**Chemical Engineering Fluid Dynamics and Heat Transfer**
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**Lecture - 47**
**Forced Convection (Contd.)**

Hello and welcome to the another lecture on Forced Convection in Chemical Engineering Fluid Dynamics and Heat Transfer. in the last class we were discussing about the boundary layer that is the velocity boundary layer and the thermal boundary layer, their co-current development. Now, we have seen that the velocity boundary layer develops when a fluid flow over a flat plate.

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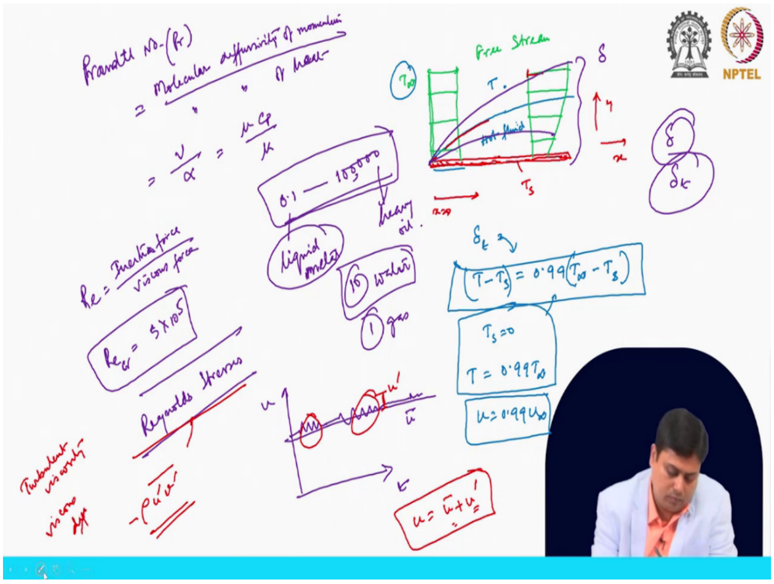


Now, at the same time when we have; so, we consider the similar situation like we have seen in case of velocity boundary layer. In that case we have seen that when a fluid approaches a flat plate there exist a boundary layer. Now, similarly when we have a hot fluid compared to its temperature is higher compared to the temperature of the solid plate, then what will happen a thermal boundary layer also will develop.

Now, if we consider in this case that there is a uniformly the, there is a plate of uniform or isothermal plate is there uniform temperature over which a fluid is flowing. The fluid particles that or the fluid layer that is adjacent to the surface of the temperature that is of the plate $T_S$, it will exchange energy until and unless these temperatures become similar or same.

Now, at sufficiently far from the surface in the y direction sufficiently far from the surface, what will happen there will be a temperature variations that is different than this fluid that is plate surface temperature or the surface temperature. So, like the velocity boundary layer there exist a thickness of this thermal boundary layer. Initially there we have the uniform temperature that is of the free stream, but then there exist a temperature gradient as we move from this (x = 0) position from the leading edge upstream to the downstream position.

Now, the thickness of this thermal boundary layer if we term that is as $\delta_t$, t stands for the thermal boundary layer, then this actually is defined. Because, in velocity boundary layer you have realized that when the velocity across this boundary layer becomes 99 percent of the free stream velocity that area was demarcated as the velocity boundary layer thickness.

Now, similarly here analogous to that when the difference in temperature; that means ($T-T_S$); here the temperature is T, this is $T_S$:

$$(T - T_s) = 0.99(T_\infty - T_s)$$

So, this temperature difference when it becomes equals to the 99 percent of the temperature difference between the $(T_\infty - T_s)$ that portion or that point is demarcated as the thermal boundary layer thickness.

So, which means in case of ($T_s = 0$),

$$T = 0.99T_\infty$$

at the outer edge of the thermal boundary layer thickness. Now, this is analogous to our

$$u = 0.99u_\infty$$

that is in the velocity boundary layer case. But this is a very special case when ($T_s = 0$),. Otherwise, the same analogy is there, but in terms of temperature difference that at any point $(T - T_s)$ this difference. If this difference is 99 percent of the temperature difference of the $T_\infty$ and the $T_s$ that line or that point is demarcated as the thermal boundary layer thickness and that varies as we go downstream or far away from the leading edge.

Now, the point is that along with this there is also velocity boundary layer, there must exist velocity boundary layer. So, this velocity boundary layer for example, is having a thickness say simply as $\delta$ if we mention it. Now, which one would be thicker and which one would dominant $\delta$ or $\delta_t$, what is the criteria to understand, what is the mechanism to understand that which one would be thicker or why not this line would go like this.

