Momentum Transfer in Fluids

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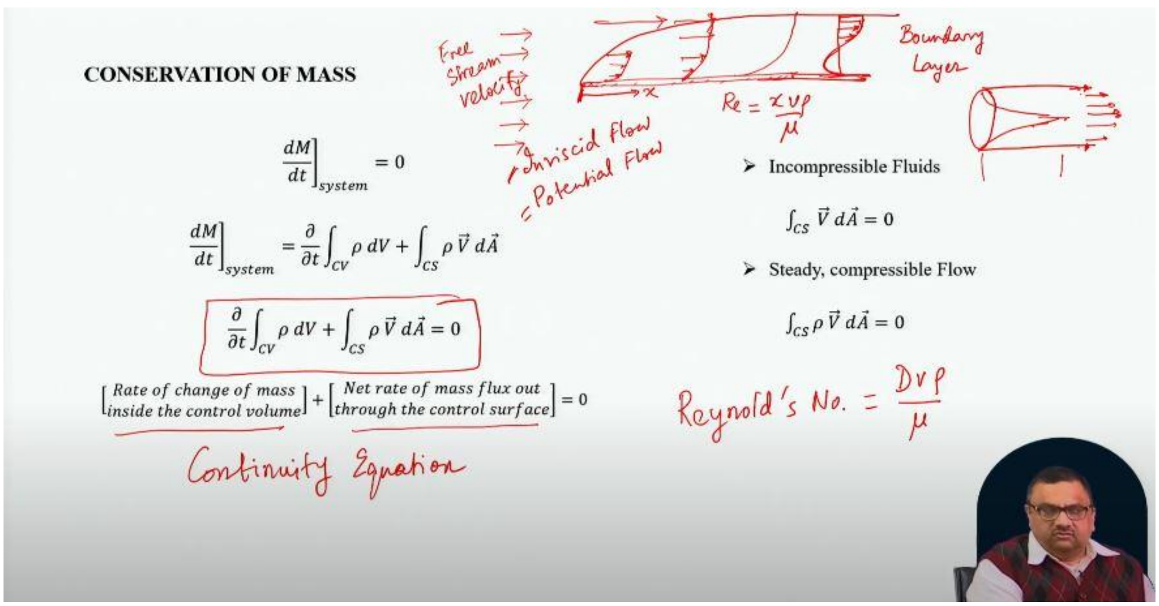
Week-03

Lecture-11

I welcome you to this course of Momentum Transfer in Fluids. We have already discussed several elementary aspects of these fluid characterizations, fluid properties, etc. Now, we are going to look into something called an inviscid flow. First of all, what is inviscid flow? We have defined viscosity. We talked about viscosity, and we have given you characteristics of different fluids that do not follow Newton's law, etc. Now, there is a class of fluid flow that goes by the name of inviscid flow.



Inviscid flow means, it does not mean that the viscosity is 0; it means that the flow is happening in mass. When one layer slides against the other, then the viscosity comes into play because then the shear stress arises, and then you have shear stress proportional to velocity gradient, and the proportionality constant is referred to as viscosity. Now, if the fluid is traveling in mass, that means there is no sliding of one layer. Rather, all the layers are moving at the same constant velocity. Before we get to this, let me tell you clearly how this inviscid flow or where this comes in.

Suppose, I have a flat plate. So, this is a flat plate, and I have a fluid coming in there. So, this is known as all the fluid layers they are moving in mass. You might have seen this type of flow in rivers and this type of flow in other places in large channels. So, all the fluids are not sliding against each other; they are moving in mass.

As it approaches this plate, now you have one condition that has to be satisfied, which means, at the wall, the velocity has to be 0, and away from the wall, the velocity will be increasing and far away from the wall; this is known as the free stream velocity. So, far away from the wall, the effect of the wall will not be there. So, far away from the wall probably this free stream velocity will be retained. So, at the wall, the velocity is 0; far away from the wall, the velocity is the same as the free stream velocity. So, in between there would be a transition.

So, I would expect that, let us say, at this location, the velocity itself is 0, then velocity increases a little bit, a little bit more, a little bit more, and then finally, the free stream velocity. So, you will see that the velocity increases from 0 velocity to the free stream velocity. So, typically, within this layer, the velocity builds up, the velocity layer sliding against the other and building up the velocity to free stream velocity. Typically, this thickness here would be less; here, it is the free stream velocity, and you will find that here, it is even more. This generally goes by the name boundary layer.

