Chemical Engineering Fluid Dynamics and Heat Transfer
Prof. Rabibrata Mukherjee
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 05
Kinematics 05 - Shear Stress

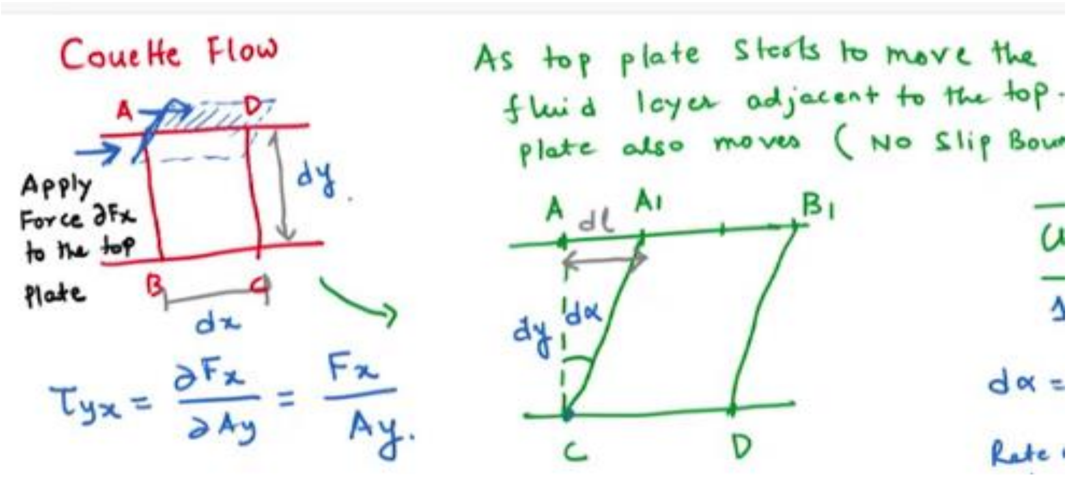
So, welcome back. In the previous lectures we have talked about the concept of substantial derivative like situations where non temporal acceleration and the conditions under which the fluid element can deform. We have quantified the total angular deformation acting on 2D fluid element resting on the xy plane and also discussed the rate of angular deformation. And based on that we are going to build some concepts on shear stress in this lecture.

Shear stresses are the stress that is caused by the forces acting over the surfaces of the fluid element. There are two types of stresses: Normal and shear stresses. The forces acting directly or the normal to the face of the control volume or fluid element leads to normal stresses. The forces acting in all other directions other than the normal direction leads to shear stresses.

whether τ_{xy} and τ_{xy} are same or not…?

From the basics you knew that τ_{yx} signifies x momentum transferred in y direction. τ_{xy} represents y momentum transferred in x direction. Now you can see that τ_{xy} and τ_{xy} are different.

Couette flow: Consider fluid element ABCD confined between two stationary parallel plates initially. Then we apply force dF_x to the top plate in the x direction, it causes the top plate to start to move. Based on no slip boundary condition, as the top plate starts to move, the fluid layer adjacent to the top plate also moves. As a consequence, only the top layers will move initially. Point A will move to A_1 and the B to B_1, the point C and D will not move from the initial position.



The value of u changes only in y direction. So here we can consider that u is a function of y only and it is a 1D flow field. Consider that you are applying force dF_x over the area dA_y.

So, basically the shear stress is the force applied divided by the cross-sectional area.so we can write,

$$\tau_{yx} = \frac{dF_x}{dA_y}$$

Consider dl is the length by which the fluid has moved when we applied the force. And force F_x has caused a velocity in the top plate as denoted by du over time dt as the point moves from A to A_1.

Therefore, AA_1 = du. dt = dl

If the angular deformation be the $d\alpha$ and the rate of angular deformation will be $\frac{d\alpha}{dt}$.

From the geometry, then we can write tan $d\alpha = \frac{dl}{dy}$

Since dα is small, here we are taking the approximation that **tan $\boldsymbol{d\alpha = d\alpha}$**.

Then the $d\alpha = \frac{dl}{dy}$

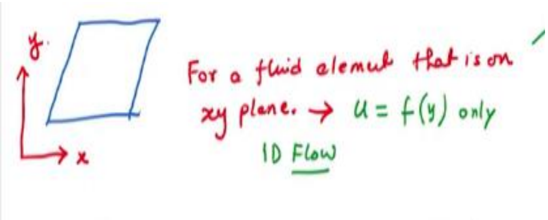
We know that, $dl = du.dt$

Therefore,$d\alpha = \frac{du.dt}{dy}$: $\frac{d\alpha}{dt} = \frac{du}{dy} = \dot{\alpha}$;rate of angular deformation.

