Chemical Engineering Fluid Dynamics and Heat Transfer

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Lecture - 53

Internal Forced Convection (Contd.)

Hello everyone. Welcome back once again with another lecture on heat transfer in Chemical Engineering Fluid Dynamics and Heat Transfer NPTEL online certification course. Over the last couple of lectures, we were discussing Convection with a focus on forced Convection. We covered external flow and we were into internal flow since the last lecture.

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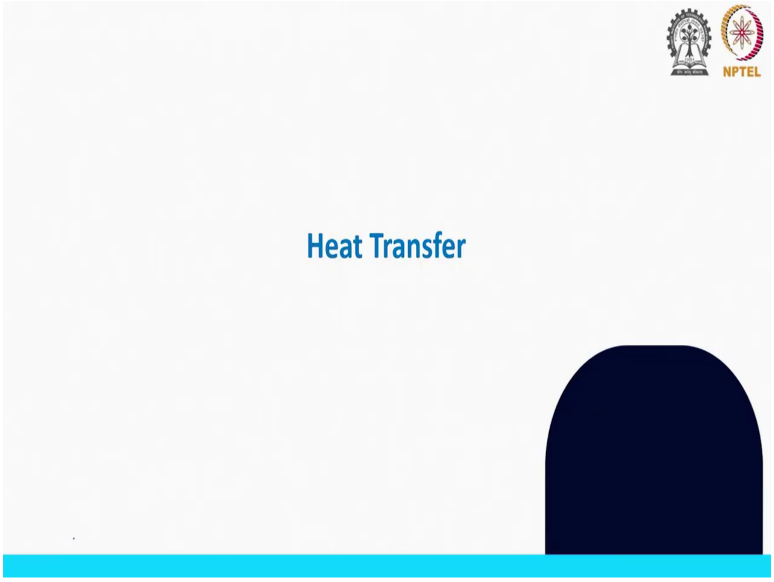

Heat Transfer

In the last lecture what we have seen is that the concept of Logarithmic Mean Temperature Difference LMTD.

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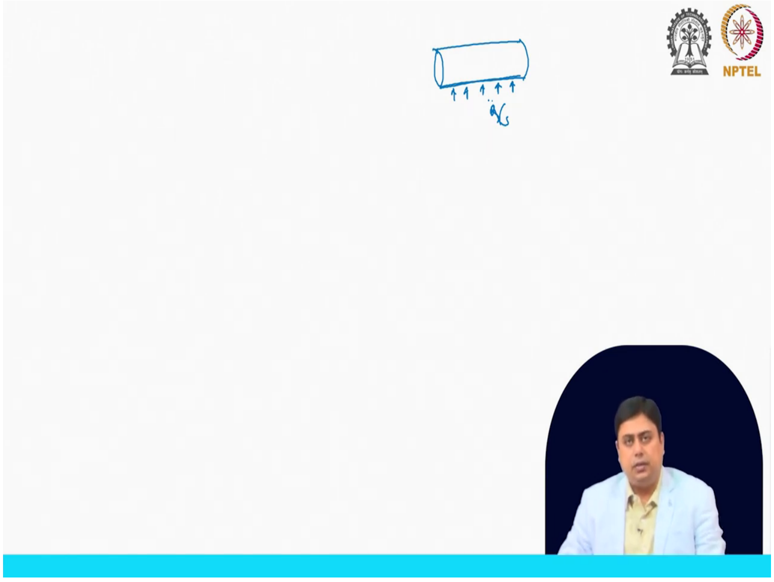

So, when there is a flow in a pipe, as we have discussed earlier there, we can have two different thermal boundary condition. One is that say this pipe surfaces or the pipe walls are kept at constant heat flux condition.

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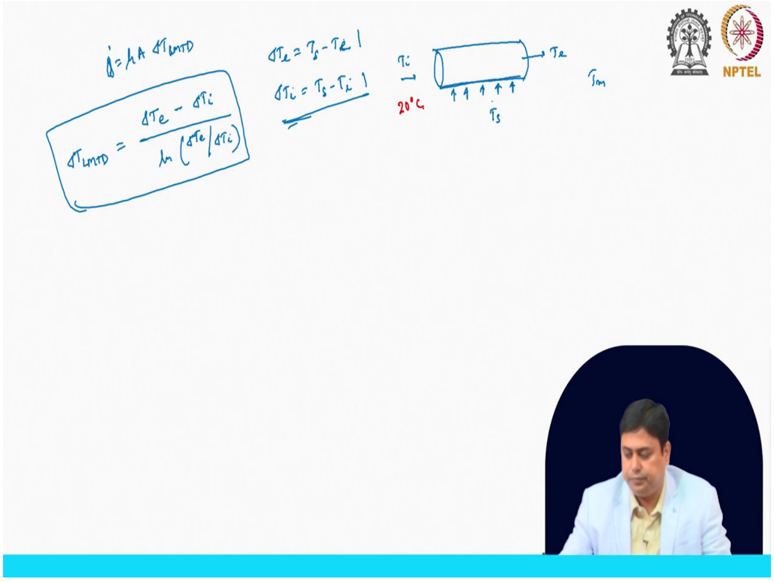

