Momentum Transfer in Fluids
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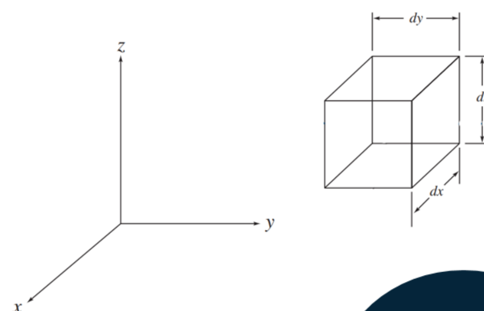
Lecture-13

I welcome you to this lecture of Momentum Transfer in Fluids. What we were discussing was elements of inviscid flow, and in this context, we are trying to have a force balance on a differential element where only the normal stresses are applicable. We are not considering the shear stresses. The reason would be obvious very soon. One reason is that we are talking about an inviscid flow and the shear stress and one layer sliding against the other imparting a shear stress.

Body force on a differential volume

$$d\vec{F}_B = \vec{g}\,dm = \vec{g}\,\rho\, dV$$

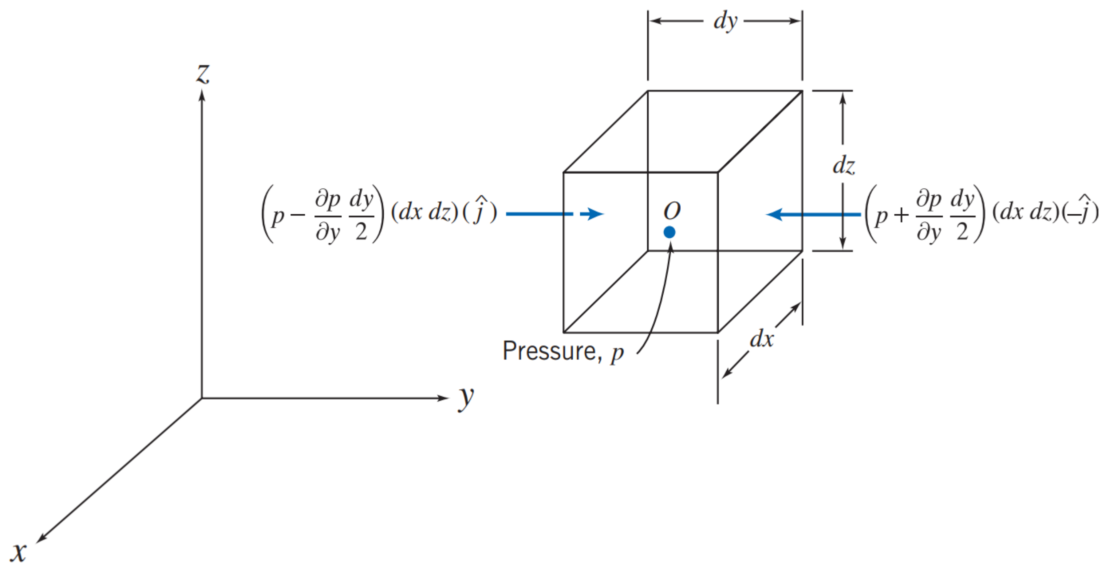
$$d\vec{F}_B = \rho\vec{g}\, dx\, dy\, dz$$



All these are applicable when there is viscosity coming into play, but here we are talking about fluid moving in mass. So, it is more like a plug flow that we are trying to address. So, here, as a differential element, there would be body forces. This we discussed at the last class that there is a body force term arising, which can be given as

$$d\vec{F}_B = \rho\vec{g}\ dx\, dy\, dz$$

This dx dy dz is the volume of the differential element which is written as dV. This V should be struck through because we differentiate V by striking through the V we call that volume and V with an arrow on top we call that a velocity field in this Eulerian context. So, we know that this is the unit of volume, so to say m^3, and then we have this density, which has the unit of kg/m^3, and so when we take the product, this meter cube and meter cube, they cancel out. So, we have kg left mass into acceleration due to gravity that is the body force imparted on that fluid inside that differential volume. When it comes to the surface force acting on this differential volume and only the pressure force not that is the normal stresses.



