Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 06
Kinematics 06 and Conservation Equation 01

In the last two classes, we have discussed about the deformation of a fluid particle that can undergo depending on what type of flow field it is inside or depending on the nature of the spatial dependence of the flow field itself. And, we have discussed about the rate of linear deformation and rate of angular deformation, rotation of a fluid element. And we also talked about how angular deformation causes shear stresses or how shear stresses are related to the velocity gradients through angular deformation for small deformations. And also, we understood the concept of rotation, vorticity and irrotational fluid.

In this lecture, we are going to talk about the concepts of mechanism or modes of representing a fluid flow and the conservation of mass in a control volume, concept of stream function. So, we are going to talk about basically flow lines.

Different types of Flow lines: streamlines/path lines/streak lines

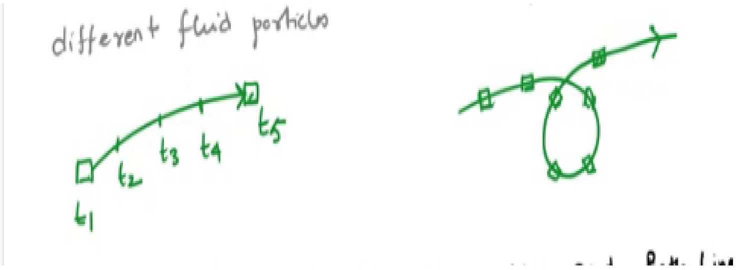
1.Streamlines: It's a Eulerian approach for describing flow velocity for geometrical representation. For a particular instance of time, a space curve is drawn, so that the tangent to that curve everywhere gives the velocity vector. streamline is a line or the tangent to which at any point at any instance of time essentially gives the direction to the velocity vector.

Equation of streamline is given as $\vec{V} \times \overrightarrow{dS} = 0$

In scalar form we can written as $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

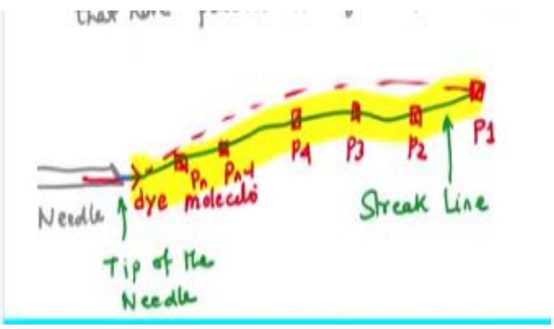
Two streamlines cannot intersect. Because at any point over the streamline the tangent gives the direction of the velocity and if two streamlines can intersect, you can essentially draw two tangents at that point corresponding to the two lines and you will get two directions of velocity which is not possible.

2.Path Line: It is a Lagrangian approach, and it describes the fluid flow by tracking the path of different fluid particles. In the case of a path line, you are tracking the motion of a particle with time.



In simple way, we can say that if we are tracking the motion of fluid particle at different instant of time like t_1, t_2, t_3, t_4 and if we are joining the position of fluid particle, we will get a hypothetical line, which is the path line. A path line can intersect. A path line depending on the circulation pattern can intersect itself. So, within a flow field you get streamlines at any instance of time and path lines are for individual particles. It turns out that for the steady flow, streamline and path line become identical.

3.Streak Lines: It is the locus of the temporary location or position of all particles that have passed through a fixed point in the flow field. It is based on both Lagrangian concept and Eulerian concept. For example, consider a flow field in which there is needle injecting few dye molecules to the flow field. After some time, if we are joining the present position of the different dye particle, you will get the streak line at that time. Because all the dye particles passed through the tip of the needle. And we can say that with time, the streak line changes. The best example for streak line in practical field is the intravenous injection. It's a combination of Lagrangian and Eulerain concept.



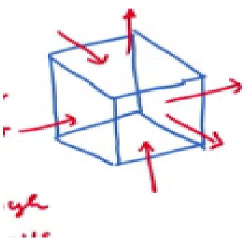
Next, we are going to discuss about the Conservation Equation.

Conservation of Mass/Continuity equation in the cartesian coordinate system

Consider a control volume in cartesian coordinate system,

Control volume is a Eulerian concept. It's a hypothetical space or volume in the flow field, which is fixed, and you can track the flow through its hypothetical walls. Here we have drawn the control volume as a cuboid and there is no necessity that a control volume must be cubical or cuboidal.

Consider that the flow is entering through the left and bottom face and the flow is leaving through the right and bottom face. So, if there is an unsteady flow that the flow rates are going to change with time. And accumulation is essentially the change of mass within the control volume with time. And even if there is accumulation or any types of chemical reaction, the total mass will be always conserved.

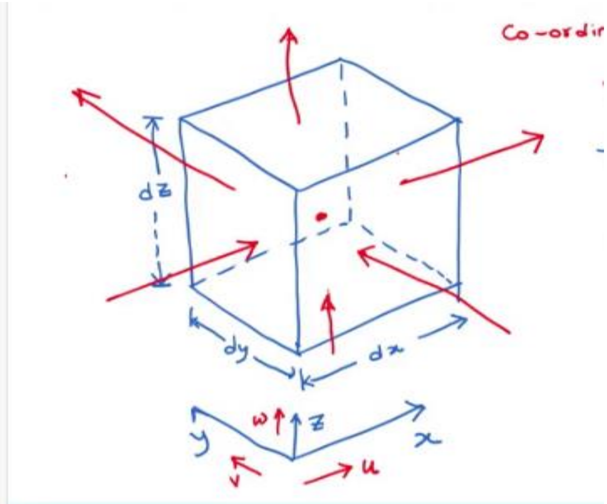


Conservation of mass:

$Mass_{in}$-$Mass_{out}$ + Generation = Accumulation

the accumulation term will be zero for the steady state flow process.

Consider a control volume with the sides are dx, dy, and dz and the centroid with coordinate (xyz)



Here we are considering, the x coordinate of any point on the left face will be $x - \frac{dx}{2}$ and similarly the x coordinate on any point on the right face will be $x + \frac{dx}{2}$ .Similarly z coordinate of any point on the bottom surface will be $z - \frac{dz}{2}$ and the z coordinate of any point on the top surface will be $z + \frac{dz}{2}$.

If we look into the control volume ,we will understand that mass entering through the left face is due to the x component velocity ,Mass entering through the front face is due to the y component velocity and mass entering through the bottom phase due to the z component velocity, similarly we also have mass leaving through right face due to x component velocity and mass leaving through the back face due to y component velocity and mass leaving through the top face right due to z component velocity.