Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 54
Internal Forced Convection (Contd.)

Hello everyone and welcome back once again with another lecture on Internal Forced Convection in Chemical Engineering Fluid Dynamics and Heat Transfer NPTEL online certification course.

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In the last class we discussed about various Nusselt number relations or Nusselt number correlations that are available or that we have seen particularly for a pipe flow problem with two different thermal boundary condition that is constant surface temperature and constant heat flux and specifically for the laminar flow conditions.

We have also seen the once of the correlations in the entry region for laminar flow in circular tube for the entry regions when the flow is not fully developed. It's not thermally fully developed. Now, the point is before we go into the turbulent flow correlations.

Let us look into that how we use this information for the laminar and the laminar circular tube this correlations and how we solve one of such problem that is that would be helpful to analyze this theory in a more clear way. So, consider that there is a flow of oil at 20 ^{0}C.

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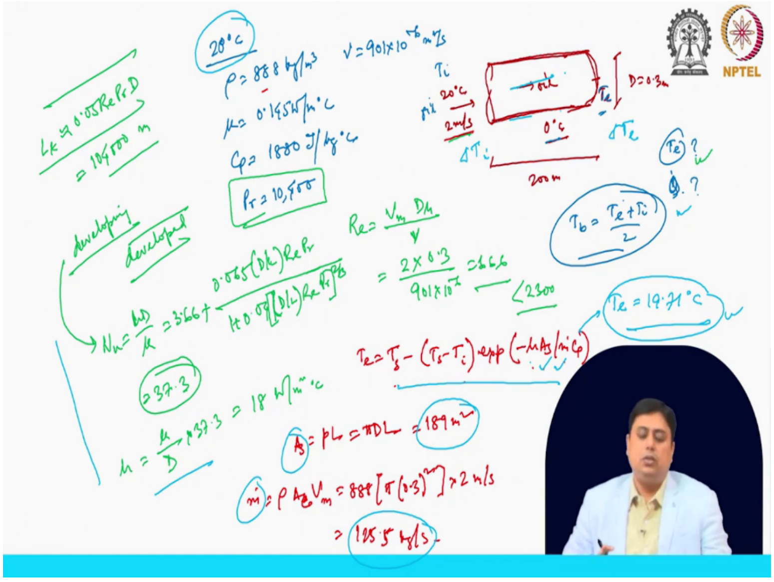

So, there is a pipe where oil is flowing at 20 ºC this pipe is say 200 m long, this is flowing at 2 m/s and the diameter of the pipe is say 0.3 m. Now, the surface condition or the temperature of this tube surface or the pipe surface are maintained at 0 ºC.

So, the pipe surface temperature is at 0 ºC, oil is flowing through it, oil is at 20 ºC. The question is I mean if all the parameters and physiochemical parameters are known that are typically given in the problem statement which I will tell you slowly. The question is the temperature of the oil at the pipe exit. What would be that temperature?

This is the first question. That what would be my T_e that if I consider this as the exit temperature inlet and exit temperature if I consider T_i and this is T_e the first question is what is T_e in this case. The second question is that the rate of heat transfer from the oil. What is the rate of heat transfer from the oil? These are the two questions simple question is asked in this problem.

Now, the point is how do we approach such problem. The first thing again the assumption that we make is that there is steady state operation is happening. The surface temperature is kept constant that is already mentioned as 0 ºC, the thermal resistance of the pipe is negligible. We are not considering conduction inside the pipe thickness or in the pipe wall, surface roughness is smooth,.

So, we are not considering any rough surface condition and the flow is hydro dynamically fully developed when it enters the fluid enters the situation or at the entry. So, now the exit temperature is not known, the surface condition is constant the surface temperature.

So, the fluid properties that we were thinking or we actually took at the bulk fluid temperature that bulk fluid temperature calculation is difficult here it's not since the exit temperature is not there or we do not know what would be the exit temperature. So, the bulk fluid temperature that we had taken is $T_b = \frac{T_e+T_i}{2}$; this calculation is not going to happen here.

So, what we do here? Again, we take an approximation that this 20 ºC would drop to a value that would not be a significantly different than the 20 ºC. Although it will differ because we have 0 ºC here and we have at the entry 20 ºC. So, whatever the temperature here it would be? That T_e and again if we take this average at the end of the calculation it would not be significantly different than 20 ºC because here we have 0 ºC.

So, at the exit maximum it can attain 0 ºC the T_e ideally, but it is it would not touch 0 ºC. So, eventually the average is 10 ºC, but instead of that assumptions what we are taking here that we take the mean fluid temperature at 20 ºC itself since the exit temperature is not known.