So, we had talked about at the end of the last class that this pressure force would be if I assume the pressure at the center of this differential element as p then this is here on the left face on the left face the pressure would be p - $\frac{\partial p}{\partial y}\frac{dy}{2}$ by noting that this from the from the center line to this face the distance is $\frac{dy}{2}$. So, what we are assuming is we are taking a Taylor series expansion and ignoring higher order terms that is we are assuming that the pressure from left face to right face it changes linearly which is perfectly valid as long as the differential volume is small that the dy is small. So, you are linearizing the pressure profile between left face and the right face. So, this is the pressure we mentioned dx dz is the area. So, pressure in N/m^2 into area dx dz that is in m^2.

So, this gives me the unit of Newton. So, this is the force acting in ĵ direction acting in the direction of y, and there is pressure always as far as point O is concerned. Pressure is acting towards that point, whereas normal stress is acting outward. So, the pressure from this side from the right face would be acting towards point O, and here again, if the pressure at point O is p here by taking Taylor series expansion and then ignoring higher order terms, we come up with this p+ $\frac{\partial p}{\partial y}\frac{dy}{2}$. So, here it is p plus, and here it is p minus. So, you need to make a note of, and once again, the area is dx dz, and here, the force is acting in -ĵ direction.

The same thing is applicable from the bottom and the top and from this side and from the other side as well. So, these forces so, we are talking about forces pressure multiplied by the area of the face. So, these forces along with the body forces. So, if we sum them up that gives me the net force acting on this differential element and that we are going to equate with the because that is force at per as per Newton's second law that has to be equal to mass into acceleration. So, that will give me the acceleration of that differential volume or, for that matter, the acceleration of the fluid.

In a static fluid shear stress is not present.
Hence net surface force is pressure,

$$p = p(x, y, z)$$

By Taylor expansion, pressure at left face

$$p_L = p + \frac{\partial p}{\partial y}(y_L - y)$$

$$p_L = p + \frac{\partial p}{\partial y}\left(-\frac{dy}{2}\right)$$

$$p_L = p - \frac{\partial p}{\partial y}\frac{dy}{2}$$

pressure at right face,

$$p_R = p + \frac{\partial p}{\partial y}(y_R - y)$$

$$p_R = p + \frac{\partial p}{\partial y}\left(+\frac{dy}{2}\right)$$

$$p_R = p + \frac{\partial p}{\partial y}\frac{dy}{2}$$

Net pressure forces on the whole fluid element,

$$d\vec{F}_s = \left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right)(dy\,dz)(\hat{\imath}) + \left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right)(dy\,dz)(-\hat{\imath})$$

$$+ \left(p - \frac{\partial p}{\partial y}\frac{dy}{2}\right)(dx\,dz)(\hat{\jmath}) + \left(p + \frac{\partial p}{\partial y}\frac{dy}{2}\right)(dx\,dz)(-\hat{\jmath})$$

$$+ \left(p - \frac{\partial p}{\partial z}\frac{dz}{2}\right)(dx\,dy)(\hat{k}) + \left(p + \frac{\partial p}{\partial z}\frac{dz}{2}\right)(dx\,dy)(-\hat{k})$$

$$\Rightarrow d\vec{F}_s = -\left(\frac{\partial p}{\partial x}\hat{\imath} + \frac{\partial p}{\partial y}\hat{\jmath} + \frac{\partial p}{\partial z}\hat{k}\right)\ dx\,dy\,dz$$

$$\Rightarrow d\vec{F}_s = -\left[\left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right)p\right]dx\,dy\,dz$$

$$\Rightarrow d\vec{F}_s = -[\text{grad}\,p]dx\,dy\,dz$$

$$\Rightarrow d\vec{F}_s = -\nabla p\ dx\,dy\,dz$$

So, what we did here which we said that Taylor series expansion exactly what I mentioned just now $p_L = p + \frac{\partial p}{\partial y}\left(-\frac{dy}{2}\right)$, we are ignoring the higher order terms. So, this is how we arrived the pressure at the left face. So, that is what we mentioned here $p - \frac{\partial p}{\partial y}\left(\frac{dy}{2}\right)$. Similarly, pressure at the right face is $p + \frac{\partial p}{\partial y}\left(\frac{dy}{2}\right)$. So, this is the pressure at the right face we have already discussed.