So, this goes by the name boundary layer. So, within the boundary layer, you will find that one fluid layer is sliding against the other whereas, outside, it is still free stream velocity is retained. So, the viscosity comes into play within this boundary layer; viscosity comes into play whereas, outside this, it would be just the free stream velocity, which means the fluid they are moving in mass. This type of flow, or what is happening outside this boundary layer, is commonly referred to as inviscid flow. In some cases, we refer to these as potential flow.

So, here, the pressure is important, but shear stress is not, because the moment you bring in shear stress, it ensures that one layer slides against the other, but the moment you bring a shear stress, then it is not an inviscid flow, if viscosity comes into play. So, within this boundary layer probably, the viscous flow governs, but outside the boundary layer, it would be inviscid flow. So, now, inside a tube, when you have a flow you would be expecting that this type of boundary layer will start forming from the wall, and here also it will start forming and at one point you will find that these two boundary layers they merge and at that point, entire channel one layer is sliding against the other that means, at the wall velocity is 0, here at the wall velocity is 0, and in between the velocity is sliding against the maximum velocity at the center and again it decreases to 0. So, we will have an entirely viscous flow inside the tube, but it takes some time. There is a boundary layer growth, and you need some entrance length for the boundary layer to fully develop and merge to the center line. Now, these dimensions are for small tubes; if you have a large tube possibly, you will have this boundary layer forming near the wall, whereas, away from the wall, you still have the inviscid flow.

So, inviscid flow you can think of it is more like a plug flow, the entire fluid is flowing like a plug that is one thing, and since we talked about this boundary layer, I must tell you that as the boundary layer continues to grow at one point you will find that this growth of the velocity profile, the boundary layer becomes too big and there is a reversal of flow that happens. So, let us say this is the velocity. Here, it is growing to the free stream velocity, but here, there is a reversal of flow. So, that causes some amount of flow reversal, and we say that it is no longer a laminar flow because laminar flow requires the fluids to move one layer sliding against the other, whereas, if there is a flow reversal. So, it will not be strictly laminar. So, then that is going to happen here in this case.

So, generally, this is laminar to turbulent. We will discuss this laminar to turbulent and their transition, etc. Typically, there is a dimensionless number that goes by the name Reynolds number, which is for a tube or for a pipe. It is given as $D v \rho / \mu$, D is the diameter of that tube, v is the velocity, and ρ is the density divided by μ is the viscosity. This is a dimensionless number, which means all the dimensions have canceled out, and this Reynolds number defines whether the flow is laminar or turbulent. So, we can expect that when the velocity increases, for example, for a flow through a tube, flow through a pipe. So, when the velocity increases, one layer sliding against the other does not continue to happen. Rather, there would be eddies formed, and eddies will have random motion, and we call that the turbulent flow.

Whereas, if you have a flow over a flat plate, typically, the Reynolds number is given as $x v \rho / \mu$, where x is the distance from the tip of the plate. So, this is x. So, x is counted from the tip of the plate distance. So, that x, when x exceeds some threshold value or v increases for some value of x, this transition from laminar to turbulent takes place. So, further downstream you see the transition taking place because this x is increasing here.

For that matter, if you increase the characteristic dimension D then also you can have turbulence because it is you are expecting one layer is slide against the other, but if this growth of this boundary layer is too large or if the tube dimension is too large you expect that this process of one layer sliding against the other and shear force dominates this effect that is not significant compared to the other effects that come in inertia coming in. So, because of increase in diameter, the inertia force dominates over the viscous force and there are other implications which are leading to some amount of instability setting in and that is no longer called laminar. So, even by change of D, change of ρ, or change of μ, that also can lead to a transition from laminar to turbulent. So, my original point was that inviscid flow. So, when it comes to inviscid flow, we are talking about a sort of plug flow that is happening outside the boundary layer, and that is a case where the channel size is large when you do not have a very small channel where one layer is sliding against the other, and the viscous flow dominates.