To understand this, the relative thickness of velocity and thermal boundary layer we again here define another dimensionless number calling the Prandtl number. And this is defined as molecular diffusivity of momentum divided by molecular diffusivity of heat, the competition between these two which we further write as:

$$Prandtl\ No.(Pr) = \frac{Molecular\ diffusivity\ of\ momentum}{Molecular\ diffusivity\ of\ heat} = \frac{\vartheta}{\alpha} = \frac{\mu c_p}{k}$$

Prandtl number which tells us that which one is dominant in such scenario that is and this is this comes after the name of the scientist Ludwig Prandtl. He introduced this concept of boundary layer you have already heard this name in the fluid dynamics part. So, here what happens Prandtl number of fluid it ranges from (0.1 – 100,000) huge variation; 0.1 for say liquid metal case this extreme is typical happens for the heavy oil case.

Now, Prandtl number just for a sake of reference is around 10 for water in the order of 10, and it is for the gas it is nearly 1, for number of gases. Now, what it means? It means for gaseous phases molecular diffusivity of momentum and molecular diffusivity of heat these are happening at a same rate.

If $Prandtl\ No.(Pr) = 1$, the thickness of velocity boundary layer and the thickness of thermal boundary layer overlap's. If $Prandtl\ No.(Pr) < 1$ say for example, in case of liquid metal or if it is very high; that means, for if it is less than 1 if this value you can see; molecular diffusivity of heat is or the rate of its transfer is much higher than the momentum. So, the heat dissipates quickly in such cases and the thickness the $\delta_t$ is much thicker than the velocity boundary layer.

But on the other case when it is heavy oil or Prandtl number is very high then the heat that is dissipated is on a very slower rate than the velocity or the momentum. If $Prandtl\ No.(Pr) > 1$, then in that case the heat diffuses very slowly compared to the momentum. And in that case the boundary layer is much thinner for the heavy oils, the thermal boundary layer is much thinner for the heavy oils relative to the velocity boundary layer for such fluids.

Now, we have been already aware about the different layers, sub layers in velocity boundary layer that is the laminar sub layer, turbulent region, buffer layer etcetera. Now, the point is that and also we have understood the concept of Reynolds number (Re) i.e.

$$Re = \frac{Inertia\ force}{Viscous\ force}$$

There exist critical Reynolds number and we have seen that for a flat plate this critical Reynolds number ($Re_{cr}$) where the transition from laminar to turbulence happens is around ($5 \times 10^{-5}$) for a flow over a flat plate. Similarly, there exist several critical Reynolds number depending on the flow scenario or the flow condition. Now, when this happens, from laminar to turbulent there exist eddies in turbulent flow or the fluctuations.

Now, also I hope you have come across the term that is called the Reynolds stresses; these Reynold stresses are typically explained in the fluid dynamics. So, Reynold stresses are the stresses resulting from these eddies, because whenever there is turbulence and if you look at the turbulent velocity profile. We see a fluctuations random fluctuations of the velocity around the mean value which is say if you if I say it is $\bar{u}$, with time and u on the y-axis there exist fluctuations or fluctuating components.

These fluctuating components around the mean value are the eddy velocities, we designate that as prime. So, eventually ($u = \bar{u} + u'$), these are the fluctuating component and this is the mean value. So, when we look into each and every velocity component that is the x, y and z direction velocity component that is u, v and w. And when we put that in the momentum equation, if you have already come across the Reynolds stresses it is fine you understand while i am going.

This Reynolds stresses is eventually are the terms that comes with this form, on the right hand side of the momentum equation, these quantities are non-zero and these contributes to the Reynolds stresses. But we will not go into the details there exists a few terms which we must remember is that the turbulent viscosity and the viscous dissipation viscous dissipation and turbulence viscosity.

So, again these are the terms I am highlighting here that you should be remembering from the fluid dynamics part or you may have to go to this term because we will not elaborate this here.

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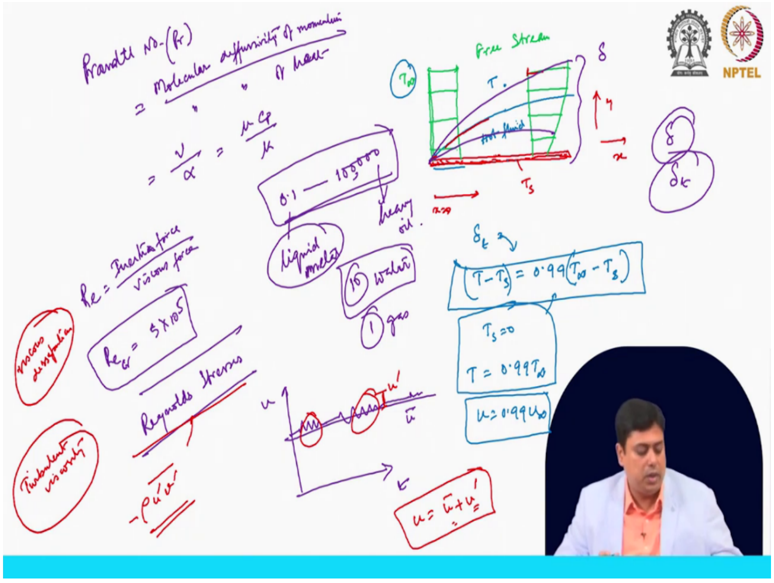