$$\dot{q} = hA\,\Delta T_{LMTD}$$

$$\boxed{\Delta T_{LMTD} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\Delta T_e / \Delta T_i\right)}}$$

$\Delta T_e = T_s - T_e$

$\Delta T_i = T_s - T_i$

In other case it can be of constant temperature condition, surface temperature condition. So, for the constant heat flux condition we have seen how to take the average or the mean fluid temperature. And when we came to constant surface temperature condition, we have seen that this arithmetic mean or the average would not be productive and we will see a proof of it today.

Along with that here we have seen the concept of logarithmic mean temperature difference. So, in this case if I consider that we have T_i that is going in of the fluid T_e that is coming out of the pipe. We have a constant surface temperature. In that case this logarithmic mean temperature difference that we have understood which is a necessary for Q overall heat transfer overall amount of heat calculation is $\dot{Q} = hA\Delta T_{LMTD}$.

$$\Delta T_{LMTD} = \frac{(T_i - T_e)}{ln\frac{T_s - T_e}{T_s - T_i}} = \frac{\Delta T_e - \Delta T_i}{ln\frac{\Delta T_e}{\Delta T_i}}$$

$$\Delta T_e = T_s - T_e$$

$$\Delta T_i = T_s - T_i$$

So, these are the temperature difference at the outlet and at the inlet.

Now, this is purely because if we take the average or the arithmetic mean of the temperatures at the exit and at the inlet, we make an inherent assumption that the temperature profile is varying linearly the mean fluid temperature the T_m that we have discussed earlier is varying linearly along the flow direction which does not happen that we have seen in the last class.

So, it actually reflects the exponential decay of the temperature difference as it goes along the surface. So, if say T_s is at higher temperature than the T_i. So, the fluid is heated a cold fluid is coming into a hot tube or pipe and it is being heated. In such cases the temperature difference that we have initially this decays exponentially and we have seen the concept of NTU that is the Number of Transfer Unit.

When the number of transfer unit is more than 5 it has been seen that if we provide that much length sufficient length, then we see that the exit temperature T_e becomes very close to the temperature of the wall or the pipe wall. That means this ΔT_e the gap decreases and it happens exponentially. And eventually reaches the surface temperature of the pipe.

So; that means, this temperature variation is the mean fluid temperature variation is not linear. So, now the point is that if we try to see this its influence in terms of an example, the example is in this case the water; water is entering this pipe at 20 ºC.

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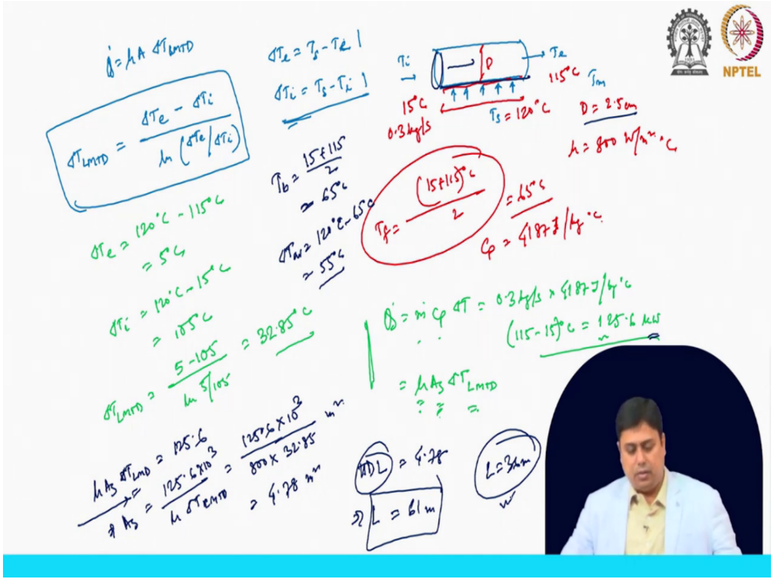

See we consider instead of 15 ºC. The surface temperature of the wall or the pipe wall temperature is 120 ºC. The water is going into the tube that is having diameter D as 2.5 cm. It is flowing at a rate of 0.3 kg/s and outside it a steam is flowing.