Before we proceed on this further, we will be focusing here on inviscid flow in this lecture, and before we proceed this further, we must understand there is some conservation rule that any fluid flow has to obey, and that is also referred to as continuity equation. So, the continuity equation means that the fluid that is going in, let us say, I pick up a differential volume, the fluid that is going in, and the fluid that is leaving the place. So, if I subtract in minus out, if the fluid is incompressible then in has to be equal to out, we cannot afford to have maybe the fluid that is moving in x direction the exactly same amount is not coming out from that other side of the face maybe some are coming out extra is coming out to the y or z direction that is different, but whatever is going in from all x, y, and z faces has to come out from the other side of x, y, and z faces. So, that is there and if the fluid is compressible that means, the density can change, if the density is not constant then in minus out would be accumulation and accumulation would be reflected in terms of the change of density of the fluid because the fluid will be denser because in is more out is less, but accumulation in terms of fluid density increases. So, that is a primary condition, any fluid flow has to satisfy, and this is referred to as continuity equation.

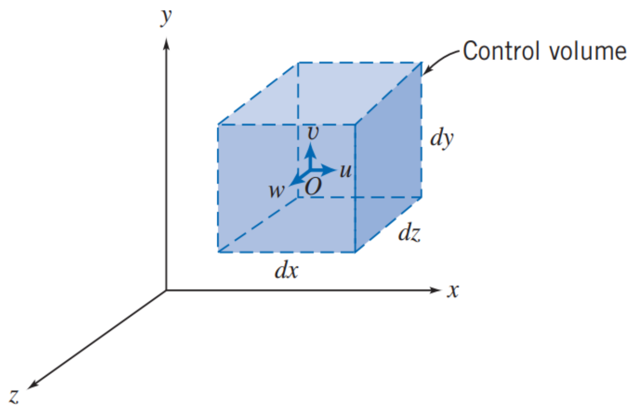
So, the fluid that enters or leaves through the surface. So, through the control surface,

$$\int_{CS} \vec{V}\, d\vec{A} = 0$$

When you have a steady compressible flow, when there is a density present there, then one has to keep track of the density as well. When it is an unsteady flow one has to look into this equation.

$$\frac{\partial}{\partial t}\int_{CV} \rho\, dV + \int_{CS} \rho\, \vec{V}\, d\vec{A} = 0$$

That means, here, it is the net mass flow that has taken place into that differential element, in minus out, in is positive out is negative net over the control surface, and net accumulation that has taken place in terms of the buildup of density.



So, that is given here. So, the rate of change of mass inside the control volume and net rate of mass flux out through that has to be equal to 0. What is the implication of it? If we take a differential element here and if we try to write density and velocity. So, this coordinate that the center of this differential element is at a location x, y, z, those are the coordinates, and the dimensions of this differential volume are dx, dy, and dz. So, this is the control volume.

So, now, if we try to find out what is the density at the left face that means the face which is here. The density of the left face. So, that would be

$$\rho]_{x-dx/2}$$

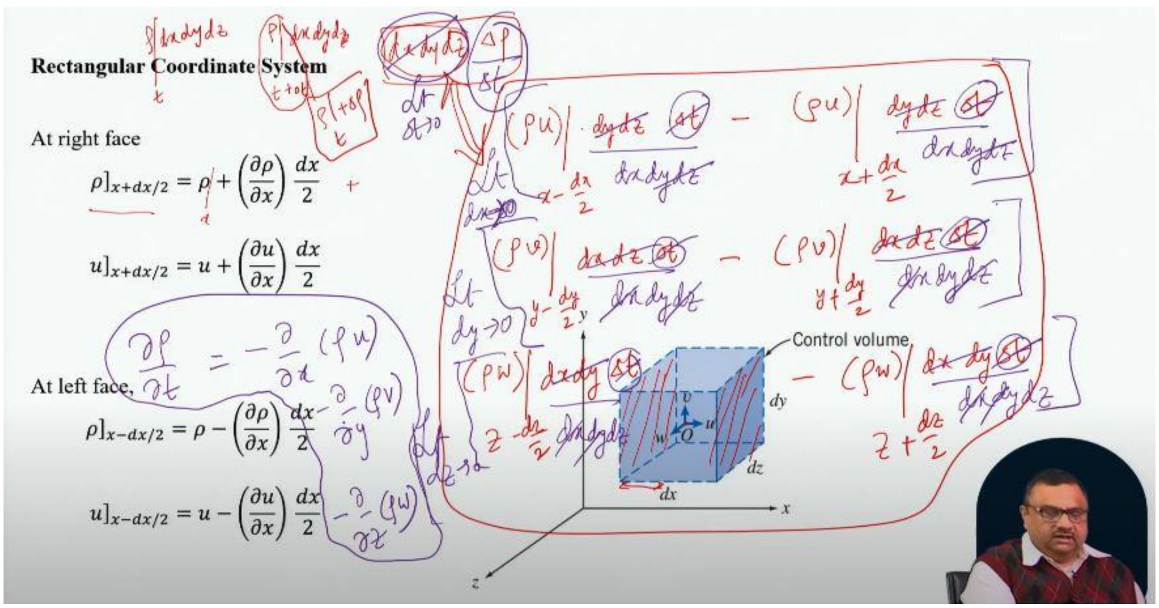
When it comes to the right face, it would be

