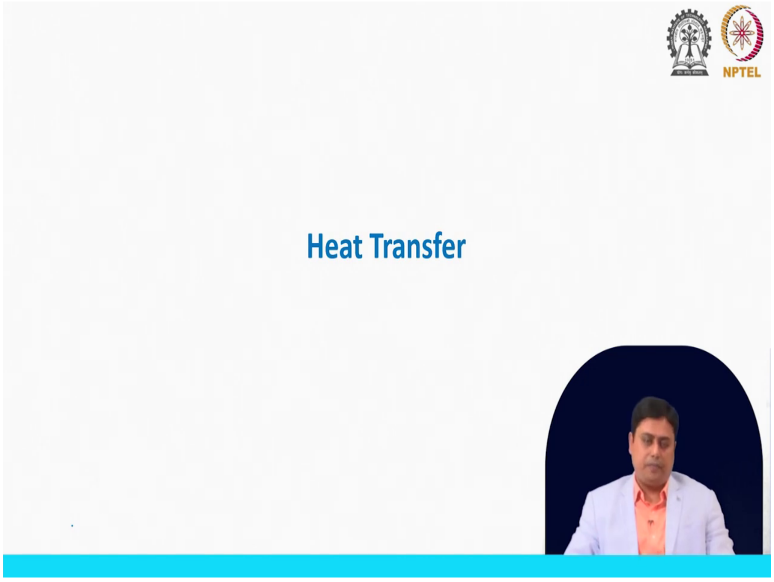
Chemical Engineering Fluid Dynamics and Heat Transfer
Prof. Arnab Atta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 51
Internal Forced Convection

Hello everyone. Welcome back once again in the another lecture on Internal Forced Convection in the NPTEL online certification course in Chemical Engineering Fluid Dynamics and Heat Transfer.

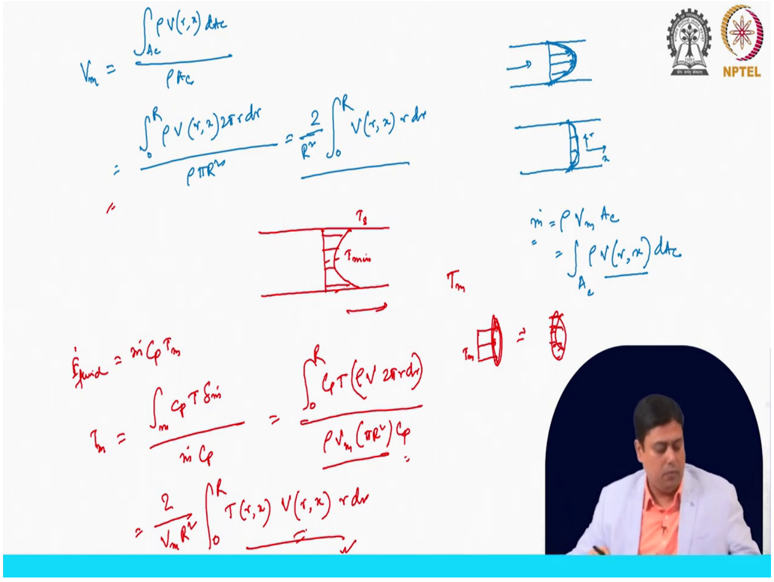
(Refer Slide Time: 00:43)



So, in the last couple of lectures, we have discussed about convection and particularly forced convection that too with external flow, where the boundary layers were growing indefinitely. But in case of internal flow, for example, pipe flow or tube flow or in a duct flow, what happens the boundary layer does not have a infinite length to grow or indefinitely grow. It cannot indefinitely grow.

So, what will happen in such cases, we will see in this class. And also, how in such flows we estimate the Nusselt number, the average Nusselt number, from there the average convection coefficient and then we can solve such related problem.

