Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 55
Internal Forced Convection (Contd.)

Hello everyone. Welcome back with the another lecture on Forced Convection in Chemical Engineering Fluid Dynamics and Heat Transfer.

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In the last couple of classes, we are discussing about Internal Forced Convection, a few correlations based on the flow regime we have seen. In the last class we have particularly seen the turbulent flow, Dittus Boelter equation and the beauty of the Dittus Boelter equation or the Colburn equation is that both are fair enough for different types of thermal boundary condition.

That means the results are not much sensitive, whatever the thermal boundary condition would be like is it for turbulent flow particularly that whether it is constant surface temperature or constant heat flux. In both the scenarios those are used and gives us reasonably accurate solution. Now, if we try to see the applications of such of those equations say in one example, we have water that we want to heat.

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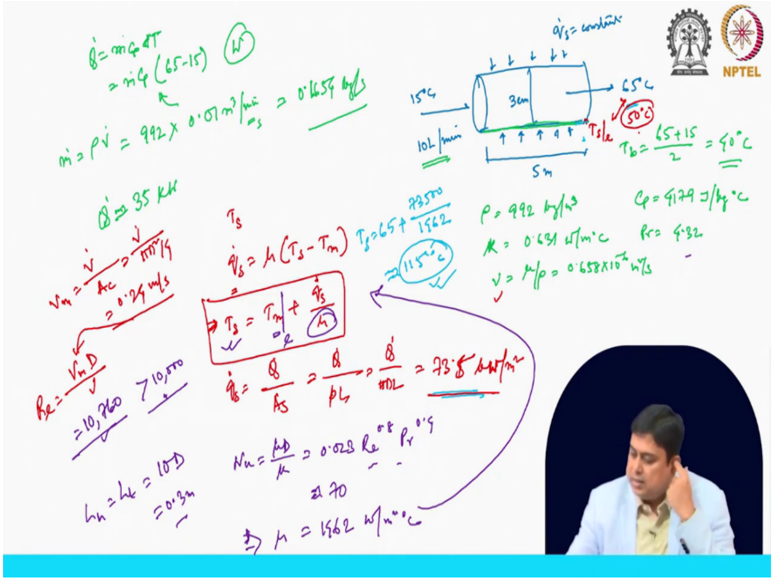

And that is why it is flown inside a pipe, the water is at 15 °C, the length is 5 m long and it is going out at 65 °C. So, 15 °C to 65 °C, we are heating water from 15 to 65 °C. The diameter of this pipe is 3 cm.

The tube is 5 m long and it is wrapped with an electric heater. So, that we can have constant surface heat flux condition which means $\dot{q}_s = constant$. There is no other kind of heat transfer that is happening or; that means, there is no heat loss, it is well in insulated. The system in this case the flow of water is say 10 L/min.

So, how much power of this electric heater we need to generate in order to attain such condition this is the unknown. So, what should be the heater rating power rating? We have to heat water from 15 °C to 65 °C in a 5 m long tube, the tube has a diameter of 3 cm, the flow of water is 10 L/min. So, the question is how much power rating should be of the heater that keeps a constant heat flux condition.

So, in this case again since we know the inlet and outlet temperature, the bulk fluid temperature is:

$$T_b = \frac{65 + 15}{2} = 40°\text{C}$$

So, we look at the tables of water properties for the water density, viscosity etc at 40 °C.

$$\rho = 992\ kg/m^3$$

$$k = 0.631\ W/m°\text{C}$$

$$\vartheta = \frac{\mu}{\rho} = 0.658 \times 10^{-6} m^2/s$$

$$c_p = 4179 \frac{J}{kg} °\text{C}\ \&\ Pr = 4.32$$

All the values are given all the necessary values are given.

So, now if we look into this problem again, we have to find out what is the current or the present Reynolds number and the amount of heat transfer that is needed or that needs to be transferred to the water in order to attain or increase its temperature from 15 to 65 °C.

In order to do that what we have to do the estimation is:

$$\dot{Q} = \dot{m} c_p \Delta T$$

$$\dot{Q} = 992 \times 0.01 m^3/min \times (65 - 15)\ (\dot{m} = \rho \dot{v})$$

$$\dot{Q} = 35\ kW$$

This much Watt we needed this is the simplest part that this much wattage of power we need constantly in order to attain this result. That means the energy that must come from this resistance heater, which would be around 35 kW numerically close to.

Now, the point is the other question that we have the first question was how much energy that is needed to do this or the rating of the heater? And the next question is that we also have to estimate the surface temperature of the pipe at the exit?

