**Chemical Engineering Fluid Dynamics and Heat Transfer**
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**Lecture - 52**
**Internal Forced Convection (Contd.)**

Hello and welcome to another class on Chemical Engineering Fluid Dynamics and Heat Transfer, we are in the heat transfer part and discussing Internal Forced Convection.

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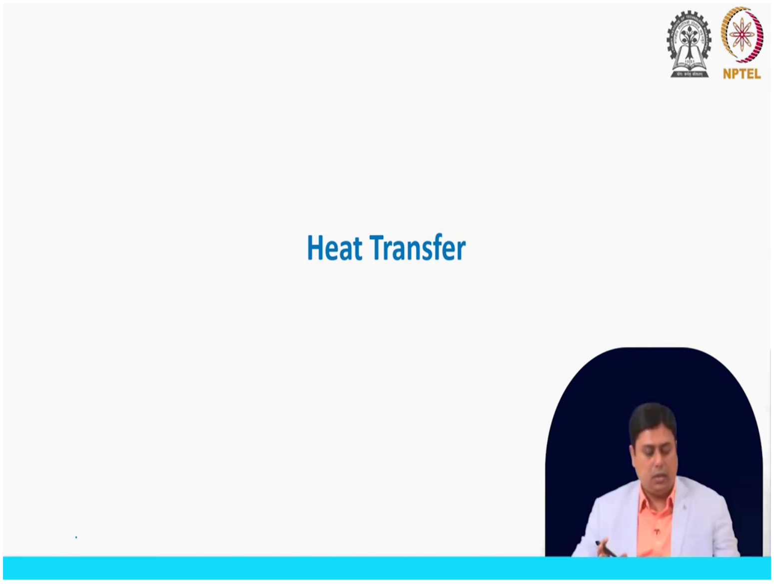


In the last class, we have gone through the essentials of internal forced convection that is understanding mean fluid temperature, mean velocity and also, we have seen what is fully developed flow that is the thermally fully developed region and hydrodynamically fully developed region. And the utility of the entrance length on the heat transfer coefficient, the convection coefficient.

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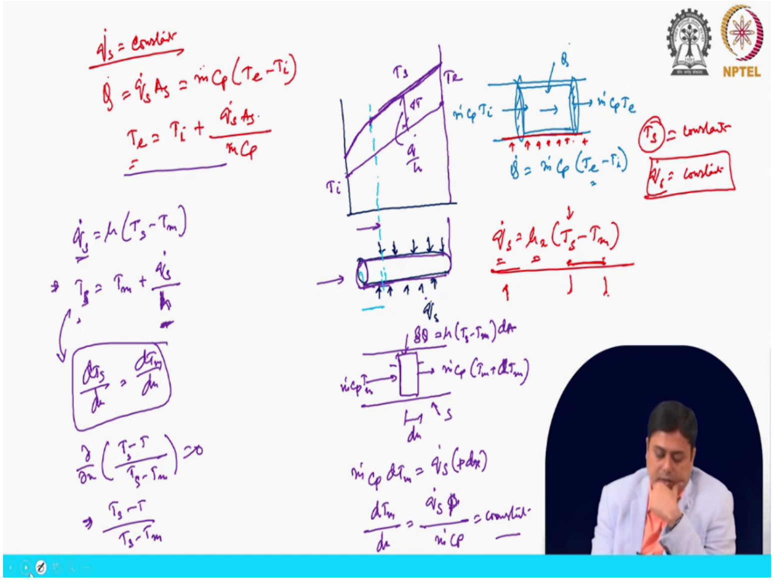


Now, generally, what happens when we have a fluid element or when a fluid is flowing through a tube, say if we consider element of it, if the energy is going into the fluid. So, what happens the energy balance if it is $(\dot{m}c_pT_i)$ temperature and $(\dot{m}c_pT_e)$, then what happens what we write is that energy balance that:

$$\dot{Q} = \dot{m}c_p(T_e - T_i)$$

The energy equation that we can write for this fluid element. Now, where this $T_e$ and $T_i$ the inlet and exit, these are the mean temperatures at those positions. That means, in this cross section, these are the mean temperature, at the exit as well as at the inlet corresponding e and i, e stands for the exit, i stands for the inlet condition.

Now, the thermal conditions at the surface usually that we can consider either of these two cases that we have seen. That either we can consider this is having a uniform or isothermal surface which is having a constant temperature that is ($T_s$ = constant). Or we can have constant surface heat flux that is $q_s = c\dot{o}nstant$

We can supply constant heat flux in order to maintain the second boundary conditions. So, constant surface temperature we can think of when say there is phase change is happening of a fluid. It is at the same temperature, but the phase is changing, say for example, boiling condensation etcetera is occurring.