So, that by which we maintain the surface temperature at 120 ºC. So, if this is the case that this water cold water is heated by a steam of 120 ºC. I mean the steam that has a temperature high temperature and steady state this tube wall attains 120 ºC.

So, the question is and if my average heat transfer coefficient is given say as 800 W/m^{2}ºC, we have to determine what is the length of this pipe that is necessary so that this water gets heated till 115 ºC. So, in a heat exchanger as I mentioned this kind of problem you would encounter in design of heat exchanger. What happens in the heat exchanger?

You have several tubes and these tubes are stacked and are enclosed by a shell. So, inside the tube some fluid flows that is to be heated or to be cooled. And outside that means, in the shell region there is a different kind of fluid with a different temperature. So, this heat transfer takes place the energy transfer takes place between the cold and the hotter fluid.

So, in such a situation inner tube which is maintained at say 120 ^{0}C, a cold water at 15 ^{0}C is flowing through it.

The objective is to heat that cold water from 15 ^{0}C to 115 ^{0}C. We have a hot tube through which it is flowing at 120 ^{0}C constant surface temperature. So, the question is how much length of the tube is necessary in order to attain that exit temperature of 115 ^{0}C. And all other material or physical properties are given for this problem.

So, how do we solve such problem? The first thing is that we consider or we assume that the steady state operation is happening. Because if it is not steady state then it would be difficult to consider that the wall surface temperature the pipe or the tube wall surface temperature is attained at 120 ^{0}C due to steam.

So, whatever the heat loss whatever the transfer that was supposed to happen through the metals thickness the pipe wall thickness that is at steady state and the inner pipe wall takes a temperature of 120 ^{0}C.

Now, steady state scenario, the fluid properties we consider are constant that it is not varying. This is the assumption although temperature is varying significantly from 15 ^{0}C to 115 ^{0}C, but for liquid cases the property does not vary much. So, it is a logical assumption.

The other assumption is that the convection heat transfer coefficient is constant in this case. Because we have an average value that is stated here as 800 W/m^{2}^{0}C and the conduction resistance of the tube imparted by this thickness metal thickness or whatever the material this is being built that is negligible; that means, and that is why I told that the steady state scenario is attained.

So, we are not considering anything that is happening by conduction that steady state temperature profile is attained. So, the pipe thickness that we are considering or the material thermal conductivity is very high and the thickness is very small. So, it quickly attains this temperature of the outside fluid temperature.

So, based on these assumptions what we look into is again the calculation of certain things. The starting with the Reynolds number because this is the strategy, we have mentioned that which we typically follow. We try to find out in which flow design this is this problem belongs to.

And then accordingly we choose appropriate correlation. And in order to calculate the Reynolds number what we need several fluid properties; that means, the density viscosity or the kinematic viscosity. So, and those fluid properties we will look into at film temperature. So, the film temperature we have considered here as the average. I mean this is the thing that we have understood earlier that the fluid properties would be evaluated at the film temperature.

So, here this is 130 ^{o}C. So, that means, at 65 ^{o}C we find out the water properties from the reference table that are usually given or the data would be given. Now, once we know those values for example, C_p values we have to find out from the table or from the given information that what is the value of C_p at 65 ^{o}C of water and that is around 4187 J/kg^{o}C.

Now, since we know the inlet and outlet temperature of the fluid or of water specifically here. So, the rate of heat transfer we can easily determine from this simple relation from the energy conservations. The amount of heat transfer that happens is essentially:

$$\dot{Q} = \dot{m} c_p \Delta T$$

$$\dot{Q} = 0.3 \times 4187 \times (115 - 15) = 125.6 kW$$

Now, this is a constant surface heat flow constant surface temperature problem. So, LMTD calculation is essential. Because with that LMTD and the amount of heat transfer that is happening here because this

$$\dot{Q} = h A_s \Delta T_{LMTD}$$

which is equals to this value, where h is given A_s comes with a term length, that is the surface area, perimeter multiplied by the length.