$$p_R = p + \frac{\partial p}{\partial y}(y_R - y)$$

So, here it is once again $p + \frac{\partial p}{\partial y}$ and the difference the right face minus the center of the differential volume. So, that difference is $+\frac{dy}{2}$ in this case when I am comparing the center with the right face. So, if we try to find out the net pressure force on the whole fluid element. So, this is coming from the left face. So, this is going there $\left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right)(dy\,dz)$ and here we have the right face, this is the right face term, and that is going there.

So, since we are acting in the x direction. So, this is $\hat{\imath}$. Here we are talking about this in the y direction we had mentioned. So, since this is in y direction. So, we have $p+\frac{\partial p}{\partial y}\frac{dy}{2}$, and this is $\hat{\jmath}$ what we have here is I should be correcting myself this is not the case one second this is going here this term is going here and this term is going there. So, this is as far as the $\hat{\jmath}$ term is concerned.

So, this is $\hat{\jmath}$ and this is $-\hat{\jmath}$. So, this is similarly we have $\hat{\imath}$ we have once again the one face and the other face and similarly bottom face and the top face for the $\hat{k}$. So, we have these three terms coming in plus this is $\hat{\imath}$ with plus that means, it is in positive x direction, and this is minus I hat negative x direction. Similarly, positive $\hat{\jmath}$ and negative $\hat{\jmath}$. So, when we sum them up one thing is for sure that we have sign on this unit vector that keeps track of the direction.

So, we do not have to subtract anything we just have to sum them. So, the sign will automatically be taken care of by this sign attached to these unit vectors. So, when we sum them up, what we see here is p dy dz, and here p dy dz, and there is a minus sign. So, this p term and this p term will cancel out. Similarly, here we have p dx dz and p dx dz, but here it is with positive $\hat{\jmath}$ and here it is negative $\hat{\jmath}$.

So, again, this p will cancel out by the same way. So, what we have here is $\left(-\frac{\partial p}{\partial x}\frac{dx}{2}\right)(dy\,dz)(\hat{\imath})$ and again this minus come in here. So, $\left(-\frac{\partial p}{\partial x}\frac{dx}{2}\right)(dy\,dz)$. So, if I put dx dy dz in one bracket if we put this dx dy dz together. So, what do I have from these two terms? I have dx dy dz I take this out. dx dy dz $\left(-\frac{1}{2}\frac{\partial p}{\partial x}\hat{\imath}-\frac{1}{2}\frac{\partial p}{\partial x}\hat{\imath}\right)$

So, minus half and minus half that becomes minus 1. So, then this becomes dx dy dz $\left(-\frac{\partial p}{\partial x}\hat{\imath}\right)$. So, the contribution of this is dx dy dz $\left(-\frac{\partial p}{\partial x}\hat{\imath}\right)$and here similarly you have to say for the second term you have again dx dy dz, but this time you have $\left(-\frac{\partial p}{\partial y}\right)$ and here again you can take this dx dy dz and you are left with $\left(-\frac{1}{2}\frac{\partial p}{\partial z}-\frac{1}{2}\frac{\partial p}{\partial z}\right)$.

So, that gives me $\frac{\partial p}{\partial z}$. So, essentially, if I sum all these six terms, what I get is dx dy dz outside and I have $\frac{\partial p}{\partial x}\hat{\imath}$ coming from here I have $\frac{\partial p}{\partial y}\hat{\jmath}$ and here I have $\frac{\partial p}{\partial z}\hat{k}$. So, these are the three terms there and dx dy dz. So, that is the net surface force acting on this differential element.

So, instead of this you can write this as grad of p this is really $\frac{\partial}{\partial x}\hat{\imath}+\frac{\partial}{\partial y}\hat{\jmath}+\frac{\partial}{\partial z}\hat{k}$ you would like to write this in a compact notation as grad of P which is having a symbol this inverted triangle. So, this is referred as grad of p. So, this is a $d\vec{F}_s = -[\text{grad p}]dx\ dy\ dz$. So, this is as far as the surface force is concerned. So, if I try to find out what is the total force.