However, the constant surface heat flux practically we can think of a situation that this tube is say perhaps is wrapped with the electric coil where a continuous supply of electricity is given in order to maintain its temperature at a particular condition of the fluid.

So, now in those cases where $q_s$ is constant there the surface temperature would vary along the surface. Because the surface heat flux is essentially:

$$\dot{q_s} = h_x(T_s - T_m)$$

Where $h_x$ is the local heat transfer coefficient, $T_s$ is the surface temperature, $T_m$ is the mean fluid temperature at particular location wherever we are calculating $h_x$ or equating that with the $h_x$.

Now, mean fluid temperature in a flow it would change during heating or cooling whatever the scenario would be. This $T_m$ would change, that would change when $q_s$ is constant. And the surface heat flux would change if surface temperature is constant.

So, from this relation what we see that when $T_m$ which is the mean fluid temperature of a flowing fluid it would change and if $T_s$ is constant. If $T_s$ is constant and $T_m$ is changing in order to maintain this balance $\dot{q_s}$ must change that means, the surface heat flux must change if we keep $T_s$ is constant.

Similarly, or on the other hand if $T_s$ is constant in the second case, as $T_m$ is changing in order to have this balance $T_s$ must change, it cannot be constant. So, this we have to clearly understand. So, let us consider the second case first that means, the constant surface heat flux condition that means, $\dot{q_s} = constant$.

If this is the case then the rate of heat transfer also that we consider is essentially:

$$\dot{Q} = \dot{q_s}A_s = \dot{m}c_p(T_e - T_i)$$

This is the value and the mean fluid temperature at the outlet that we can calculate is essentially:

$$T_e = T_i + \frac{\dot{q_s}}{mc_p}$$

So, mean fluid temperature at the exit what we see is linearly increasing. So, if we try to see now the situation in a schematic for a pipe where we have constant surface heat flux, this is constant. So, for this length L what we see that from and if this is my $T_i$, inlet temperature which is lower.

And if this is my total length and if this is my $T_e$, there is a linear relation between these two points from this expression that ($T_e = T_i$ + constant part). Considering that this is the case we are resolving constant heat surface where heat flux surface heat flux condition.

So, that means, mean fluid temperature increases linearly in the flow direction. The flow direction is in this direction where $A_s$ in this case is the area of the perimeter or the perimeter which is constant in case of I mean it is a constant, but multiplied by the tube length.

It is a constant multiplied by the tube length is the perimeter, in this case the A s. Now, the surface temperature in case of the constant surface heat flux condition what we can write is:

$$\dot{q}_s = h(T_s - T_m)$$

that means, surface temperature is essentially:

$$T_s = T_m + \frac{\dot{q}_s}{h}$$

Now, consider the case of fully developed region. In fully developed region surface temperature $T_s$ that will increase linearly in the flow direction because h is constant. Already we had $\dot{q}_s$ is constant in this case this is the conditioner we are talking about. So, in the fully developed region when h is constant because the reason we have seen earlier, the surface temperature also linearly varies, but that is in the fully developed region. So, this $(T_s - T_m)$ will have a linear slope, ok.

So, if this is my fully developed zone or say the length that is necessary in order to be the fully developed part. If this is the length that is necessary for the flow to be fully developed then in this region. So, from here there is a linear profile till this linear profile is for the $T_s$ in the fully developed region. And this is the $\Delta T$ is essentially $\frac{\dot{q}_s}{h}$ . But in the entrance region the surface temperature would vary like this. Because there h is varying, h is not constant until it reaches the thermal entry length.

So, this gives sense schematic or the idea how it is happening or what that what is the thing that is happening. And so what we can see that if we quickly balance a small element in the fluid like we did in the earlier case that this is $\left(\dot{m}c_pT_m\right)$ that is going in and $\left(\dot{m}c_p(T_m + dT_m\right))$ in this case. This is the surface temperature $T_s$, the amount of heat that is the $\delta Q = h(T_s - T_m)dA$.

So, this is the $T_m$ and this is $(T_m + dT_m)$ of a element that is $T_x$. If we do the steady state energy balance for this slice what we can write:

$$\left(\dot{m}c_p dT_m\right) = \dot{q_s}(Pdx)$$

where p is the perimeter of the tube. That means:

$$\frac{dT_m}{dx} = \frac{\dot{q_s}P}{\dot{m}C_p}$$

So, what we see the change of mean temperature of the fluid with respect to x is constant. And again from the surface temperature $T_s$ expression:

$$\frac{dT_s}{dx} = \frac{dT_m}{dx}$$

And also, based on the dimensionless temperature profile that remains unchanged in the fully developed region what we can write?