So, here L is unknown, LMTD is known here because all the values are given. So, once LMTD is calculated (hA_s) is already h is given in this case. So, we can find out what is the surface area that is needed to attain this much amount of heat transfer. And accordingly, we can finally, calculate what is the length of the tube that is necessary. So, LMTD in this case what would be the value?

$$\Delta T_e = 120 - 115 = 5°\text{C}$$

$$\Delta T_i = 120 - 15 = 105°\text{C}$$

$$\Delta T_{LMTD} = \frac{(5 - 105)}{ln\frac{5}{105}} = 32.85\ ^0C$$

$$hA_s\Delta T_{LMTD} = 125.6$$

$$\rightarrow A_s = \frac{125.6 \times 10^3}{h\Delta T_{LMTD}} = \frac{125.6 \times 10^3}{800 \times 32.85}$$

$$A_s = 4.78m^2$$

which means for this problem for the water from 15 ^{0}C to 115 ^{0}C this much temperature I mean heating from 15 to 115 ^{0}C we need a surface area of the tube to be 4.78 m^2.

Now, as this is a uniform cross sectional tube it has a constant perimeter multiplied by the length. So, the length that is required; that means, it is (πDL), I mean the surface area that is needed in this case. The (πDL) is the surface area across which this heat transfer is happening. So, $(\pi DL = 4.78)$, the diameter is known 2.5 cm, it is given in the problem statement. So, we find the value of (L = 61 m).

So, I hope this process is clear to you. Now instead of LMTD if someone had used mean temperature or the average temperature. So, arithmetic average temperature, the bulk temperature of the fluid as we have understood from the previous calculations. So, bulk temperature $(\frac{15+115}{2} = 65°\text{C})$ that is the film temperature.

Now, if someone had taken the arithmetic mean temperature difference that considering the mean temperature as varies linearly. So, in that case:

$$\Delta T_{average} = 120 - 65 = 55°\text{C}$$

Now, if you do the same problem. Now instead of LMTD if you provide this as the $T_{average}$ like we calculate usually in case of constant surface heat flux condition, by this way then this L value would be substantially different and this L in this that case would be around 36 m by that calculation.

So, just replace here instead of 32.85 by 55 and rest of the calculation remains same. One would land up with the calculation that L necessary to attain this temperature from 15 to 115 ºC is 36 m which is completely gross and poorly calculated. So, this shows the utility or importance of LMTD calculation. In case particularly I mean LMTD comes into play when we have constant surface temperature condition.

And the pipe internal flow constant surface temperature condition we must calculate LMTD logarithmic mean temperature difference. Now, coming to so this is the utility of it. Now how this is comes out to be in case of Nusselt number calculation. So, we have seen several correlations in case of external flow. Similarly, in laminar flow in tube we know all the velocity profile that is necessary as well as the pressure drop expression.

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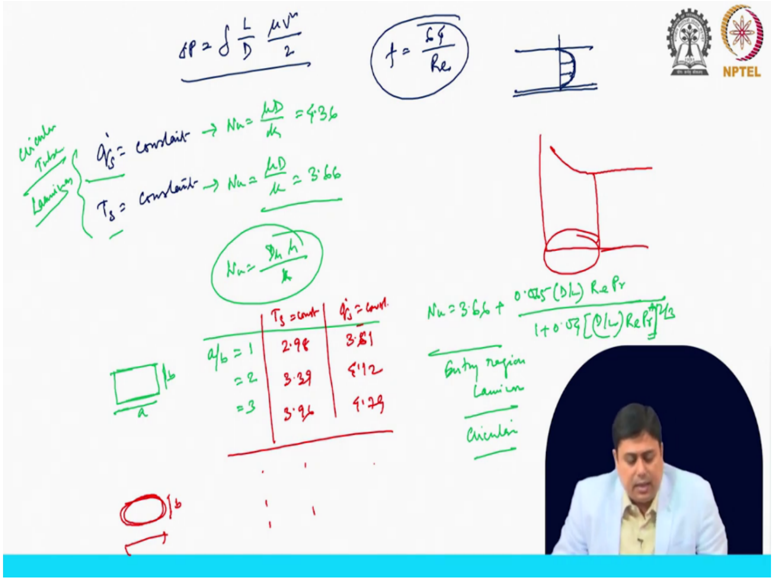


$$\delta P = f \frac{L}{D}\frac{\mu V^2}{2}$$

$$f = \frac{64}{Re}$$

Circular Tube Laminar

$\dot{q}_s$ = constant → $Nu = \frac{hD}{k} = 4.36$

T_s = constant → $Nu = \frac{hD}{k} = 3.66$

$$Nu = \frac{D_h h}{k}$$

	T_s = const	$\dot{q}_s$ = const
$a/b = 1$	2.98	3.61
$= 2$	3.39	4.12
$= 3$	3.96	4.79

$$Nu = 3.66 + \frac{0.065 (D/L) Re Pr}{1 + 0.04 [(D/L) Re Pr]^{2/3}}$$

Entry region Laminar

Circular

So, pressure drop expressions if I write you would probably remember that:

$$\Delta P = f \frac{L}{D}\frac{\mu V^2}{2}$$

where f is the friction factor. This f is related with the for a circular tube in case of laminar flow is by this expression, for circular tube laminar flow friction factor is this.