$$\rho]_{x+dx/2}$$

So, now, if we take Taylor series expansion of ρ and we ignore higher-order terms. So, what that means is that I am assuming that the ρ is varying from the left face to the right face, the ρ is varying linearly, there is higher order terms are neglected. When can we make such an assumption? When this differential element is very small, so that we can assume that within this maybe ρ has a parabolic nature or second order equation, some polynomial expression will fit, but since the dimension is small I can linearize it and get away with it. So, that is why I am ignoring the higher order terms and ρ is written as ρ at x, plus $\left(\frac{\partial\rho}{\partial x}\right)\ \frac{dx}{2}$. The next term would be dx/2 whole square by 2 factorial $\left(\frac{\partial 2\rho}{\partial x2}\right)$ etc.

So, those are ignored. Similarly, the velocity at the left face it is $u]_{x-dx/2} = u]_x - \left(\frac{\partial u}{\partial x}\right)\ \frac{dx}{2}$, which is again by Taylor series expansion it takes only take the first order term. That means, again, I am assuming that u is changing linearly from left face to right face since the differential size of this differential element is small, dx is small, and so u at two different faces here. So, as far as the x direction is concerned what is in? In is $\rho u]_{x-dx/2}$ into the area, and what is the area? Area here is this side is dy, and this side is dz. So, this into dy.dz that he gives me the net mass flow that is happening from the left face, and if I multiply that by Δt, this much of mass entering from the left face and how much is leaving the right face that would be $\rho u]_{x+dx/2}$ dy.dz.Δt. So, that would be the mass, see u is the velocity, u unit is m/s that into dy.dz, dy.dz are length units, meter × meter, meter^2.

So, m/s × m × m, so that gives me m^3/s. So, essentially udydz gives me the m^3/s, volumetric flow rate that multiplied by ρ, ρ has a unit of kg/m^3. So, kg/m^3 × m^3/s, so that gives me kg/s. So, kg/s this is the mass flow rate as far as the right face is concerned that multiplied by Δt in second. So, that gives me over duration Δt, so many kg of mass has flowed out from the right face and this gives me so much of k so much of kg of mass that has entered from the left face.



So, left minus right, so entered minus left, so this gives me an idea how much fluid in-out, as far as the x face as far as the area is concerned whose perpendicular is in x direction. The same thing will happen with the y direction, and the same thing will happen with the z direction. So, it would be again (ρv), and small v is in y-direction.

$$(\rho \mathrm{v})\Big|_{y-\frac{dy}{2}} \mathrm{dxdz}\Delta \mathrm{t} - (\rho \mathrm{v})\Big|_{y+\frac{dy}{2}} \mathrm{dxdz}\Delta \mathrm{t}$$

So this would be in the y direction. Similarly, in the z direction, you will have

$$(\rho w)\Big|_{z-\frac{dz}{2}} dxdy\Delta t - (\rho w)\Big|_{z+\frac{dz}{2}} dxdy\Delta t$$

So, over duration Δt this is in minus out as far as this box is concerned x direction y direction z direction. This in minus out has to be equal to accumulation, and what would be the accumulation? What is the volume of this block that is dx.dy.dz, that is the volume of this block? And let us say over this duration, Δt, there is a change in density, which is given by $\Delta\rho$. So, that is, initially, the density was ρdxdydz, dx.dy.dz is the volume that is d with v struck through. So, dx.dy.dz is the volume of this differential element.