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0$$

This is essentially a constant value.

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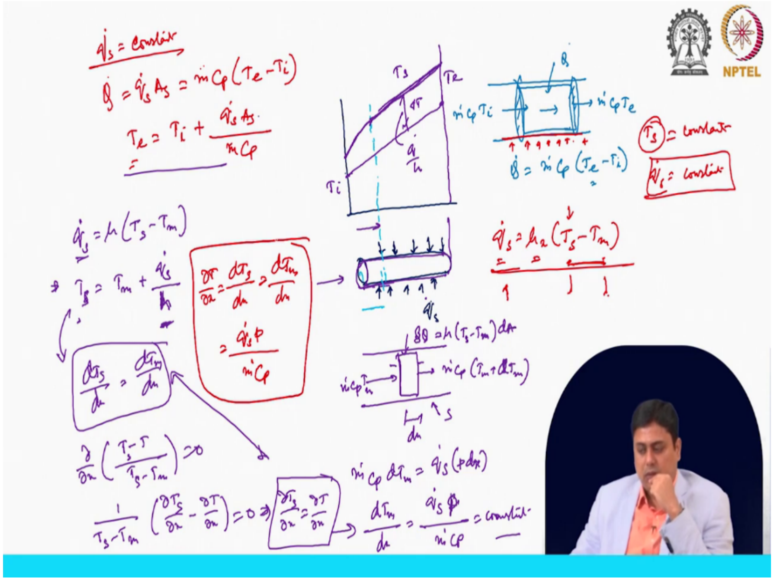


And further if we look at in a different way:

$$\frac{1}{T_s - T_m}\left(\frac{\partial T_s}{dx} - \frac{\partial T}{dx}\right) = 0$$

Or,

$$\frac{\partial T_s}{dx} = \frac{\partial T}{dx}$$

So, if we combine all these what it leads to:

$$\frac{\partial T}{dx} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q_s} p}{\dot{m} c_p}$$

So, we conclude that in the fully developed flow in a tube that is subjected to constant surface heat flux temperature gradient is independent of x and the shape of temperature profile does not change along the tube. So for fully developed flow in a tube subjected to constant surface heat flux the temperature profile does not change along the tube or along the flow direction and the temperature gradient is independent of x.

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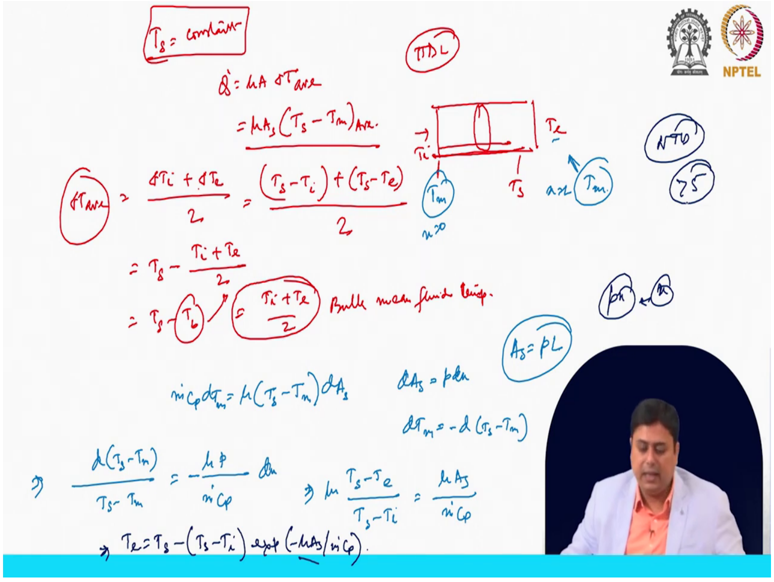


Now, if we consider constant surface temperature condition. So, in that case again from the Newton's law of cooling the rate of heat transfer either to the body or from the body that we estimate is $\dot{Q} = hA\Delta T$.

Now, here the $\Delta T$ is the $\Delta T_{average}$; that means:

$$\dot{Q} = hA_s(T_s - T_m)_{average}$$

where h is the average heat transfer coefficient, $A_s$ is the heat transfer area which is equals to $\pi DL$, in case of circular pipe of length L, diameter D and $\Delta T_{average}$ is the appropriate average temperature difference, the difference bit of average temperatures at of the fluid and the surface. Because at the surface although this is constant the fluid temperature changes along the cross section.