Net force acting on the fluid element,

$$d\vec{F} = d\vec{F}_s + d\vec{F}_B$$

$$\Rightarrow d\vec{F} = -\nabla p\ dx\ dy\ dz + \rho\vec{g}\ dx\ dy\ dz$$

$$\Rightarrow d\vec{F} = (-\nabla p + \rho\vec{g})\ dx\ dy\ dz$$

$$\Rightarrow d\vec{F} = (-\nabla p + \rho\vec{g})\ dV$$

$$\Rightarrow \frac{d\vec{F}}{d\cancel{V}} = (-\nabla p + \rho\vec{g})$$

According to Newton's second law, differential force acting on a fluid particle is

$$d\vec{F} = \vec{a}\ dm$$

$$\Rightarrow d\vec{F} = \vec{a}\ \rho\ d\cancel{V}$$

$$\Rightarrow \frac{d\vec{F}}{d\cancel{V}} = \rho\vec{a}$$

For a static fluid net acceleration, $\vec{a} = 0$

$$\Rightarrow \frac{d\vec{F}}{d\cancel{V}} = 0$$

$$\Rightarrow (-\nabla p + \rho\vec{g}) = 0$$

$$-\nabla p \quad + \quad \rho\vec{g} \quad = 0$$

$$\begin{Bmatrix} \textit{net pressure force per} \\ \textit{unit volume at a point} \end{Bmatrix} + \begin{Bmatrix} \textit{body force per unit} \\ \textit{volume at a point} \end{Bmatrix} = 0$$

So, net force acting on this element is, one is the surface force, another is the body force. Now, surface force we have already seen it is $-\nabla p\ dx\ dy\ dz$ that we saw just now in the previous slide and $d\vec{F}_B$ we have already seen at the end of last lecture and today also we briefly introduced this $\rho\overrightarrow{g}\ dx\ dy\ dz$. So, we have here $d\vec{F} = d\vec{F}_s + d\vec{F}_B$. So, if you take dx dy dz outside you have $\left(-\nabla p + \rho\overrightarrow{g}\right)$. So, $d\vec{F}$ is equal to instead of dx dy dz you can write this as dV that is again V is struck through this dV is the differential volume.

So, dv can go to the denominator on the left hand side and you have $\frac{d\vec{F}}{dV} = \left(-\nabla p + \rho\overrightarrow{g}\right)$.

Now, what is $\frac{d\vec{F}}{dV}$? You can see here if we go by the Newton's second law $d\vec{F}$ the differential force acting on that differential volume would be acceleration into mass and mass is dm in this case and this dm can be written as ρ dV. dV is the differential volume unit is meter cube and ρ has a unit of density kg per meter cube. So, this product gives me dm unit is kg mass into acceleration.

So, that is $d\vec{F}$. So, then if that is so, then if this dV I take to the denominator here, $\frac{d\vec{F}}{dV} = \rho\vec{a}$, a is the acceleration as a vector. So, so $\frac{d\vec{F}}{dV} = \rho\vec{a}$. So, instead of this $\frac{d\vec{F}}{dV}$ we can write $\rho\vec{a}$. So, this is equal to $\rho\vec{a}$ and then we have some special cases one is when so, this is our master equation. A master equation in the sense it is from Newton's second law, I am balancing force into acceleration.

If someone wants to know how much fluid will accelerate, this is the master equation, so, to say. Of course, we are ignoring one layer sliding against the other there is a viscous effect that part is not considered here. So, you have this $\left(-\nabla p + \rho \vec{g}\right) = \rho\vec{a}$. Now, I have three cases I briefly mentioned at the end of last class. One case is fluid statics. Where the acceleration is 0, there is no acceleration, and fluid is held static.

So, in that case we will directly write this as 0. So, from here it will be $\left(-\nabla p + \rho \vec{g}\right) = 0$. So,

$$\begin{Bmatrix} \text{net pressure force per} \\ \text{unit volume at a point} \end{Bmatrix} + \begin{Bmatrix} \text{body force per unit} \\ \text{volume at a point} \end{Bmatrix} = 0$$

The other option is that you have an acceleration in the form of, let us say, fluid placed in a tank, and the tank is placed on a truck, and they all accelerate. So, that is also an acceleration fluid is having an acceleration, but in mass there is an acceleration.

