Chemical Engineering Fluid Dynamics and Heat Transfer

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Lecture - 49
Forced Convection (Contd.)

Hello everyone, welcome back once again with another lecture on Forced Convection in the part heat transfer of the NPTEL online certification course on Chemical Engineering Fluid Dynamics and Heat Transfer.

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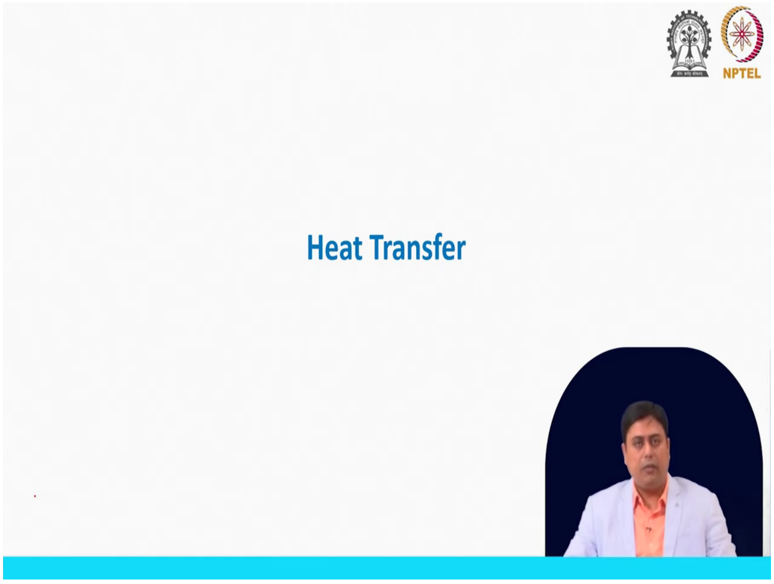

So, till the last class what we have seen? The fundamentals of convection several equations related to the governing equations of that convection heat transfer the convective flow the analogies between the thermal boundary layer and the velocity boundary layer and also what we have seen is that in convection we always need the information of the drag force or the drag coefficient because that directly influences the Nusselt number.

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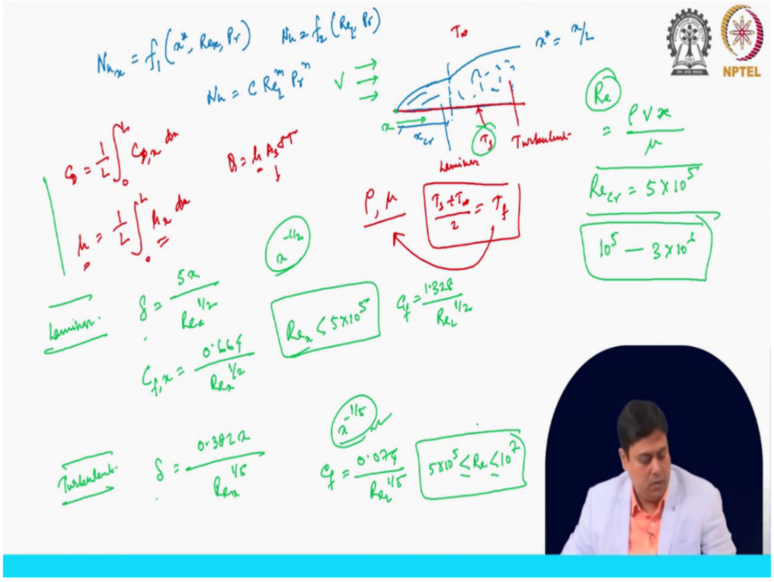

So, we have seen the form of local Nusselt number and that is a function of we have seen this form $Nu_x = f(x^*, Re_L, Pr)$. So, this is the local Nusselt number it is dependent on the dimensionless characteristic length for a flat plate it is the distance from the leading edge this x^* non-dimensional length scale it is done if the flat plate length is L. We have seen this non-dimensionalization it is also dependent on the Reynolds number as well as the Prandtl number.

Now, this Reynolds number is also the local Reynolds number depending on the length the length scale or the distance from the leading edge in case of a flat plate scenario. Also, we have seen when we integrate it over the entire domain the average Nusselt number it is a function $Nu = f(Re_L, Pr)$.