Because it is a constant heat flux condition so, the surface temperature of the tube would change along the flow directions and the question is what is the surface temperature T_s at the exit. Or specifically the inner surface temperature of the pipe at the exit where the fluid temperature is 65 °C.s

So, the surface temperature T_s at any location that we can determine that is from its local heat transfer rate, which is say if i have the constant value:

$$\dot{q}_s = h(T_s - T_m)$$

$$\rightarrow \; T_s = T_m + \frac{\dot{q}_s}{h}$$

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{\dot{Q}}{pL} = \frac{\dot{Q}}{\pi DL}$$

Once we replace those values and $\dot{Q} = 35kW$; what we get it around 73.4 or 73.5 kW/m^2 the surface heat flux, which is constant. So, this is the constant value that has to be imparted at any location. Now, the point is that we did not realize till now that, what is the flow condition? That is it laminar or is it turbulent?

So, to do that we have to calculate Reynolds number,

$$Re = \frac{V_m D}{\vartheta}$$

$$V_m = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4}$$

After putting all the given data in the above equation once we calculate we see that this V_{mean} is around 0.24 m/sec.

So, what we find after replacing all these values the Reynolds number is in the range of this value 10,760, which is greater than 10,000; that means, the flow is fully turbulent. If the flow is fully turbulent then the entry length that is necessary in both cases are similar hydrodynamic as well as the turbulent that we consider as 10D for turbulent flow which comes out to be 0.3 m. The total length is 5 m, this entry length is 0.3 meter.

So, what we can logically assume that the flow is fully turbulent over the entire length there is a negligible influence of the entry length. If we consider that then what we can do we can use:

$$Nu = \frac{hD}{k} = 0.023Re^{0.8}Pr^{0.4}$$

Putting all the given values in above equation we get (Nu = 70) ;once we find it out we have to get this h to calculate the surface temperature. For h we are looking into all these steps and from Nusselt number to calculate h we need to calculate Nusselt number. So, from this Nusselt number value D is known, k is known. So, we find the value of h which is around 1462 W/m^2°C, h value is known; we use this in this expression to find out the value of T_s at exit.

Now, T_{mean} at exit is 65 °C at steady state. Therefore, the final expression would be:

$$T_s = T_m + \frac{\dot{q}_s}{h} = 65 + \frac{73500}{1462} = 115\ ℃$$

. So, here we see that at the exit surface temperature is 115 °C which is 50 °C higher than the fluid temperature or the water temperature. This difference is 50 °C.

Now, this temperature change would not further be different in the complete fully developed region that we have to understand. Once it reaches the fully developed condition this temperature difference between the surface and the mean fluid would not change along the length of the pipe or in the direction of the flow. Provided you reached or the flow has reached the fully developed condition. That means, thermally fully developed and hydrodynamically fully developed.

After that ΔT the mean fluid temperature is changing, surface temperature is also changing. But the ΔT of these two changes that means, the mean and the surface temperature at any particular location after the fully developed region is constant. I hope this solution is clear to you. The next problem we will see not now in the circular zone or circular tube.

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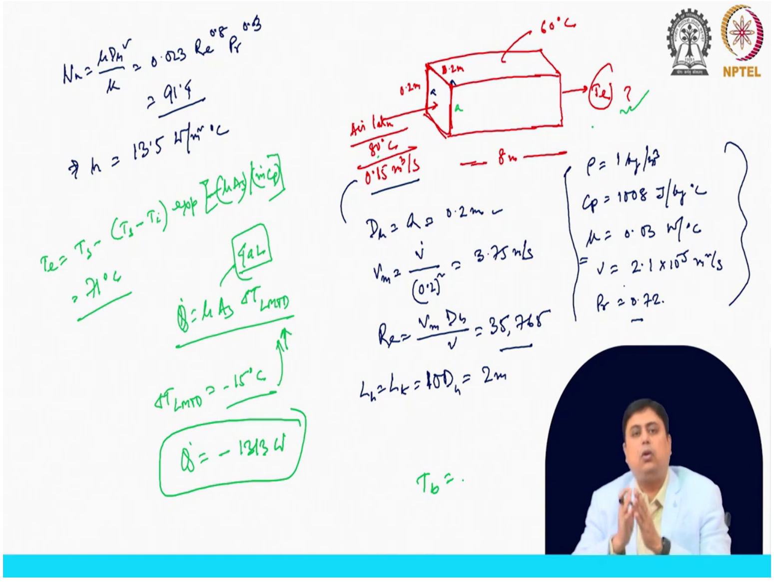

There is a heat loss happening from a duct that is say we have a rectangular or a duct of 8 m long a square duct consider in this case, which is of (0.2 m x 0.2 m) cross section. Air is going inside this duct at 1 atmosphere and 80 °C. The surface temperature of this duct is 60 °C. The temperature at which the air exit is T_e so, hot air at atmospheric pressure at 80 °C enters a 8 m long duct uninsulated square duct of cross section (0.2 m x 0.2 m).

And the flow rate is 0.15 m^3/s. Now, this duct apparently is maintained I mean it has been seen that once this condition is there at steady state the surface temperature is at 60 °C is isothermal, which means there is a heat loss from the air to the surroundings and at steady state this temperature has attained at 60 °C.