(Refer Slide Time: 01:51)



But before going into the details what we typically do, when we have a pipe flow. Usually in case of that we have seen in case of laminar flow, the velocity profile is parabolic in nature and in case of turbulent flow, this we know the fluid is the relatively blunt in shape, the profile.

Now, the point is when we talk about mean velocity or say the mean temperature at any cross section, what do we mean by that and how do we estimate such scenario? So, whenever we talk about say the fluid velocity, fluid viscosity in such pipe flow problem, we talk about the mean velocity or average velocity.

Now, by the conservation of mass principle, what happens, when we talk about mass flow it is essentially the V_{mean} and the cross-sectional area through which the if the fluid is flowing. Which is essentially if we look at it very closely what we see is that, it is nothing, but this V which is varying with respect to r and x. If we consider this is the radial direction and this is the x, which is a function of it and in a cross section of dA_c.

$$\dot{m} = \rho V_m A_c$$

$$= \int \rho V(r,x) dA_c$$

So, the mean velocity of any incompressible fluid, incompressible flow in a circular tube of radius r, once we replace all these conditions here what we typically get, the mean velocity is essentially then it becomes:

$$V_m = \frac{\oint \rho V(r,x) dA_c}{\rho A_c}$$

$$= \frac{\int_0^R \rho V(r,x) 2\pi r dr}{\rho \pi r^2} = \frac{2}{R^2} \int_0^R V(r,x) r dr$$

So, if the mass flow rate or the velocity profile that are known, then the mean velocity can be calculated from such expression.

Now, if the fluid that is flowing this is completely for the hydrodynamic part, now along with that if there is this is the heat transfer is happening; that means, the fluid is at a different temperature than the pipe wall temperature or the tube wall surface temperature.

So, if the fluid is heated or cooled whatever the situation would be and it flows through a tube, then the temperature of the fluid at any cross section would not be say a flat profile. There will be some gradient. Actually, there are say for example, if this is flowing, so the profile can look like, this is the surface temperature. So, it can have a minimum temperature at the centre.

Now, in that case, this is actually the thing is that is happening. So, and in that case the fluid temperature, the fluid is being heated, in this case the surface temperature is high, and this profile case it is mirror image on the other side if the fluid is being cold.

That means, at the centre it can have a maximum temperature like in the velocity profile, and the surface temperature would be at a lower our case. So, the point is that at this cross-sectional area, if we try to understand what is the T_{mean} that is the mean temperature at any cross section.

So, similar to this idea or analogous to the hydrodynamic part, if we look at the conservation of energy principle that has to be satisfied. That means, the energy transported by the fluid through a cross section in actual flow that must be equals to the energy that would be transported to the same cross section if the fluid were at a same temperature or a constant temperature of T_{mean}.

Which means through a cross section when the fluid is flowing, considering that it has a uniform temperature T mean, in this cross section the energy the amount of energy that it flows, it is equals to the same that is having a different profile, the same energy it would transport along the axial direction.

So, if we try to equate that then what happens the energy of the fluid that it transport is eventually the mass flow rate C_p and T_{mean}. C_p is the specific heat of the fluid. So, $E_{fluid} = \dot{m}c_pT_m$ at any cross section along the tube, it presents the energy flow that is happening with the fluid at a given cross section.

Now, if we try to find out the mean temperature of a fluid with constant density and specific heat, considering those are not changing in a circular pipe of again radius R. In that case the T_m, the mean temperature similar to that thing that we have done earlier is the small element of mass that is there.

$$T_m = \frac{\int c_p T \delta\dot{m}}{\dot{m}c_p}$$

For a radial cross section of our for circular tube, above equation becomes:

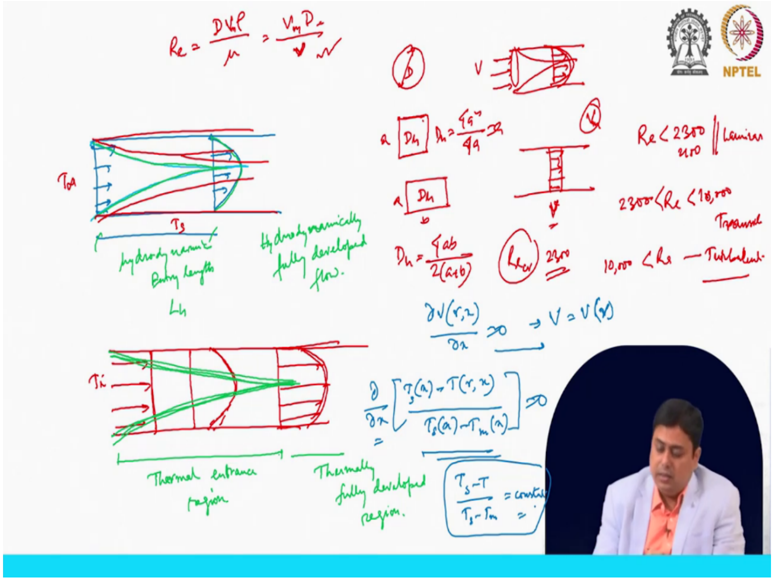
$$= \frac{\int c_p T(\rho V 2\pi r dr)}{\rho V_m(\pi r^2)c_p}$$

On simplifying this, what we get?

$$\frac{2}{V_m r^2}\int_0^R T(r,x)\,V(r,x) r dr$$

So, that is why it is essential that we at first find out the hydrodynamic part and then we go for the energy part. Because to solve the energy part we need the information of the velocity profile. And that is why fluid dynamics is extremely essential in the analysis of convection. So, by doing such a thing, we can find out what is the average or the mean velocities.

(Refer Slide Time: 11:21)



Now, for an internal flow we know how to calculate the Reynolds number. Again, we go back to the similar strategy that means for forced convection, what we have we saw? That, for any given problem or any given situation we found out what is the Reynolds number of the problem. What is the flow condition is it laminar or turbulent?

Once we identify that then we have chosen a appropriate or suitable correlations that helps us to estimate the Nusselt number or the average Nusselt number. So, similarly, when we try to find out the Reynolds number for internal flows, we know how to calculate the Reynolds number which is essentially $Re = \frac{DV\rho}{\mu}$.

And specifically, here we can say that mean velocity. Because now we know how to calculate the mean velocity because is essentially, we talk about say actually the profile is like this, the when the flow is happening. But while calculating $\frac{DV\rho}{\mu}$, we essentially take a single V which is the average velocity.

This is the actual scenario, but what during our calculation we assume that this is a kind of a flat velocity profile of V. Technically speaking, this is ideally V_m, the V_{mean} average velocity.

Now, since at the inlet we calculate this, we always assume that there is a flat velocity profile of V. But along the cross section, since the velocity changes due to no slip boundary condition and the development of boundary layers that we have seen in case of hydrodynamic boundary layer or say the velocity boundary layer. So, we have to calculate V_m, the V_{mean}, $V_{average}$ and the other properties are usually known. So, this further can be written as $\frac{DV}{\vartheta}$, the kinematic viscosity, where D is the diameter of the tube if it is a circular cross section.

So, for circular cross section, this is the diameter. If this is a duct of square cross section, then we have to find what is the hydraulic diameter. So, these things we knew already. I am not going into the details of calculating such things.

Similarly, for a duct flow of rectangular cross section, we also again have to calculate the hydraulic diameter in such case. If this is a and b, we know the hydraulic diameter is essentially $D_h = \frac{4ab}{2(a+b)}$; weighted cross section by weighted perimeter of the flow.

In this case D_h, for the circular for the square cross section $D_h = \frac{4a^2}{4a}$, which is essentially a. So, this D is replaced depending on the shape of the geometry through which the flow is happening. Accordingly, find out what is the Reynolds number of the problem.