Now, experimentally it has been determined that the average Nusselt number usually vary in a simple power law function this form also we have seen, ($Nu = CRe_L^m Pr^n$) and this is the complete length over the entire domain. Now, the point is this value of C is geometry or the domain dependent constant the value of L, m and n differs depending on the flow scenario or the flow condition and this values of m and n varies in between 0 to 1.

The point is now we also mentioned that when this scenario this see the boundary layer develops, we know that depending on the upstream velocity there exist a critical length till which the flow is laminar and after that there is a chaotic motion of the or the random motion of the flow that is your eddies.

So, this is the laminar part and this is the turbulent part after a critical length x_{cr} for a isothermal condition this surface of the temperature is T_s. Now, the point is that this free stream temperature and T_s if there these are significantly different which is the common cases mostly occurred cases and at the same time the fluid properties are changing in that duration in that range.

In that case while calculating the dimensionless numbers or any other quantity the derived parameter which temperature we should look into or say the properties if it is say density viscosity these are dependent on the temperature, then which value we should take? Because on one side we have T_s on other hand we have T_∞ and these are drastically different.

So, in those cases we have also discussed that we typically consider film temperature which: is

$$T_f = \frac{T_s + T_\infty}{2}$$

we name this as the film temperature. So, in the film temperature we consider whatever the properties are there or the values are there for this particular fluid and accordingly we calculate all other derived parameter. Now, the drag coefficient when we take the average drag coefficient value what we did? See from local to the average value the way that it is typically calculated is:

$$C_D = \frac{1}{L}\int_0^L C_{D,x}$$

Similarly, h is also calculated on the same line:

$$h = \frac{1}{L}\int_0^L h_x dx$$

Once you have an expression of h_x you integrate it over the domain and find out the average heat transfer coefficient and once this average drag and convection coefficients are known, then we can calculate the rate of heat transfer between the bodies from this simple relation which is ($hA\Delta T$), this h value comes from this integrations of the average quantity and A or A s this is the surface area across which the heat transfer is happening.

Now, when there is this kind of flow that flow over a flat plate or specifically if we say that parallel flow over flat plates in parallel to the orientation of the plate. In this case as I mentioned earlier, we are not going into the details that there are several regions several layers turbulent, laminar flow conditions etcetera that comes from the fluid dynamics understanding.

So, the transition from laminar to turbulent depends on the surface quality or the surface geometry surface roughness upstream velocity it also depends on the magnitude of the upstream velocity the surface temperature as well and of course, the type of the fluid where it would change.

Now, since all the parameters are involved, it is best described as we typically do in terms of Reynolds number. So, this Reynolds number tells which is that we write:

$$Re = \frac{\rho V x}{\mu}$$

V is the upstream velocity and others are the fluid property and x is the distance from the leading edge. What is has been seen that also we have discussed is Reynolds critical for such cases is ($Re_{cr} = 5 \times 10^5$). This is the value of critical Reynolds number for a flat plate.

Now, this value may differ because this value is from the assumptions that the plate is smooth if the plate is rough depending on the degree of roughness this value can change from (10^5 to 3×10^6) it may vary in this range depending on the surface roughness and the turbulence level of the free stream. Now, we know what are the drag coefficient values for such cases of the friction coefficient values.

So, for laminar condition for laminar condition, we can know that this boundary layer thickness is essentially:

$$\delta = \frac{5x}{Re^{\frac{1}{2}}}$$

And friction coefficient the local friction coefficient is:

$$C_{f,x} = \frac{0.664}{Re_x^{\frac{1}{2}}}$$

where the Reynolds number is in the laminar region for this condition.

Now, corresponding turbulent values the velocity boundary layer thickness is:

$$\delta = \frac{0.382x}{Re_x^{\frac{1}{5}}}$$

so look at the variation. For turbulent flow the boundary layer thickness varies with the ($x^{-1/5}$). And in the case of laminar flow, it varies with respect to $x^{-1/2}$ where x is the distance from the leading edge, the distance from the leading edge. So, friction local friction coefficient is proportional to the $Re^{-0.5}$ for laminar flow.