So, let us say the acceleration of the truck is given by $a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}$. So, in that case, this acceleration vector would be then this acceleration. So this is one possibility, and the third possibility is that the fluid itself is moving in a pipe or in a channel. So, in that case, this acceleration we have this expression for this acceleration, but since we are having into this Eulerian or we are into this continuum approach, this acceleration, these are simple acceleration. That means, you can write a_x as $\frac{du}{dt}$, a_y as $\frac{dv}{dt}$ and all those.

So, these are just a time derivative of velocity, but when we are talking about a fluid flow, we will get into that in a moment there we have this acceleration, which is defined as the again it is a derivative of the velocity definitely, but we call it substantial derivative. It is not just a derivative in a context we have seen for particle acceleration. So, this is something that we need to keep in mind. We will have a detailed discussion on how we arrive at this substantial derivative, but this is essentially the master equation. This is essentially the master equation only this acceleration term changes from whether when you have fluid statics when the fluid is hauled by placing in a tank and tank is pulled by a truck. So, that acceleration has still we have that acceleration we can relate to particle acceleration, but if the fluid itself is accelerating what do you mean by fluid accelerating? Let us say I have a fluid flowing through a conical element, through a conduit where the cross-sectional area is, let us say, decreasing, and I am having some flow taking place.

So, the flow that is going in let us say Q at a particular time. So, that same flow has to go in same flow has to come out also as Q, but here the cross sectional area is more here the cross sectional area is less. So, here you have the velocity which is Q by A which is large and here we have a area which is small. So, Q by A large that will make this factor small whereas, Q remains same, but A decreases.

So, this makes this factor large. So, what that means is velocity is increasing as the fluid travels through this conical-shaped fluid channel. So, you can see that the velocity is increasing. So, this is also acceleration by every meaning of the term. So, this acceleration if you want to define you need to go to this substantial derivative why we will get to that in a moment. So, if you have fluid statics, that means if you have acceleration as 0, that means that is what we said here $\left(-\nabla p + \rho \vec{g}\right) = 0$.

$-\frac{dp}{dx} + \rho g_x = 0$ *x-direction*

$-\frac{dp}{dy} + \rho g_y = 0$ *y-direction*

$-\frac{dp}{dz} + \rho g_z = 0$ *z-direction*

Assuming gravity is working along z-direction,
$g_x = g_y = 0, g_z = -g$

$-\frac{dp}{dx} = 0$ *x-direction*

$-\frac{dp}{dy} = 0$ *y-direction*

$-\frac{dp}{dz} = \rho g$ *z-direction*

Assumptions:
- ✓ Static fluid
- ✓ The only body force is gravity
- ✓ The z axis is aligned opposite to gravity

$$\frac{dp}{dz} = -\rho g \equiv -\gamma$$

If we break that $-\nabla p$ now $-\nabla p$ is what? $-\nabla p = -\hat{\imath}\frac{\partial p}{\partial x} - \hat{\jmath}\frac{\partial p}{\partial y} - \hat{k}\,\frac{\partial p}{\partial z}$ and similarly g is also a vector you will have g_x g_y g_z and similarly if you had a term acceleration there also you have a_x, a_y, a_z the components. Now, when you equate them, of course, you will equate a component with a component. So, if you equate the components of x-direction and if you assume that a_x term is 0, fluid statics condition prevails. So, then you have a_x, a_y, a_z all 3 are 0.