So, we take all the fluid properties at 20 ºC and we see that water or the particularly for this oil since this is the oil is flowing at for this oil at 20 ºC the values are like this density is lesser than water, thermal conductivity is 145 W/mºC.

$$k = 0.145 \frac{W}{m} °\text{C}$$

$$\vartheta = 901 \times \frac{10^{-6} m^2}{s}$$

$$c_p = 1880 \frac{J}{kg^0\text{C}}$$

$$Pr = 10400$$

Now, in order to classify this problem whether it is laminar or turbulent we first calculate the Reynolds number which is

$$Re = \frac{V_m D_h}{\vartheta} = \frac{2 \times 0.3}{901 \times 10^{-6}} = 666$$

This is less than the critical number that we have seen in case of pipe flow 2300. So, the flow is completely laminar which means, we calculate the thermal entry length because in the problem statement we mentioned we have assumed that it is hydrodynamically developed, hydrodynamically fully developed flow when it enters the tube, but that does not ensure that the fluid is thermally fully developed as well.

So, we look into the distance that is the thermal entry length that is necessary and from the appropriate correlation for laminar flow what we see it is this relation:

$$L_t \approx 0.05RePrD$$

So, once we replace all these values what we find that the length that is necessary is 104000 m which is much much greater than the actual length of the pipe and this is happening because of this high Prandtl number of the oil, this is significantly higher.

So, what we assume here again this is just because see this thermal entry length is there. So, if we assume that the thermally developing flow it is not the thermally developed flow because for thermally developing and developed and other case it is developed. For fully developed thermally developed flow we know the relations as well as for the developing flow. So, here we choose the developing flow or the relation of Nusselt number for the entry length region in a different word.

For that, if we look at the relations previously, we see that:

$$Nu = 3.66 + \frac{0.065(D/L)RePr}{1 + 0.04\left[\frac{D}{L} Re\, Pr\right]^{\frac{2}{3}}}$$

So, in this expression now we put all the values because everything is known here L, D, Reynolds number, Prandtl number all the things are known.

Once we put it here, we get the value around Nusselt number of 37.3. Had we got this L_t below 200 m we could have used that if that length is not substantial. For example, somehow by adjusting the Prandtl number or the different type of oil we get L_t is 1 meter and the length here is 200 meter. We could have considered or assumed that this flow is completely thermally developed and had used a different Nusselt number relation that is $Nu = 3.66$ that simple relation.

But since it is not the case the flow is still developing it is still thermally developing we had to use the relation for the entry region only and we get the Nusselt number value. Once we get the Nusselt number value we calculate the value of h:

$$h = \frac{k}{D} \times 37.3 = 18W/m^2°C$$

So, now the question is what would be my T_{exit}? If you remember the expression that we had Texit is essentially:

$$T_e = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right)$$

Now, T_s, T_i, h and A_s is known, we have to calculate $\dot{m}$; As = πDL. So, we calculate that around 189 m^2.

Similarly, $\dot{m}$ the mass flow rate.

$$\dot{m} = \rho A_c V_m = 888 \times [\pi 0.3^2] \times 2 = 125.5kg/s$$

This is the mean velocity through which the oil is flowing. So, what we have? A_s value is known, $\dot{m}$ value is known, all other parameters are known here. For the calculation of h we came across these steps. We replace these numerical values in T_e expression and we find out that T_e = 19.71 °C.

So, it is entering at 20 °C leaving at 19.71 °C. So, our assumption of taking all the fluid properties at this 20 °C is fine because the bulk fluid temperature once you take the average of these two would not vary much from the 20 °C or the fluid properties would not change much for these decimal changes.

The point interesting thing is here is that for the oils since the Prandtl number is very high you can see that it takes time or the hydrodynamic boundary layer quickly develops, but the thermal boundary layer does not develop even at 200 m distance. It slowly develops, the dissipation rate is extremely low, the heat diffusion rate is extremely low. So, once this is done then the next part is how do we calculate the heat transfer rate.

So, this is the second part. So, again on the first part exit temperature is this. Now, the other scenario could have been that if this exit temperature is completely different then this 20 °C and say it comes out to be 5 °C for example. So, the bulk fluid temperature could have been 12.5 °C and in that case possibly the fluid properties would have changed a bit.

If that is the scenario then we had to redo the simulations or redo this calculation once again with this new values of the fluid properties and again it goes in a iterative way. So, but in this case, we are in safe region. So, we need not worry about it we go to the next step that is the amount of heat transfer or the heat transfer rate in this case. Now, this is a constant surface temperature problem.

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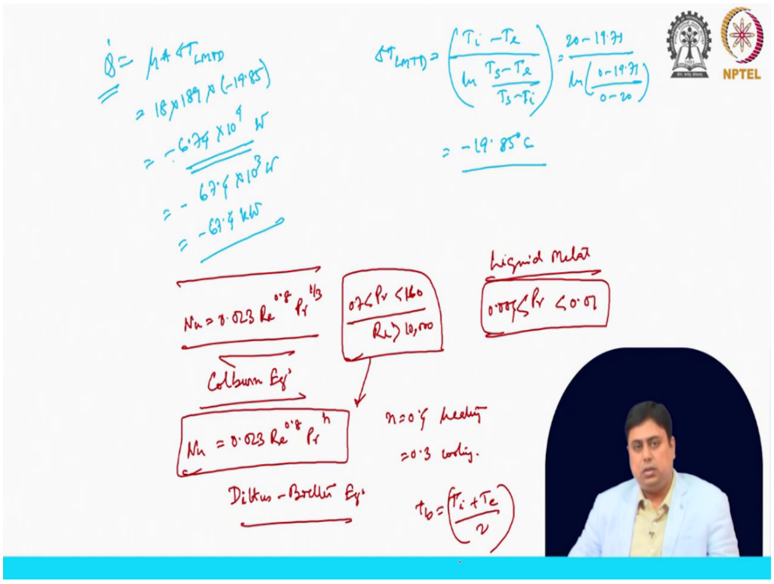