So, that means, the friction coefficient is nearly infinite at the leading edge that is ($x = 0$) and decreases by a factor of $x^{-1/5}$ for turbulent cases and $x^{-0.5}$ in case of laminar cases. So, that means, the local friction coefficients are higher in turbulent flow than that of in the laminar flow. These things all we are aware these are the hydrodynamic points that we are telling here.

Now, once we integrate it over the entire domain in this case in laminar case the friction coefficient value becomes. So, these numbers you have to remember $Re^{0.5}$ where this condition remains. In this case the friction coefficient becomes once we integrate it like we have mentioned here it becomes 1/5 where Re is eventually in this range this is the relations. So, these are the relations over the entire plate.

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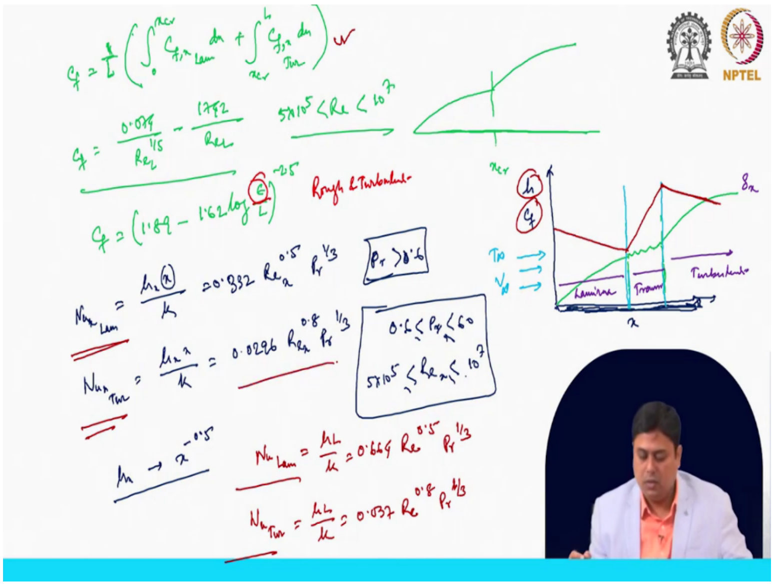

So, now if we look at the cases where initially at the flat plate, I mean say the flat plate is sufficiently long like we discussed there, so some part it is laminar boundary layer and some part we have turbulent boundary layer. It is not that the complete length of this plate is covered by one of the boundary layers. Say the scenario is something hybrid in that case what we typically do?

We find the value of C_f or the friction coefficient for the entire length by piecewise integration:

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x\,laminar} dx + \int_{x_{cr}}^{L} C_{f,x\,turbulent} dx\right)$$

We replace those previous expressions here and we find out the complete expression and it appears that expression looks like something in this form:

$$C_f = \frac{0.074}{Re_L^{\frac{1}{5}}} - \frac{1742}{Re_L}$$

For the range ($5 \times 10^{-5} < Re < 10^7$). Average friction coefficient over the entire plate is then determined in case of such hybrid function where the presence of laminar boundary layer section cannot be neglected. In such case we use this kind of an expression.

Now, again as I mentioned that these are derived or these are estimate these estimates the friction coefficient for the smooth plate. Now, for the rough surfaces this expression is modified something like this:

$$C_f = \left(1.89 - 1.62 log\frac{\epsilon}{L}\right)^{-2.5}$$

is the coefficient with the roughness factor which is epsilon here this roughness factor.

So, rough surface and turbulence and turbulent flow condition. So, these are the hydrodynamic part. Now, coming to the heat transfer coefficient part for laminar flow it has been seen that the expression for local Nusselt number in case of laminar flow is:

$$Nu_{x,laminar} = \frac{h_x x}{k} = 0.332 Re_x^{0.5} Pr^{\frac{1}{3}} \qquad [Pr > 0.6]$$

And the corresponding Nusselt number for turbulent is 0.0296.

$$Nu_{x,turbulent} = \frac{h_x x}{k} = 0.0296 Re_x^{0.8} Pr^{\frac{1}{3}} \qquad [0.6 \leq Pr \leq 60]$$

$$[5 \times 10^5 \leq Re \leq 10^7]$$

So, what we see that h_x is proportional to the $Re_x^{0.5}$ in case of laminar flow and in case of turbulent flow $Re_x^{0.8}$.