So, this is the î. So, î is 0. So, î, î we are comparing. So, what we get is

$$-\frac{dp}{dx} + \rho g_x = 0 \;\; \text{x-direction}$$

$$-\frac{dp}{dy} + \rho g_y = 0 \;\; \text{y-direction}$$

$$-\frac{dp}{dz} + \rho g_z = 0 \;\; \text{z-direction}$$

Now, mind it when it comes to acceleration due to gravity, there is nothing no acceleration due to gravity in x and y direction, and this direction in z direction there is a gravity, but mind it z if you look at our earlier picture where we defined the differential volume z was acting upward, but acceleration due to gravity is acting downward. So, that is why we have $g_z = -g$ whereas, $g_x = g_y = 0$. So, what that means is

$-\frac{dp}{dx} = 0$ x-direction

$-\frac{dp}{dy} = 0$ y-direction

$-\frac{dp}{dz} = \rho g$ z-direction

So, the immediate implication is if you have a fluid element, you will see, I mean, because of gravity; I mean, when we are talking about fluid statics, that means fluid is at rest. So, in the x and y direction, if you go in the transverse direction, there is no change in pressure; pressure will only change in the z direction. So, that is one thing, and this is a classical so, the static fluid's only body force is gravity z-axis is aligned opposite to gravity.

So, all these assumptions are there. So, now, this exercise $-\frac{dp}{dz} = \rho g$, this is if you have this ρ as constant, if the density is constant then you can write this Δp that as hρg. So, this then you do the integration treating ρg as a constant. So, it is integration of dp and on this side it is integration of dz. So, that is how you get p= ρgΔz or p= hρg, which we have studied in the context of fluid statics, which we have for manometer, etc. We know that the pressure difference is given by between 2 points vertically separated that is given by hρg. But one major assumption there is ρ is constant, ρ is not varying with z, but if the fluid is compressible then the ρ varies with pressure.

So, if you are talking about variation in pressure then this ρ has to be put as a function of pressure. For example, simple case is if you have this ρ as let us say an ideal gas, and if it is an ideal gas this ρ has to be replaced by a term called pM/RT, this is a standard physical chemistry expression we have for density p is the pressure. So, p is the pressure, M is the molecular weight if you are talking about air, the average molecular weight of nitrogen and oxygen it will come 28 and 32 in appropriate ratios that is of course, gram per mole you have to convert that to kg per mole and then this R is universal gas constant 8.314. So, that is in joule per mole kelvin. So, that has to be taken into account, and p is the temperature on an absolute scale. So, if you have temperature is 25 degrees, so 273 plus 25 which gives me 298. So, that gives me the density. Now, if you have the density varying with p, you cannot take rho g outside the integration just now what I said before. So, then you have to $\frac{dp}{p}$ has to be considered and the rest is going there $-\frac{M}{RT}gdz$.

So, then you do the integration. M is the molecular weight of air. It does not change, g acceleration due to gravity, R universal gas constant, T is the temperature in absolute scale. So, dz here if you do the integration. So, now, you can work with this height difference, but then it will not be hρg here it will get ln of p. So, pressure will take a form of some exponential, if we are trying to find out p at a higher altitude based on the pressure at a lower altitude p would be p_0 into e to the power some expression will come. So, these are the subtle things you need to remember as we proceed.

Fluids in Rigid-Body Motion

According to Newton's second law, differential force acting on a fluid particle is

$$d\vec{F} = \vec{a}\, dm$$

$$\Rightarrow d\vec{F} = \vec{a}\, \rho\, d\forall$$

$$\Rightarrow \frac{d\vec{F}}{d\forall} = \rho\vec{a}$$

$$\Rightarrow (-\nabla p + \rho\vec{g}) = \rho\vec{a}$$

$$\underset{\left\{\begin{matrix}\text{net pressure force per}\\ \text{unit volume at a point}\end{matrix}\right\}}{-\nabla p} + \underset{\left\{\begin{matrix}\text{body force per unit}\\ \text{volume at a point}\end{matrix}\right\}}{\rho\vec{g}} = \underset{\left\{\begin{matrix}\text{mass per}\\ \text{unit volume}\end{matrix}\right\} \times \left\{\begin{matrix}\text{acceleration of}\\ \text{fluid particle}\end{matrix}\right\}}{\rho\vec{a}}$$