Now, for such flow, internal flow we know also that if Reynolds number is less than 2300 or in some text book you would find 2100, the flow is called laminar. Anything in between 2300 and 10000, we call this is a transition region. And Reynolds number greater than 10000, we call this is turbulent fully turbulent.

$$Re < 2100/2300 \qquad - laminar$$

$$2300 < Re < 10000 \qquad - Transition$$

$$10000 < Re \qquad - turbulent$$

So, which means in this case, the critical Reynolds number for the internal pipe flow is 2300. This is what we have to at first understand or we have to check for the problem, and then we have to add out the suitable correlations that we will see in next couple of slides.

Now, it is also clear to us that there is a necessity of considering entrance length. Because when there is a flat velocity profile at the inlet, it takes the hydrodynamic entrance region in order to be a fully developed flow. Because till this position our boundary layer grows and it merges at this position. And after that we call the flow has to be fully developed.

This length that is necessary, we call the hydrodynamic entry length which is say L_h. Then, the flow becomes hydrodynamically fully developed. We say hydrodynamically fully developed flow. Now, imagine we also have a temperature difference between the surface or the pipe and the fluid temperature. So, this is T_s, now my fluid is at T_∞. Just like in the case of external flow. So, here also, there will be development of the thermal boundary layer along with the velocity boundary layer.

So, there also what we will find? That in this case the temperature initially, we can consider that this is what was coming in as a T_{inlet}. And then the temperature profile develops the temperature profile develops and as it grows eventually it becomes a smoother profile and becomes stable.

So, here also, from both the sides, the thermal boundary layer grows from the top surface as well as the bottom surface and it merges at a certain point. This portion analogous to the hydrodynamic entry length is called the thermal entrance region.

And then, it becomes thermally fully developed region. So, in case of hydrodynamic boundary layer, what was happening that:

$$\frac{\partial V(r,x)}{\partial x} = 0 \rightarrow V = V(r)$$

That means, velocity is then varying only in r directions after the fully developed region is attained.

Similarly, for the fully developed or thermally fully developed flow, the condition happens is that:

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right] = 0$$

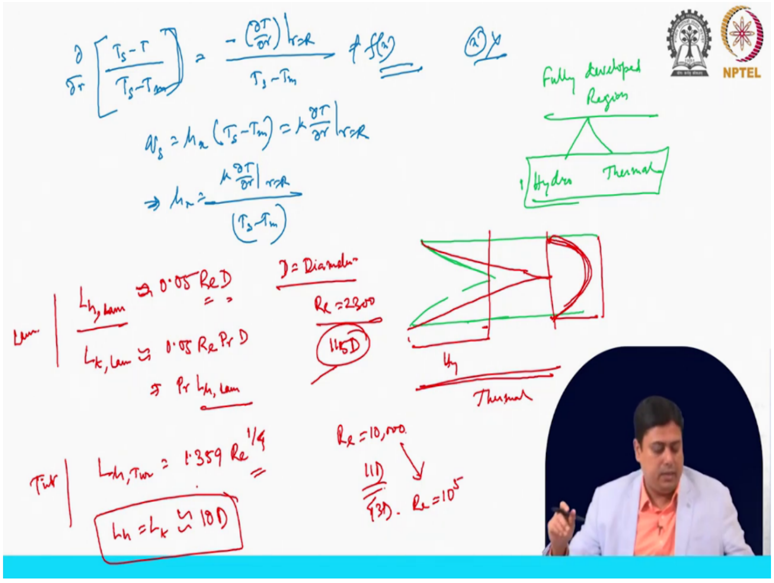
So, velocity profile remains unchanged in the hydrodynamically fully developed region. Friction factor also remains constant in that region consequently.

Now, similarly, the same argument goes for the thermally fully developed region. And what happens? The heat transfer coefficient in thermally fully developed region becomes a constant value. So, in thermal fully developed region, this derivative with respect to x is 0. So, which means this:

$$\frac{T_s - T}{T_s - T_m} = constant$$

or independent of x specifically, with respect to x this is constant value.