The point that here we now should understand that based on this generic concept, we will try to derive or we will try to see the convection equations, although again we will not go into the derivations of those equations. But conservation of mass equation and conservation of momentum equation where it will lead and what are the forms of those, because those are important in order to understand several concept.

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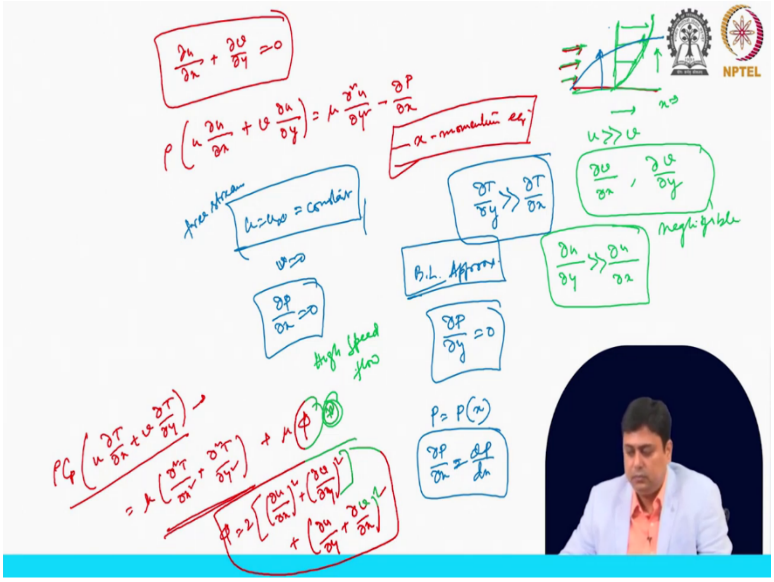


For example, that conservation of mass equation for a differential element, it would lead to a form which is in a two dimension if i write is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

The conservation of mass relation which is also known as the continuity equation. Similarly, the conservation of momentum equation would lead to a form which would look like:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial p}{\partial x}$$

in the x-direction, we call this as the x-momentum equation or conservation of momentum in x direction. Similarly, for all the directions we can find out the form.

Now, there are few things that we assume in the boundary layer to simplify the situation. For example, the velocity component in the flow direction is usually much higher or larger than in the normal direction. If the flow is happening over a flat surface, the velocity in the flow direction and this is the normal direction. The velocity in the flow direction is usually much higher than the normal direction; so, what we consider $(u \gg v)$.

So, $\frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ these are negligible, also what happens u it varies greatly with y. There is a profile of u that varies greatly with in the y direction from 0 at the surface to the free stream value. However, that u along the x is typically small, the variation of u along x is usually small. So, which means what we again further consider that $\frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}$ ; this is another assumption that we make.

This you have possibly seen in the hydrodynamic boundary condition, hydrodynamic boundary layer equation development, governing equation development. Now, along with these two, when there is thermal boundary layer; so, heat conduction will primarily happen in the direction normal to the surface quite obvious. So, which means here further we are considering; so, further we are considering that $\left(\frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x}\right)$ this analogous assumption like the velocity boundary layer.

So, which means the velocity and temperature gradients normal to the surface are much greater than that of along the surface, these simplifications we call as the boundary layer approximation. And now, if we somehow can neglect the influence of gravity or if we consider there is no influence of gravity in certain cases and there is no other body forces and the boundary layer assumptions are valid.

Then what will happen? The y-momentum equation would simply look like $\frac{\partial P}{\partial y} = 0$. Which means the variation of pressure in the direction normal to the surface is negligible; the variation of pressure normal to the surface is negligible which means P = P(x) and it means

$$\frac{\partial P}{\partial x} = \frac{dP}{dx}$$

Now, the velocity component in the free stream region what we see that $(u = u_\infty)$, which is constant in the free stream region.

And at the same time what we have seen that if there is a flow over a flat plate in such cases, and if we consider that (v = 0), then this x-momentum equation gives that $\frac{\partial P}{\partial x} = 0$. So, which means for a flow over a flat plate, pressure remains constant over the entire flat plate both inside and outside boundary layer.