So, that is why the mean fluid temperature we are considering and also since it changes along the x-direction. So, throughout the case we take an appropriate average of it. Now, in case of constant surface temperature one of the way to estimate $\Delta T_{average}$ is taking the arithmetic mean difference.

$$\Delta T_{average} = \frac{\Delta T_i + \Delta T_e}{2}$$

$$= \frac{(T_s - T_i) + (T_s - T_e)}{2}$$

So, at the inlet the average of $\Delta T_i$ is $(T_s - T_i)$ plus the difference at the exit is $(T_s - T_e)$ and its arithmetic average which becomes:

$$= T_s - \frac{T_i + T_e}{2}$$

$$= T_s - T_b$$

$T_b$ is the bulk fluid temperature equals to $\frac{T_i+T_e}{2}$. We can also say this as the bulk mean fluid temperature. This is the arithmetic average of mean fluid temperatures at the inlet and at the outlet.

Now, in this case what happens? The arithmetic mean temperature difference is simply the average of temperature difference between the surface and the fluid at the inlet and at the exit. The inherent assumption in this case is that the mean fluid temperature varies linearly along the tube. If this linearly varies then such assumptions or this definition work best.

But that does not happen; that this mean fluid temperature does not vary linearly in this case. Therefore, we need a better way to find out what is the $\Delta T_{average}$. So, now consider again energy balance of an differential element and in that case what we usually have seen that:

$$(\dot{m}c_p dT) = h(T_s - T_m)dA_s$$

So, with the increase in energy of the fluid that is equals to the energy transferred to the fluid from the tube surface by convection where $dA_s = pdx$ .

Now, also we have seen this is $dT_m$. So, $dT_m = d(T_s - T_m)$, because surface temperature is constant. So, what we can write? Since $T_s$ is constant in this way. And then if we try to rearrange this:

$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hp}{\dot{m}c_p}dx$$

Once we integrate it from (x = 0) to (x = L), 0 means where $T_m$ is $T_i$, at (x = 0), $T_m$ is basically the $T_i$. And at exit or (x = $T_m$) is basically $T_e$.

$$ln\frac{T_s - T_e}{T_s - T_i} = \frac{hA_s}{\dot{m}c_p}$$

where again $A_s$ is nothing but (pL) surface area of the tube and h is the constant average convection heat transfer coefficient. So, once we find this once we have this, then if we solve for $T_e$ we can write an expression that gives us the value:

$$T_e = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right)$$

So, this relation we can use to determine the mean fluid temperature at any x by replacing As is equals to (pL) by (px) at any position x if we consider (As) as (px) from this expression, we can find that mean temperature. So, that means, the temperature difference between the fluid and the surface decay exponential from this relation we can clearly see.

And this rate of decay depends on the value of this exponent. And this dimensionless number is called the Number of Transfer Unit, NTU and this is the measure of effectiveness of heat transfer process. So, what has been seen that if number of transfer unit if it is more than 5, then it is seen that the exit temperature of the fluid becomes almost equals to the surface temperature.

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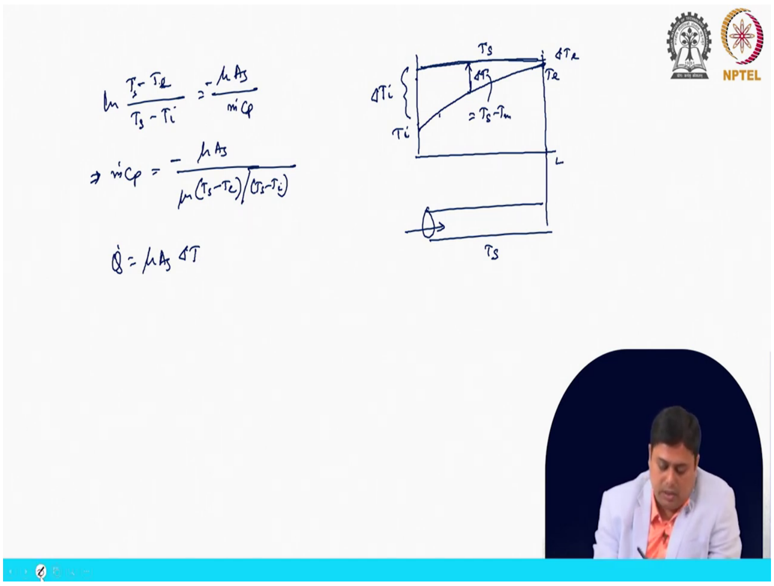

That means schematically what is there in this case when we have a constant surface temperature condition the fluid is flowing from (0 to L). So, this is our $T_i$ and this is our $T_{exit}$. Now, in this case the temperature varies in this way where we have it is not exactly overlapping. This is my constant surface temperature condition.