So, friction factor is only a function of Reynolds number irrespective of surface roughness in case of laminar flow. So, we know these expressions we are not going into the details. What I will tell you here is that when it comes to the temperature profile. Because when the flow is happening through a pipe, we know what is the velocity profile, where the maximum value is and what is its expression along with the ΔP expressions etc..

The thing is that if this fluid temperature and the surface temperature is different just continuing with our previous discussion, what happens with the Nusselt number calculations? Again, we are not going into the derivations; if you are interested you can look into the test book and go to the derivations. But what happens here for a circular tube laminar flow and when $\dot{q_s} = constant.$ As I mentioned there can be two condition. One is the surface temperature constant and the other thing is the constant surface heat flux. In the first case the Nusselt number correlations comes out to be:

$$Nu = \frac{hD}{k} = 4.36$$

Two conditions are here circular tube and laminar flow plus thermal boundary condition is $\dot{q_s} = constant.$

And this we have seen in the previous class in a schematic form. That once it is fully developed this does not vary. And in case of constant surface temperature this expression of Nusselt number is:

$$Nu = \frac{hD}{k} = 3.66$$

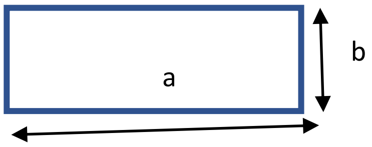
Now, in all these cases, in this particular this example that we have seen in the last few seconds back is that the value of k of the fluid that is we are evaluating at the bulk temperature of the fluid or the film temperature, when it comes for the constant surface heat flux condition.

So, k is evaluated, that is used in Nusselt number at the bulk mean temperature that is the arithmetic average of the mean fluid temperature at the inlet and at the exit of the tube. Now, for laminar flow the effect of surface roughness on friction factor we have seen negligible so as for the convection heat transfer coefficient in all the cases. So, roughness does not have much influence or negligible influence in case of laminar flows. So, these are the cases for the circular tube laminar flow.

So, now if the tube is not circular in such condition what can happen? The similar equations we can use, but with the difference in the value of the friction factor values. So, in this cases what happens is that the Nusselt number is usually calculated in terms of hydraulic diameter. Like we calculate Reynolds number using the hydraulic diameter.

So, similarly Nusselt number is calculated once this Nusselt number once we find out from the expressions, based on our Reynolds number relations that the Reynolds number is based on the hydraulic diameter and then what we calculate h from this expression.

So, Nusselt number relations for the non-circular tubes. So, the Nusselt number relations I mean in this case for the non-circular tube there are different values. For example, for a rectangular case when we have this:



	T_s = constant	**q_s = constant**
$\frac{a}{b} = 1$	2.98	3.61
= 2	3.39	4.12
= 3	3.96	4.79

The point is that you need not remember this values. This values we can always refer back wherever or whenever it is needed from the textbook or the reference books. But I am showing you here just for the sake of your understanding that how this values actually differs.

So, something like this the value changes and this kind of values you would find out in the textbook or the reference books. So, similarly for a elliptical cross sectional tube or pipe, if it is not circular depending on this a and b similar correlations are available the Nusselt number would be constant value. So, the point is laminar flow we have seen the correlations.

Now, in case of developing laminar flow in the entrance region. Because you remember that in those cases the value were not constant that we have seen. The h values were different and then it became constant after the thermal boundary layer fully thermal developed condition achieved. So, in those cases this in the entrance regions again several correlations are available. I am just writing for the sake of your observation here which you need not again remember or memorize.

The point is that it is something like this. It is a complex relation:

$$Nu = 3.66 + \frac{0.065(D/L)RePr}{1 + 0.04\left[\frac{D}{L} Re\ Pr\right]^{\frac{2}{3}}}$$

This is for the entry region and laminar flow. So, similarly several such correlations are available for different kinds of and particularly this is for the circular tube, different geometry shape and different combinations or different combinations of flow.

With this I will stop here. In the next class I will be back with the turbulent correlations. The pipe flow turbulent conditions and how then the Nusselt number looks like and will solve a few problem, till then.

Thank you for your attention.