We have considered that fore dF_x is applied over the top plate and the force has resulted into shear stress τ_{yx} .The angular deformation $\dot{\alpha}$ has been caused due to the shear stress τ_{yx} that has been generated due to application of force dF_x.

Therefore, we can say that $\tau_{yx} = f(\dot{\alpha}) = f\left(\frac{d\alpha}{dt}\right) = f(\frac{du}{dy})$

You knew th at for in the case of Newtonian fluid, the relation between the angular deformation and applied shear stress is linear, and proportionality constant is the coefficient of viscosity.so the equation becomes, $\tau_{yx} = \mu\ (\frac{du}{dy})$

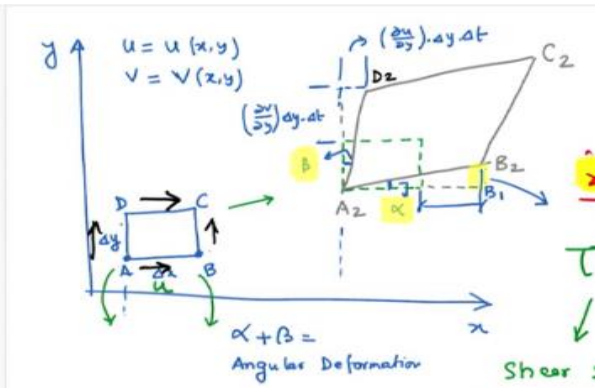


The example of Couette flow is for a 2D fluid element that is on xy plane with the condition that u is a function of y only. So, the above relation is valid for this specific example and the nature of the flow is 1D flow.

If you have a fluid element between two vertical plates (Couette flow),so we can write

$$\tau_{yx} = f(\frac{dv}{dx})$$

In the next example, consider 2D fluid element in xy plane.



Over a xy plane,

the total angular deformation of a 2D fluid element, $\dot{\gamma_{xy}} = \dot{\alpha} + \dot{\beta} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$

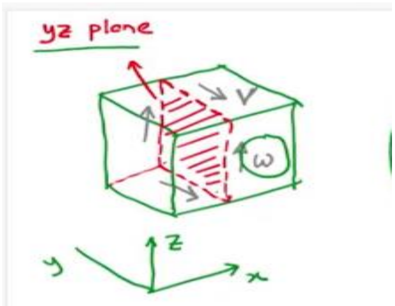
But, in the case of 2D flow field, total shear stress experienced by the fluid element on xy plane or at constant z will be the

$$\tau_{xy} = f(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$$

The total shear stress has two components. One component is due to the u varies as function of y and v varies as function of x.

Next, we are going to discuss the shear stress experience by a cuboidal fluid element in 3D flow field.

And we know that,τ_{yz}: It is the stress experienced by yz plane of the fluid element. We are considering the yz plane of the fluid element is the plane one which is passing through the centroid of this fluid element. Velocities are acting along the edges of the plane contributes to the total shear stress acting over the corresponding plane. For this particular plane the two velocity components that are active along the sides, that are w along the z direction, and v along the y direction causes the deformation of the yz plane. So, the total shear stress will be the sum of variation was function of y and variation of v as a function of y.

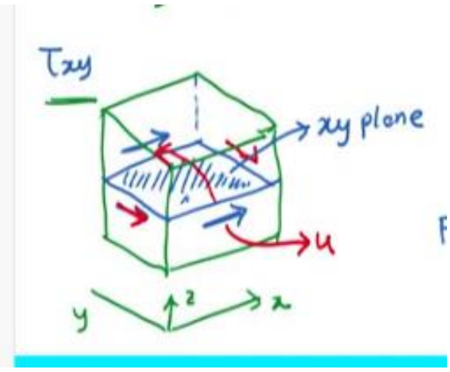


$$\tau_{yz} = f(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z})$$

Similarly, if we are considering xy plane, the deformation of xy plane is due to the variation of u along the y and variation of v along the x.so therefore,

$$\tau_{xy} = f(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})$$

This value τ_{xy} represents the total deformation caused to xy plane due to the variation of velocities u and v along the edges of the xy plane at particular value of z.



In the case of Newtonian fluid, the functionality becomes linear, and it becomes.

$$\tau_{xy} = \mu(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})$$

But in the case of 3D flow field τ_{xy} it is the total deformation caused to an xy plane at a particular z, due to the variation of x component velocity in the y direction, i.e. $\frac{\partial u}{\partial y}$ and the y component velocity in the x direction, i.e. $\frac{\partial v}{\partial x}$.

If we are looking into the terms, the $\frac{\partial u}{\partial y}$ captures the x momentum transferred in y direction and the term $\frac{\partial v}{\partial x}$ captures y momentum transferred in x direction. The total stress experienced at a particular plane is a combination of these two terms.

Thank you.