So, for laminar flow h_x the local transfer coefficient the convection heat transfer coefficient that means varies with ($x^{-0.5}$) . So, here we have ($x^{0.5}$) when it is divided by another x it becomes proportional that h_x is then proportional to x by this relation $h_x \sim x^{-0.5}$.

So, which means the h_x similar now we can see the analogy like in the case of hydrodynamic boundary layer the friction coefficient was infinite near the or at the leading edge where ($x = 0$). Similarly, the convection heat transfer coefficient local convection heat transfer coefficient is infinite at the leading edge.

And then it takes a value or decreases along the length of the path and it decreases gradually with a slope of (-0.5). So, schematically if I try to show you the trend. So, if this is my x the plate plate length is like here this is the plate and the value of h and C_f together the flat velocity profile of the upstream velocity profile is there which is at T_∞ & V_∞ in order to clearly differentiate.

Now, this what happens? We have seen that. So, here let us further introduce another region that instead of directly going from its have a laminar section and then there is a turbulent section and then we have a turbulent boundary layer. So, this is the part till which we have laminar and from here the turbulent boundary condition starts. So, till this part I have laminar this is transition and here we have turbulent boundary condition.

Turbulent boundary layer and this is my say δ_x the thickness that varies along the x direction. Now, how the profile would look like h and C_f that is the heat convection heat transfer coefficient, convection coefficient and the friction factor it happens like this it is just a schematic. So, it decreases like this and then in the transition region it reaches a maximum value that is at the onset of turbulence and then again it decreases as with x with a different slope.

This is for h and C_f both local friction and heat transfer coefficient for flow over flat plate that is isothermal the plate is isothermal. So, if we have understood this thing and then what happens? The natural question comes ok these are the local values. So, what about the average properties?

So, say Nusselt number for laminar case now instead of local value we are integrating over the entire plate or till the length where this laminar boundary layer exist which is:

$$Nu_{laminar} = \frac{hL}{k} = 0.664 Re^{0.5} Pr^{\frac{1}{3}}$$

In case of the turbulent flow this takes a form:

$$Nu_{turbulent} = \frac{hL}{k} = 0.037 Re^{0.8} Pr^{\frac{1}{3}}$$

these are the average Nusselt number over the entire plate when the plate is having a uniform temperature and constant temperature.

Similar to the process that we have followed in case of hybrid condition that if the plate is sufficiently long where both laminar and the turbulent boundary layer must be considered that laminar part cannot be neglected. In those case similar to this idea, we also integrate it over the entire domain in a piecewise function.

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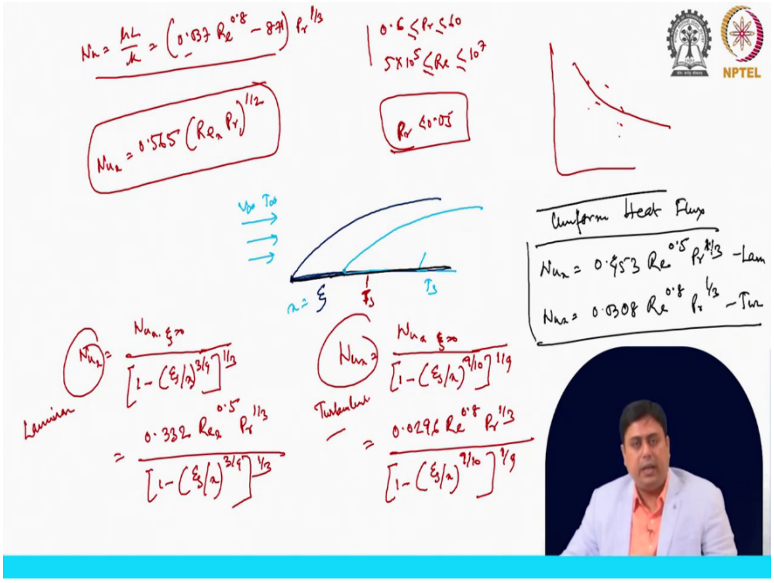

In that case again a form that appears Nusselt number:

$$Nu_{turbulent} = \frac{hL}{k} = (0.037Re^{0.8} - 871)Pr^{\frac{1}{3}}$$

$$[0.6 \le Pr \le 60]$$

$$[5 \times 10^5 \le Re \le 10^7]$$

Now, this constant values that we have seen in both the equations would be different if the critical Reynolds number is different because this is specifically for the flat plate with a smooth surface and the plate is isothermal.