The component equations of vector equation

$$-\frac{dp}{dx} + \rho g_x = \rho a_x \quad x\text{-direction}$$

$$-\frac{dp}{dy} + \rho g_y = \rho a_y \quad y\text{-direction}$$

$$-\frac{dp}{dz} + \rho g_z = \rho a_z \quad z\text{-direction}$$

In the next case, I mentioned the three cases: fluid is in fluid statics is prevailing, and fluid is put on a truck, and truck is the truck is accelerating. So, in that case you have an acceleration term, but these acceleration is simple acceleration. So, in that case again

$$\left(-\nabla p + \rho\overrightarrow{g}\right) = \rho\vec{a}$$

So, last time we had these all these were 0s, but now we will have here this is equal to rho into a x this we will have

$$-\frac{dp}{dx} + \rho g_x = \rho a_x \quad \text{x-direction}$$

$$-\frac{dp}{dy} + \rho g_y = \rho a_y \quad \text{y-direction}$$

$$-\frac{dp}{dz} + \rho g_z = \rho a_z \quad \text{z-direction,}$$

because this acceleration itself is given by $a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}$.

So, î components will be matched with the î component, ĵ component would be matched with the $\hat{j}$ component, and $\hat{k}$ component would be matched with the $\hat{k}$ component. So, that is how we have these terms. So, now, if we have these then the question is what is the, I mean how do we how we use this information. One classical problem that can lead to this type of situation here is when you have a liquid placed in a tank, and the tank is being hauled at an acceleration let us say in this direction. So, it is it is being hauled a_x is existing a_y does not exist, this I am treating these as the x direction and the truck and the water is pulled in a tank and tank is hauled at with an acceleration a_x.

Shape of free surface of liquid in rigid body linear acceleration

$$\nabla p + \rho\vec{g} = \rho\vec{a}$$

$$-\left(\frac{\partial p}{\partial x}\hat{\imath} + \frac{\partial p}{\partial y}\hat{\jmath} + \frac{\partial p}{\partial z}\hat{k}\right) + \rho\left(g_x\hat{\imath} + g_y\hat{\jmath} + g_z\hat{k}\right) = \rho\left(a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}\right)$$

Since $p \neq f(z), \frac{\partial p}{\partial z} = 0,\ g_x = g_z = 0,\ g_y = -g$, and $a_y = a_z = 0$

$$-\frac{\partial p}{\partial x}\hat{\imath} - \frac{\partial p}{\partial y}\hat{\jmath} + \rho g_y\hat{\jmath} = \rho a_x\hat{\imath}$$

$$\left(-\frac{\partial p}{\partial x} - \rho a_x\right)\hat{\imath} + \left(-\frac{\partial p}{\partial y} - \rho g\right)\hat{\jmath} = 0$$

$$-\frac{\partial p}{\partial x} = \rho a_x, \qquad -\frac{\partial p}{\partial y} = \rho g$$

So, the liquid level when it was static let us say this was the liquid level, but then when it is hauled by this acceleration a_x you will find that this liquid level takes the shape of a shape like this which you you must have experienced I mean if you put some water in a glass and if you pull the glass suddenly you will find that the water level changes. So, so, if this is a classical example of liquid placed in a tank and tank is being hauled. So, this is this is not exactly a fluid flow through that conical section that that I that I mentioned earlier, but here also the acceleration is involved. So, suppose I want to know how far this liquid level will go up, where such type of things are important, there is there is something called, I mean if you if you go to a process industries you will find there are bucket conveyor system is there. That means, a fluid is placed inside buckets and then these buckets are placed on a, I mean sort of trolleys, they are placed on a moving platform and these platform would be moving either horizontally because it may be a sludge it may be something which you do not want to you do not want to use the pump to flow that through a pipe.

Instead, you prefer to have it put on these cans put on these buckets, and the bucket will be hauled by some roller mechanism. So, this may not have to be horizontal maybe it is going upward, these pulleys can be at an angle also. So, that is how and you can put the buckets there. So, one may like to know at what velocity I should be or what I have what acceleration I can give. So, that this this type of spilling of liquid does not happen or that can be limited.

So, these are the common calculations one needs to do in the process industry. So, the treatment that I am the general treatment that I am presenting here with this general equation from Newton's second law can be applicable there. I will continue this lecture in the in the next session. So, till that time, have a look into it, think about it where all these type of applications are important and we will get back to this to the theories of it in my next session. Thank you very much for your attention.