So, initially we had $\Delta T_i$ and here the $\Delta T_e$ and this is the $\Delta T = T_s - T_m$ which exponentially decay and reaches the exit temperature of the fluid. This is the fluid temperature curve which is not linear and that is why that simple arithmetic mean average would not have been an appropriate approximation that we are now seeing from this expression it reaches for NTU more than 5 the exit temperature reaches nearly the constant surface temperature.

So, NTU has importance, we will not go into the details of it, but the point is once we have understand this importance say we have seen now the expression:

$$ln\frac{T_s - T_e}{T_s - T_i} = \frac{hA_s}{\dot{m}c_p}$$

$$\dot{m}c_p = \frac{hA_s}{ln\frac{T_s - T_e}{T_s - T_i}}$$

Now what we know that $\dot{Q} = q_s\dot{} A_s = \dot{m}c_p(T_e - T_i)$

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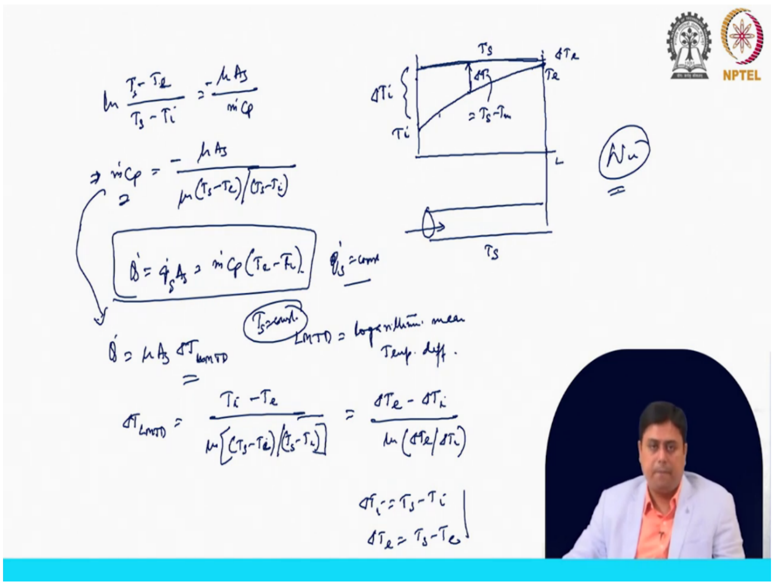

Above equation is for constant surface heat flux condition when this was for the constant surface heat flux condition, when this was constant. Now, once we substitute this ($mC_p$) in this equation what happens is that:

$$\dot{Q} = hA_s \Delta T_{LMTD}$$

LMTD we call this as Logarithmic Mean Temperature Difference.

$$\Delta T_{LMTD} = \frac{(T_i - T_e)}{ln\frac{T_s - T_e}{T_s - T_i}} = \frac{\Delta T_e - \Delta T_i}{ln\frac{\Delta T_e}{\Delta T_i}}$$

These are the temperature difference at the inlet at the inlet between the surface at the inlet fluid temperature and at the outlet the outlet fluid mean temperature and the surface temperature the difference of it. So, once we replace this m $\dot{m}c_p$ in this equations on we try to find it out for the constant surface temperature. What we see is that instead of the $T_{average}$ this relation is more appropriate in order to find out the amount of heat or the rate of heat transfer that is happening.

And this introduces the concept of LMTD which is extremely useful for the case of internal forced convection. Because this concept in future you would see the applications in shell and tube heat exchanger design and various other purposes. So, I stop here today. Based on this concept we will solve a couple of problem in the next class and then we move on to the natural convection part.

After understanding these applications of constant surface heat flux and constant surface temperature we will see that how to apply this concept for problem solving and then we move on to the some overview of the natural convections and the related values. But we have come here, but till this class is the concept of LMTD. And now we will see in the next class before you solve the some problem that how the Nusselt number is varying in these two conditions.

That is the constant surface heat flux and the constant surface temperature conditions. Because we have not till now seen how Nusselt number calculate can be estimated for these conditions for both laminar and turbulent flows. So, those we will see in the next class and we will solve a few one or two problem and then we move on to the next section.

With this I thank you for your attention and please rehearse these things before we go to the next class.

Thank you.