Now, in case of liquid metals liquid metals why we are specifying liquid metals? Because liquid metals have very high thermal conductivity and Prandtl number in those cases are significantly lower than 0.6 values ($Pr < 0.05$). So, what kind of relation then you could be applied here to find the heat transfer coefficient for such fluids?

In those cases, it has been seen that the local Nusselt number follows this kind of a relation:

$$Nu_L = 0.565(Re.Pr)^{\frac{1}{2}}$$

because in those cases it has very small Prandtl number. So, the thermal boundary layer develops much faster than the hydrodynamic or the velocity boundary layer.

So, if we assume that the velocity in the thermal boundary layer is constant that is of the free stream value then we solve the energy equation and find this expression. Now, there these are if you look at this each expression has their limitation or the operating range in terms of the Reynolds number, Prandtl number etcetera. Now, the point is it is desirable to have a generic expression that applies for all the fluids all the conditions.

Now, those are critical those are at an advanced stage and based on the curve fitting phenomena; that means they do several experiments with different fluids they find the best feed condition or the best feed curve and propose some relation. So, those things we need not remember those things are there in the reference book textbook whenever we need for design purpose we go into that reference or particularly in that topic and we choose the relevant expression to find out the value in such cases.

But these expressions within this operating limit they fairly do well in this case. The other thing that can happen in case of the thermal boundary layer , we have some unheated portion at the beginning. So, this ξ length that we have is unheated portion. So, velocity boundary layer will develop from here, but the thermal boundary layer will develop from the where the temperature is different in such way because this is the T_s, we have the free stream values.

Now, in such cases so, if we have a flat plate whose heated section is maintained at a constant temperature from a distance ($x = \xi$). Now, in those cases the Nusselt number for both laminar and turbulent they also have a different just correction that we have seen from the earlier cases. So, in those cases this is what happens?

$$Nu_x = \frac{Nu_{x,\xi=0}}{\left[1 - \left(\left(\frac{\xi}{x}\right)\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}}$$

$$= \frac{0.332 Re_x^{0.5} Pr^{\frac{1}{3}}}{\left[1 - \left(\left(\frac{\xi}{x}\right)\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}}$$

Above equation valid for the laminar case.

For the turbulent case similarly, the similar expression is there, but the denominator changes:

$$Nu_x = \frac{Nu_{x,\xi=0}}{\left[1 - \left(\left(\frac{\xi}{x}\right)\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}}$$

$$= \frac{0.0296 Re_x^{0.8} Pr^{\frac{1}{3}}}{\left[1 - \left(\left(\frac{\xi}{x}\right)\right)^{\frac{9}{10}}\right]^{\frac{1}{9}}}$$

Above equation is valid for the turbulent part. Because at ($\xi = 0$) this eventually both the relations falls back to our previous expressions. The other part the final part in this lecture is that. So, this all the things we are talking about T_s is constant the temperature of the surface is constant.

When the flat plate is not having a constant surface temperature, but we have uniform or say constant uniform heat flux condition. In those case the Nusselt number relation is for laminar flow:

$$Nu_x = 0.453 Re^{0.5} Pr^{\frac{8}{3}}$$

$$Nu_x = 0.0308 Re^{0.8} Pr^{\frac{1}{3}}$$

Second equation valid for turbulent flow. This is when we have uniform heat flux condition instead of a uniform temperature. So, what we see in this lecture is that the value of C, m and n from the generic expression.

As I mentioned these are system dependent as well as the flow condition dependent. Depending on how the boundary conditions what is the boundary condition or how the flow operating condition these parameters changes, but the format remains almost similar.

So, with this concept I stop here there are several things you have to remember, there are several expressions that I have not mentioned here need not be because those can be referred as and when required. So, we will see one or two problems in the next class related to utilization of these expressions.

Till then thank you for your attention.