momentum at the x component of the momentum and del del T of that. So, this is the time, this entire thing is the time rate of change of momentum of the control volume. Now, I have all the terms necessary for the Newton's second law of motion for an open system. So, my left-hand side or one side will contain this term, the other sides are going to contain all these terms taken together.

$$\text{Rate of momentum ACCUMULATION x-component: } \Delta x \Delta y \Delta z \frac{\partial(\rho v_x)}{\partial t}$$

And the next step is straightforward next step is straightforward. So, what we do is as before we divide all sides by del x del y del z, take in the limit when del x del y del z I mean all of them approach 0 and you convert the difference equation into a differential equation. The differential equation after some in a rearrangement mostly by the equation of continuity and the definition of the substantial derivative would give rise to this compact vector equation which is termed as the equation of motion. Now, what is the left-hand side? It is nothing, but rho times acceleration.

$$\rho \frac{Dv}{Dt} = -(\nabla p) - (\nabla . \tau) + \rho g$$

So, that is the force. So, rho times dv dT d capital D dT refers to the substantial derivative. Then the next one is del of rho which is essentially the pressure per unit volume. So, the left-hand side is mass per unit volume which is rho and acceleration which is d dT of v, but we note that this d dT of v is the substantial derivative. The next one what we have is the pressure force per unit volume, then one after that which is the shear force per unit volume and then the last one that we have is the gravitational force per unit volume.

So, this is the equation of motion. This in this equation of motion everything is expressed in per unit volume basis. So, that is to be kept in mind that what we have is everything per unit volume basis. So, starting with this equation of motion which is very general one can add additional simplifications that it is a constant rho and constant mu situation and that we are dealing with a Newtonian fluid. So, if it is a constant rho and constant mu system then this term, this can be and it is a Newtonian fluid. So, if it is a Newtonian fluid then this expression which I have in the blue border it is known as the Navier Stokes equation.

$$\rho \frac{Dv}{Dt} = -(\nabla p) - \mu \nabla^2 u + \rho g$$

So, we have conceptually derived the Navier Stokes equation we understand what is the significance of each term I will go through it once more again, but this is what is known as the

Navier Stokes equation. So, the term over here this is the convective transport of momentum. So, if you expand that d dT of v it is going. So, that is essentially the substantial derivative if you use the definition of the substantial derivative this refers to this part will have v appearing in explicit form referring to the motion of the actual motion of the particle. So, the left-hand side of Navier Stokes equation will always be one term that refers to the transient part, I will show in the in the expanded part form of it and the rest of the terms are going to be the convective transport of momentum.

The term the second term on the right-hand side is essentially the conductive or molecular transport of momentum this is the viscous term this term is due to the viscosity of the fluid the nonzero viscosity of the fluid. So, all those molecular transport or viscous transports is represented by the second term on the right-hand side. This term on the other hand is due to the application of a pressure gradient, flow initiated due to the application of a pressure gradient and this being the flow which could be due to gravity for as an example. Now, if the viscous forces are not present if you are dealing with an inviscid fluid then essentially the second term on the right-hand side of the second term on the right-hand side would disappear and what you get is this expression which is known as the Euler equation. So, the Euler equation and how Bernoulli's equation is obtained from that will be dealt with in the subsequent in the later part of the course, but this is what Navier Stokes equation is all about.

$$\rho \frac{Dv}{Dt} = -(\nabla p) + \rho g$$

$$[\partial \rho / \partial t + (\nabla \cdot \rho \mathbf{v}) = 0]$$

Cartesian coordinates (x, y, z):
$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$
Cylindrical coordinates (r, θ, z):
$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$
Spherical coordinates (r, θ, ϕ):
$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$

If I show you the Navier Stokes equation the x, y and z component of the Navier Stokes equation for a Cartesian coordinate system or for a cylindrical coordinate system or a spherical coordinate system initially they would look ominous. You would think how am I going to deal with this complicated looking equation, but what I will show you in the next class is that it is extremely easy to figure out to use Navier Stokes equation for solving fluid mechanics problems. The only thing that you have to do if it is let us say we are dealing with one dimensional flow, you have to choose the right component of the Navier Stokes equation and write the entire Navier Stokes equation. For example, let us say this is what equation of continuity ok. Let us start with the equation of continuity first.

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g}]$$

Cartesian coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

So, equation of continuity for Cartesian coordinates, cylindrical coordinates and spherical coordinates the expression is given. So, one can start with that and obtain the form of the equation that can be used for the equation of continuity, but let us concentrate more on the Navier Stokes equation. This is what Navier Stokes equation looks like and whatever I have said before would be would be would be more prominent in this. So, this is equation of motion for a Newtonian fluid and because it is a Newtonian fluid then you could see that it is mu and the dependence is clear over here with constant rho and constant mu. Now, when you when you think of let us say the flow is principally in the x direction.

So, if the flow is in the x direction you are going to choose the x component of the Navier Stokes equation or if it is in the y then or z you took the appropriate form of the Navier Stokes equation. Now, when you look at this first term del v x del t is the time varying part. So, this is the transient term in Navier Stokes equation. In the subsequent three terms on the left-hand side, you would see the appearance of velocity explicitly explicit appearance of the velocity. So, these three are collectively called as the convective momentum transport, convective momentum transport part.

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Spherical coordinates (r, θ, ϕ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] + \rho g_\theta$$

This is the transient term, this is the transient part of the Navier Stokes equation these three are the convective transport, this is the flow induced by applied pressure gradient. Many times, seen our previous treatment we have seen that the flow is initiated by a pressure gradient or the flow is initiated by a body force acting on it. For example, for an inclined plate or for a tube which is held vertical and you have both the applied pressure gradient and the body force. So, these two terms refer to that whereas these terms with mu appearing outside of the third bracket is the viscous transport of momentum.

So, this m square refers to the momentum. So, the mu containing term inside the third bracket is the viscous transport part. So, you identify which one is going to be the right component of Navier Stokes equation for a specific problem. Write the entire Navier Stokes equation and see whether it is a steady state problem or not. If it is a steady state obviously, then the first term on the left hand side is going to be equal to 0 the velocity does not change with time. Then you look at each of these terms read the problem find out whether v x is a function of x, y or z.

Depending on your understanding or the actual flow situation let us say v x is a function only of y it is not a function of x and z. So, if it is not a function of x then the second term and the fourth term will disappear. You also have to think whether there is a v y component present in the flow. So, you can choose about the all the terms decide about all the terms on the left-hand side. Then you come to the right-hand side, is there an applied pressure gradient in the system if so, you have to keep del p del x or you can neglect del p del x if it is not there.

Is it a horizontal situation, inclined situation or it is a vertical situation depending on that the component g x will have different values. So, you are going to choose the right expression or right value of g, it could be 0 or it could just be g you can choose these g x. And then inside the third bracket term again figuring out v x is a function of what x y z or any combination of them, you are going to keep the terms which are there and which are which could be neglected. After you do this exercise then you have your governing equation.

You do not have to go through any shell momentum balance. That is the beauty of working with a generalized approach not thinking about the formation of a shell drawing of a shell figuring out which surface is associated with convective flow, which surface is associated with the conductive flow. Pick the right equation right component cancel the terms that are not there and what you get is the governing equation. We will solve some problems which we have done previously using Navier Stokes equation and using shell momentum balance in the subsequent class and to show you how easy it is to use Navier Stokes equation. We will do that we will take it up in the next class. Thank you.