So, at time t, the density into volume that gives me kg right density is in kg/m^3, and this dx dy dz is m^3. So, this is kg. So, ρdxdydz gives me so, many of kg at time t present in the box, and at time Δt it would be ρ at t+ Δt, dx.dy.dz. So, let us say I write this ρ at t+ Δt as ρ at t plus $\Delta\rho$.

Let us say I write this change over duration Δt as change in density as $\Delta\rho$. So, what is the net change in mass over duration Δt, that would be the net $\Delta\rho$dxdydz, $\Delta\rho$ has a kg/m^3 unit and this is m^3. So, so, many kg has been has accumulated over duration Δt and that is reflected by the change of density because the volume Δ is Δy.Δz, that is not going to change it is not a deformable differential element. So, this is a rigid differential element.

So, this is the change. So, then this in minus out that I have here in x, y, and z direction has to be equated to this, and if you bring in this dx dy dz when you bring into this side. So, what I do here is I bring in this dx dy dz part and this dx dy dz part I bring into the denominator. So, this becomes dx dy dz this is gone here. So, this is dx dy dz, and further this Δt, I remove from here and it goes there in the denominator. So, this delta t this is going out this Δt is going they have all gone to the denominator on the left hand side.

So, this Δt is gone and this Δt is gone. So, what I see here is that the dy dz goes out with dy dz, this dy dz dx dz goes out with dx dz, and dx dy goes out with dx dy. So, you are left with on this side ρu at x minus dx by 2 and ρx plus dx by 2, this is divided by dx and this is divided by dx is in the denominator. So, if you put limit dx tending to 0 you, if you put limit dx tending to 0 we could have we could have used this as Δx then we could have written it as delta x tending to 0. So, limit delta x tending to 0 within this block, and here we put limit dy tending to 0, and here we put limit dz tending to 0, and this whole thing this whole thing and then this whole thing.

So, if we if put this we will end up with this term here as -$\frac{\partial}{\partial x}\rho$u, this term the second rho that gives me $-\frac{\partial}{\partial y}\rho$v and this gives me third term is $-\frac{\partial}{\partial z}\rho$w and here in this if we add limit delta t tending to 0 this gives me $\Delta\rho/\Delta t$. So, this whole thing this right hand side is equal to $\Delta\rho/\Delta t$. So, this is a primary condition simply arising from conservation equation.

So this is referred to as the continuity equation, and if I come, I give you a field variable, the velocity with their u, v, and w, some expression, and if those do not satisfy this condition, then the fluid flow is not possible. So, this is a primary condition, also referred to as the continuity equation, which arises from the conservation mass conservation equation.

So, it is in a simplified form. This $\vec{V} \cdot d\vec{A}$ is written as.

$$\int_{CS} \rho\vec{V} \cdot d\vec{A} = \left[\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z}\right] dx\, dy\, dz$$

I mean it in a compact form. This is how you will write in an expanded form I have already shown here. So, you can see here that if we take all these right-hand sides to the left. So, all these minus would be equal to plus, and this is equal to 0, and that is that is exactly what we see here:

$$\frac{\partial}{\partial t}\int_{CV} \rho\, dV = \frac{\partial \rho}{\partial t} dx\, dy\, dz$$

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

So this is referred to as the continuity equation. So, I give you a velocity field. Let us say some u i hat plus v j hat plus w k hat, mind. It is a velocity field, not the velocity of a particle. The velocity of a particle does not have any such restriction, but when it comes to the Eulerian framework and continuum assumption as valid, then this u v w some relation between u v w has to be considered, that is that has to have to be followed has to be obeyed that is mass conservation and that that condition is this. So, u v w their relation is that has to satisfy this equation. Any arbitrary choice of u v w will not get you that may be a vector, but that may not satisfy the condition of a field variable called velocity in the Eulerian framework in with the continuum hypothesis. So, I will proceed on this further, and I will work on the extension of this.

Now, this equation has to be valid to at the very outset I mean inviscid or non-inviscid this is this is a very basic framework one has to satisfy. I will proceed further on inviscid flow, but before we proceed on inviscid flow this condition was important. This condition is a primary condition that has to be satisfied for any fluid flow irrespective of what you have inviscid or viscous. So, that that I must point out. So, I will continue this discussion on inviscid flow. That is all as far as this lecture module is concerned. Thank you very much for your attention.