So, the calculation of $\dot{Q} = hA\Delta T_{LMTD}$; involves LMTD. In this case we have to calculate the LMTD and in this case again the ΔT_{LMTD} if i write it is again in the same procedure that is:

$$\Delta T_{LMTD} = \frac{(T_i - T_e)}{ln\frac{T_s - T_e}{T_s - T_i}}$$

This is one of the form of the LMTD and we can find it out what is the value.

$$\Delta T_{LMTD} = \frac{(20 - 19.71)}{ln\frac{0 - 19.71}{0 - 20}} = -19.85°C$$

So, the amount of heat transfer that is happening:

$$\dot{Q} = 18 \times 189 \times (-19.85) = -6.74 \times 10^4$$

So, it means the oil will lose this much of energy the negative sign implies that oil will lose this much of energy which is around (-6.74×10^4) W or we can say right -67.4 kW.

So, oil will lose around -67.4 kW of heat as it flows through a pipe in that condition kind of immersed in ice condition because that is the 0 °C ; that is how we can maintain practically because this temperature can be maintained at constant surface temperature when the phase changes are occurring that we have mentioned in the previous lectures. So, this is the answer.

Now, in this case the another interesting thing practically that you must notice is that in this case if someone had used arithmetic average instead of the logarithmic mean temperature difference. He or she could have land up with the similar value or the similar result because the ΔT changes at the inlet and here are not much different. Here the ΔT is around 20 °C at the inlet and 19.71 °C at the outlet.

So, it's close to the inlet temperature difference as well which means the mean fluid temperature did not change much or it has not changed in a non-linear fashion. It has slightly changed, but that change is not significant to have an major impact on the final result. I hope these explanations and the implications are clear to you. Now, coming to the turbulent flow. So, these are all about the laminar flow and specific to the circular case circular tube we have seen the example.

Now, in case of turbulent flow the relation by which this varies the Nusselt number is that we see Nusselt number is:

$$Nu = 0.023 Re^{0.8} Pr^{\frac{1}{3}}$$

For Prandtl number in the range of 160 to 0.7 and Reynolds number greater than 10,000. For fully developed turbulent flow in smooth tube or pipe a simple relation of Nusselt number was proposed and that look like in this format that is

$$Nu = 0.023 Re^{0.8} Pr^{\frac{1}{3}}$$

which is known as Colburn equation. Now, this equation later was modified for better accuracy and it was done in a fashion that this part remained same and in state of 1/3 it was written as n and n was taken as 0.4 for heating cases and it is 0.3 for cooling cases.

$$Nu = 0.023Re^{0.8}Pr^{n}$$

When the fluid is heated you take n is 0.4 when the fluid is cooled you take n as 0.3 in this relation. And this equation is known as the famous Dittus-Boelter equation and it is preferred over the Colburn equation for its wider range of application and more accurate prediction.

So, these equations were further improved in a case to case basis because these are the generic expressions which has an accuracy I mean at the maximum deviation of around 25% in any application, but where that difference seems to be significant people or the researchers have developed more accurate predictions which are more complex in nature and maybe specific to that application or that kind of application.

So, we will not go into those specific applications because those again can be learned or can be referred whenever it is needed for designs, but the thing that we must remember is this Dittus-Boelter equation. It is for the turbulent pipe flow smooth pipe here we have considered and it is applicable for both heating and cooling of the fluid.

Now, there are other applications also available when the Prandtl number is different because here we have limited the Prandtl number or the applicability of this relation for better accuracy within this range of Prandtl number that is 0.7 to 160. Now, for the liquid metals when the Prandtl number is much lower Prandtl number as for the liquid metal we have seen this example or we have seen this numbers the Prandtl number is below 0.01 and it can go up to 0.004.

In those cases, different equations are available or different correlations are available. So, in this Prandtl number again the fluid properties that we calculate are evaluated at the bulk mean fluid temperature that is T_b :

$$T_b = \frac{T_i + T_e}{2}$$

So, again for the rough surfaces there are modifications to this equation and again as I mentioned those are very specific and those increases the number of equations for the sake of our observation. So, we will not go into those specific values, but the point is that if there is a roughness a separate set of correlations are available. So, for each and every specific kind of a problem the equations or the correlations are available which are to be selected.

But our strategy remains same that we at first identify whether the flow is laminar or turbulent and accordingly we choose our correlations to calculate the Nusselt number. With this understanding I will stop here I will be back with a problem that will be solved within this understanding on the turbulent flow in the next class and then we move to the condensation and boiling overview on this convection part.

Till then thank you for your attention.