So, you have to calculate what is the value of T_e the exit temperature of air and the rate of heat loss from the duct to the surrounding? These are the two things we have to calculate. So, we consider assumption is that we air is an ideal gas, which is not true, but we assume that this is an ideal gas because then it helps in the calculations and the surfaces of this duct are smooth we are not considering any rough wall because then the correlation should change.

So, now we do not know again in this case similar to the previous example in case of laminar flow where, we saw that since we do not know the exit temperature, we cannot calculate the bulk fluid temperature or the bulk air temperature mean temperature. So, that means, temperature at the inlet at 80 °C this we consider that it would drop somewhere in between 80 and 60 °C it cannot go below 60 °C. So, again in that range the fluid temperature of the air properties are not changing much.

So, we consider the all the air properties at 80 °C and one atmosphere. And accordingly, we look at the values or we take the values from the text book or from the table that row in that case is:

$\rho = 1\ kg/m^3$,$c_p = 1008\, J/kg$°C, $k = 0.03\ W/m$°C, $\vartheta = 2.1 \times 10^{-5} m^2/sec$, Pr = 0.72

Above values are given or you have extracted based on the table that is provided at the appendix of any text book or reference book related to this subject. So, now here we have to calculate the Reynolds number because we have to identify or we have to classify whether this problem is laminar or turbulent.

In order to do this here since this is a non circular cross section, we have to calculate hydraulic diameter for the calculation of Reynolds number. Hydraulic diameter in this case is weighted cross section by weighted perimeter.

$$D_h = a = 0.2\ m$$

$$\dot{V_m} = \frac{\dot{V}}{0.2^2} = 3.75 m/s$$

$$Re = \frac{V_m D_h}{\vartheta} = 35765$$

we find out that comes in the range of 35000, which is way greater than 10000 which means it is completely turbulent and the entry length again in this case is 10D or 10D_h rather here, which is 2 m and the length is 8 m.

So, again we consider here the flow is fully developed turbulent flow . The fully developed turbulent flow so, this is hydrodynamic as well as the thermal. So, fully developed turbulent flow and accordingly now here we use the Dittus-Boelter equation, but here this air is cooled air is dissipating its energy to the surrounding. So, my Nusselt number relation:

$$Nu = 0.023 Re^{0.8} Pr^{0.3}$$

We replace this values Reynolds number, Prandtl number the value that we get is 91.4. Once we have this, we can calculate the value of h because D_h is known, k value is given; the k value is known here we replace this and find out that $h = 13.5\ W/m^2$°C. And again, we use that relation to calculate the exit temperature that T_{exit} is essentially:

$$T_e = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right)$$

Substituting all the values in the above equation we get $T_e = 71$°C. So, exit temperature is known to us now that was the first part. Once this exit temperature is known the second question was that how much heat loss is happening? It is further simple heat loss is happening:

$$\dot{Q} = hA_s\Delta T_{LMTD}$$

Constant surface temperature condition and we have to evaluate LMTD value here. And the surface is essentially the surface area the perimeter multiplied by L in this case perimeter is (4aL).

So, if you calculate LMTD in this case you would land up with the value which is minus 15 °C around. Because, now you inlet exit everything is known surface temperature all are known you replace this value here as value as I mentioned (4aL)

$$\Delta T_{LMTD} = -15°\text{C}$$

$$\dot{Q} = -1313\ W$$

that minus signifies that heat is being lost from the fluid for which we are calculating that is the air.

So, air will lose heat at a rate of 1313 Watt as it flows through the duct of length 8 m. Now, here also if you see that we got a value at the exit is 71 °C it was entering 80 °C.

So, eventually T_{bulk} or the mean fluid temperature by this process is: $T_b = \frac{80+71}{2} = 75.6$ °C, which is not much different from 80 °C and the fluid properties are not expected to change drastically. So, it is also the point where we are safe in this calculation or in this assumption.

So, that means, throughout this internal flow force convections what we have learnt several correlations based on the flow classification either fully develop laminar flow or fully develop turbulent flow.

If fully develop laminar flow constant surface temperature constant surface heat flux condition in that case of fully developed turbulent flow the thing that we have is Dittus Boelter equation for better accurate result than the Colburn equation. And on both the thermal conditions a thermal boundary conditions in those two cases it appears or gives the similar result.

So, this concludes and again there are several correlations available either for surface roughness in both the flow regimes for different flow scenario for example, not only through the circular tube, but say through the annular of concentric tubes. In those cases, the hydraulic diameter is essentially calculated which is the outer diameter minus inner diameter and that is replaced in the case of Nusselt number calculation.

So, those specific cases those who are interested and whenever it is required can refer to the textbook and got those information’s, but those analysis are analogous or similar to what we have done here today. With this I conclude the forced convection part and in the next class I will take up with the natural convection.

Thank you for your attention.