Chemical Engineering Fluid Dynamics and Heat Transfer
Prof. Rabibrata Mukherjee
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 08
Conservation Equation 03 - Conservation of Momentum

So, welcome back to the 8th lecture. In this lecture, we are going to continue with the Conservation Equations. In the last two lectures, we understood the conservation of mass, continuity equation. And, we have obtained expression for continuity equation in the Cartesian coordinate system for an incompressible fluid based on the shell balance around a cuboidal control volume. But, similar to what we did for conservation of mass we will try to do it through the based on the momentum that is entering and leaving through different faces of the control volume.

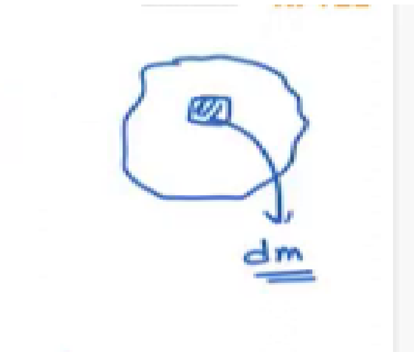
Conservation of momentum:

It's actually a Direct consequence of Newton's second law.

$\vec{F} = \frac{d\vec{P}}{dt}|_{\text{system}}$ $\vec{F}$:Force applied, $\vec{P}$:Linear Momentum

$\vec{P}_{system} = \int \vec{V}\, dm$ $\vec{V}$:Velocity

For a system with infinitesimal mass dm, the newton's second law can be written as



$$d\vec{F} = dm.\frac{d\vec{V}}{dt}|_{\text{system}}$$

$\frac{d\vec{V}}{dt}|_{\text{system}}$→Acceleration for a rigid object

Therefore, for a flowing system $d\vec{F} = dm.\frac{D\vec{V}}{Dt}$

For a rigid object the term $\frac{d\vec{V}}{dt}\Big|_{\text{system}}$ represents the temporal acceleration. Substantial derivative or the material derivative which represents acceleration for a flowing system, that essentially combines the concepts of both temporal acceleration as well as spatial or convective acceleration.

If we are writing the x component balance for a flowing system,

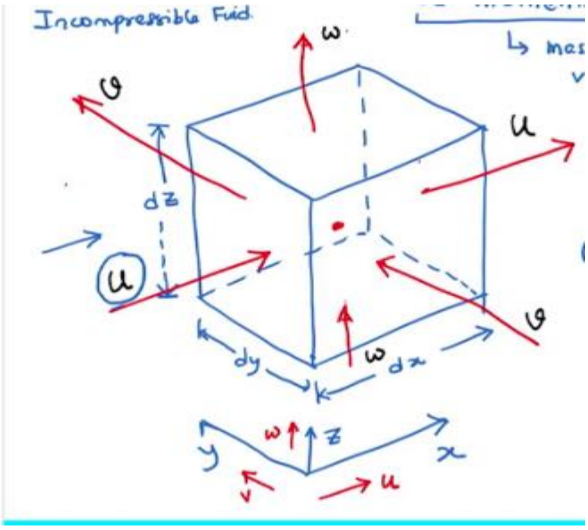
$$dm\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{du}{dz}\right] = dF_x = dF_{Bx} + dF_{Sx}$$

dF_x is the net force acting in the x direction, generally there are two types of forces: body force and surface force

F_{Bx}: x component of Body force and F_{Sx} : x component of Surface force

Let's check what type of forces are body forces and what type of forces are surface forces. Body forces include gravity which will acts over the entire mass and another example of body force are the electromagnetic force. So, Body force are the forces that act over the entire mass or entire body or object and forces that act on the surface are essentially the surface forces. Stresses are the best example for the surface force. And other examples of surface forces are the pressure, surface tension and relaxed forces. You must have noted, in the classical fluid dynamics, we would not have detailed discussion about surface tension. But, if we are considering the smaller length scale like microfluidics or nano fluidics, the surface to volume ratio increases and there will be significant importance for surface than the bulk.

Consider a control volume of centroid (x,y,z) with the edges dx, dy and dz and with the flow of an incompressible fluid,



As we know momentum for a rigid object will be the product of mass and velocity, for a flowing system mass will be constant and there will be velocity in all three directions.

If we are looking into the x component momentum balance:

that x momentum enters the control volume through three different faces. So, the x momentum enters to the left face due to x component velocity, and also x momentum enters through the front face due to the y component velocity and x momentum also enters through the bottom face due to the z component velocity.

x momentum that is entering to the control volume means the mass that is flowing due to the x component velocity.

mass that is flowing due to the x component velocity=$(\rho.u).dy.dz = m_x$

Mass flow due to x component velocity entering through the left face of the control volume $= \left[u.(\rho u)|_{x-\frac{dx}{2}}\right].dy.dz$

x momentum getting pushed into the control volume due to y component velocity through the front face $= \left[v.(\rho u)|_{y-\frac{dy}{2}}\right].dx.dz$

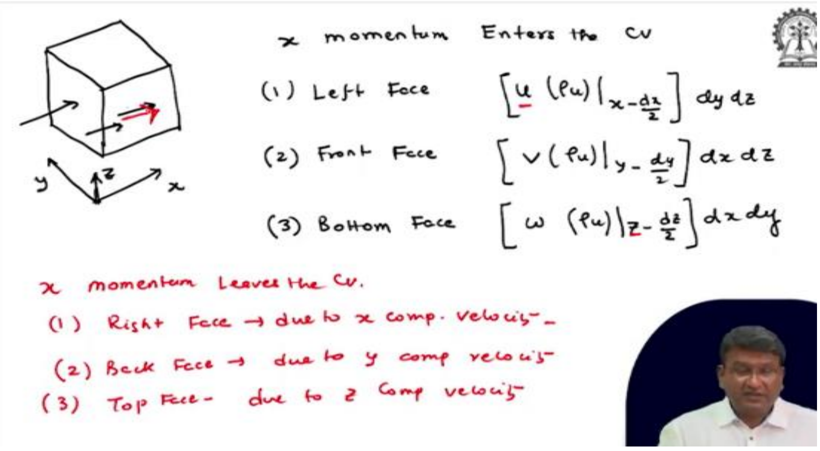
x momentum getting pushed into the control volume due to z component velocity through the bottom face $=\left[w.(\rho u)|_{z-\frac{dz}{2}}\right].dx.dy$

Similarly, x momentum leaving the control volume through

Right face: due to the x component velocity $\left[u.(\rho u)|_{x+\frac{dx}{2}}\right].dy.dz$

Back face: due to the y component velocity $\left[v.(\rho u)|_{y+\frac{dy}{2}}\right].dx.dz$

Top Face: due to the z component velocity $\left[w.(\rho u)|_{z+\frac{dz}{2}}\right].dx.dy$



.

So, we can conclude that the x momentum will enter the control volume through all the three faces. One face due to that x component of the velocity and the two other faces are due to the y and z components of velocity.

Thank you.