However, this is a restrictive condition, where we are considering several assumptions that the boundary layer approximation is valid in such cases. And this analysis is done at the outside of the boundary layer that is in the free stream region. So, inside and outside this boundary layer, our pressure remains constant through the entire plate.

And along with that when we look into the energy equation, the energy equation follows an expression which is:

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \varphi$$

in two dimension with negligible shear stress and the constant properties, the form that we see in this case is like this. Now, this tells that the energy convected by the fluid out of the control volume is equals to the net energy transferred into the control volume by conduction, this is the convective part, this is the diffusive part.

The net energy convicted out of the system is equals to the net energy transferred into the system by conduction. Now, here what we have considered is that the viscous shear stresses are negligible. If it is not negligible, then this expression is further modified with the term viscous dissipation function $\varphi$. And this $\varphi$ has an expression which is just for your information it is quite lengthy and we need not require here, but still, it is for your information something like this.

$$\varphi = 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right]$$

Now, viscous dissipation is important, this is important in certain cases, particularly high speed flow. When the viscosity is very high; for example, the flow of oil in the bearings journal bearings. So, then it actually has a significant role in increasing the fluid temperature due to the conversion of kinetic energy to the thermal energy by viscous dissipation.

So, in case of high viscous flow and high speed flow, both should occur simultaneously or the cases should be analogous, that high speed and highly viscous flow in those cases viscous dissipation cannot be neglected.

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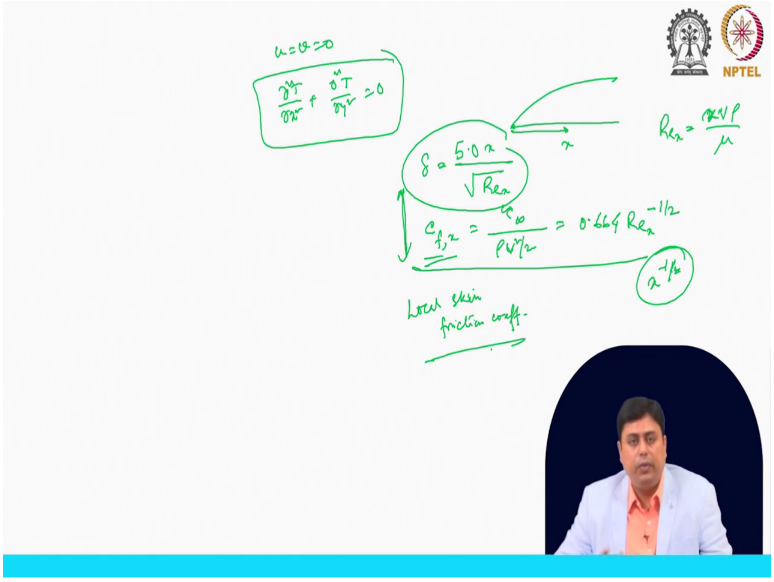

Now, for stationary fluid when (u = v = 0), the energy equation eventually if you look at this expression, it reduces to only this part, because u = 0 and v = 0. In that case, it becomes a simply heat diffusion equations that we have studied earlier, two dimensional heat conduction equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Now, so, what we have seen? We have seen continuity equation, momentum equation and energy equation.

Now, if we have to solve this for the boundary layer, we need appropriate boundary condition. Now, with the appropriate boundary condition and say for a specific system that is flow over a flat plate. If we solve it and if we try to find out what is the value of $\delta$ after rigorous derivations, what we will see that the velocity boundary layer thickness.

In this case, which I hope by now you have seen this format or in this form that:

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

where, this x is the distance from the leading edge of a flat plate. And the local friction coefficient friction coefficient at any given x which is this value, it comes out to be:

$$c_{f,x} = \frac{\tau_{\infty}}{\rho u^2/2} = 0.664 Re_x^{-\frac{1}{2}}$$

So, what happens here and this relation, what it tells that the wall shear stress and the friction coefficient decreases along the plate with ($x^{-0.5}$); those who have forgotten is x is the distance from the leading edge, v is the velocity, free stream velocity, $\rho$, the density of the fluid, $\mu$ is the viscosity of the fluid. So, these two expression tells the variation of $\delta$ that is the velocity boundary layer and the local skin friction, this is the local skin friction coefficient. Similarly, in the next class we will see what happens with the thermal boundary layer, because we are now looking into this in parallel and we are trying to have the analogies.

So, this is why this initial couple of classes we are devoting to find or to refresh our memory on the hydrodynamic boundary layer, what are the variations, what is the governing equations. Along with that we have introduced the energy equations to have the relevance in the heat transfer part. And we are now looking into the solution for or specific case, which is the flow over a flat plate.

So, we will come back to this in the next class with the other relations related to the thermal boundary layer.

Till then thank you for your attention.