(Refer Slide Time: 22:22)



So, that means, here we can further write that this:

$$\frac{\partial}{\partial r}\left[\frac{T_s - T}{T_s - T_m}\right] = \frac{-\left.\frac{\partial T}{\partial r}\right|_{r=R}}{T_s - T_m} \neq f(x)$$

And surface heat flux that we can calculate:

$$q_s = h_x(T_s - T_m) = k\left.\frac{\partial T}{\partial r}\right|_{r=R}$$

$$h_x = \frac{\left.\frac{\partial T}{\partial r}\right|_{r=R}}{(T_s - T_m)}$$

So, what we see? That here this is independent of x; so, what we eventually conclude from here that in thermally fully developed region of a tube once it is attained, local convection heat transfer a coefficient or say the local convection coefficient is constant and does not vary with x. So, which means both friction factor and the convection coefficient remains constant in the fully developed region of a tube.

So, when now coming to the explanation of fully developed region is specifically when it attains both the condition that is fully developed, hydrodynamically it is fully developed as well as it is thermally fully developed. When both are fully developed. So, in a tube depending on the Prandtl number that we have understood, that whichever is thicker or thinner; the boundary layer thickness.

Once it is attained say if the fully developed region is this one hydrodynamically and this is thermal, after this the velocity profile does not change with x also the thermal profile does not change with x.

So, after this region, we typically say that the flow is now fully developed, if there is heat transfer is occurring. Otherwise, for simple hydrodynamic case, we also call fully developed region once it attains or it crosses the hydrodynamic entry region.

So, in convection part or in heat transfer part, if we call a flow as fully developed, it means it is hydrodynamically fully developed as well as thermally fully developed. Once both these conditions are attained, then we call that fluid as or that flow as fully developed flow.

Now, the value of the entry length, it has been seen that and also has been proposed that for laminar case the length that is required h stands for the hydrodynamic entry length, it is numerically close to 0.05ReD. Re is the Reynolds number, D for this is for particularly pipe flow problem. D is the diameter of the pipe or the tube.

$$L_{h,lam} = 0.05ReD$$

This, in case of laminar flow when t stands for the thermal entry length that is necessary for laminar flow, it is similarly in the same order, but it is associated with Prandtl number as well. So, which means this is essentially:

$$L_{k,laminar} \approx 0.05RePrD$$

So, whatever the hydrodynamic boundary layer entry or hydrodynamic entry length is necessary, that is multiplied by the Prandtl number of the flowing fluid or the flowing liquid that would give us the estimate of the length of thermal boundary entry length or thermal entry length in order to be a fully developed flow.

Now, in case of turbulent flow, now if we if we give some number here, what we see, say for example, Reynolds number is 20. If we multiplied by 20, what we see? That this length that is necessary for laminar flow is around a 1 diameter from the entry position.

Now, in the case, when it is the limiting one; that means, 2300. The hydrodynamic entry length is around (115D). And accordingly, the thermal boundary thermal entry length would be Prandtl number multiplied by this length for critical Reynolds number.

Now, this is for the laminar flow. In case of turbulent flow, similar relations are a similar propositions are there. And in that case the hydrodynamic for the turbulent flow entry length is:

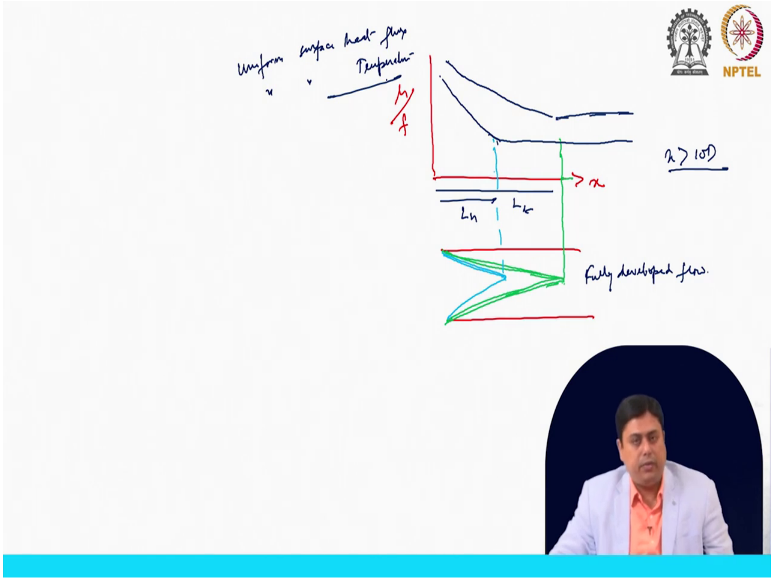
$$L_{h,turbulent} = 1.359Re^{\frac{1}{4}}$$

This is one of the relation that has been given by some scientist. So, hydrodynamic entry length is much shorter in turbulent flow, in order to be a fully developed turbulent flow. And it dependence on the Reynolds number is much weaker to the power 1 by 4. So, what we see that at Reynolds number 10,000 it is around 11D and it increases to say around 43 D if Re is 10^5. So, from 10,000 to 10^5 it increases from 11 D to 43 D.

Now, the point is that for practical purposes, for practical purposes it is generally agreed that the entrance effect are confined within the tube length to a certain length that is of 10 times of the diameter in case of turbulent flow and that does not change much of it.

And that is why in case of turbulent flow both L_h and L_t; that means, the hydrodynamic and thermal entry length are considered to be around 10D. So, what are the observation? What are the things that we can observe here? It is that the Nusselt number and the convection heat transfer coefficients as much higher that would be much higher in case of the entrance region.

(Refer Slide Time: 32:07)



So, if we look at this graph that could be more clear that when we have entry region the green one is for thermal and the blue one is for the velocity. And this corresponds to some position here and this corresponds to a position here on the graph where this is in the x direction.

If we plot h or f; that means, the convection coefficient or the friction factor, in case of thermal cases or the hydrodynamic case, what we observe is that h_x is essentially vary, and then becomes constant after the thermally developed region. And the friction factor it changes and then it becomes constant after this case. So, this is the L_h and this is our L_t. After this, it is fully developed flow.

So, in the entrance region, we see that the h is much higher than the rest of the portion. So, the Nusselt number and subsequently, the convection coefficient convection heat transfer coefficient are both higher in the entrance region. The Nusselt number reaches a constant value at a distance less than 10 times of diameter in case of say fully developed turbulent flow.

And the Nusselt number for uniform surface temperature and uniform surface with heat flux conditions are identical in the fully developed region, and they are nearly identical in the entrance region as well. So, Nusselt number is essentially insensitive to the type of thermal boundary condition and turbulent flow correlations can be used for either type of boundary conditions, in those cases.

So, there are some important conclusions that we make based on this understanding, which again let me reiterate, that the Nusselt number and the convection heat transfer coefficients are higher in the entrance region. Nusselt number reaches a constant value at a distance that is around 10 times of the diameter at a distance from the entry length, and where from where the flow can be considered as fully developed or assumed to be fully developed for $(x > 10D)$.

And Nusselt numbers for uniform surface temperature or uniform surface heat flux, in both the conditions are identical in fully developed region. And those are also nearly identical in the entrance region or in a sense the Nusselt number is insensitive to the thermal boundary conditions.

So, with this, I will stop here. And in the next class, we will see some general thermal analysis with those both conditions that is the uniform surface heat flux and uniform surface temperature cases, how we estimate the Nusselt number, With this I stop here. And I thank you for your attention. We will see you in the next class.