Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 44
Transient Heat Conduction (Contd.)

Hello and welcome back once again with another lecture on Transient Heat Conduction in the online NPTEL course Chemical Engineering Fluid Dynamics and Heat Transfer.

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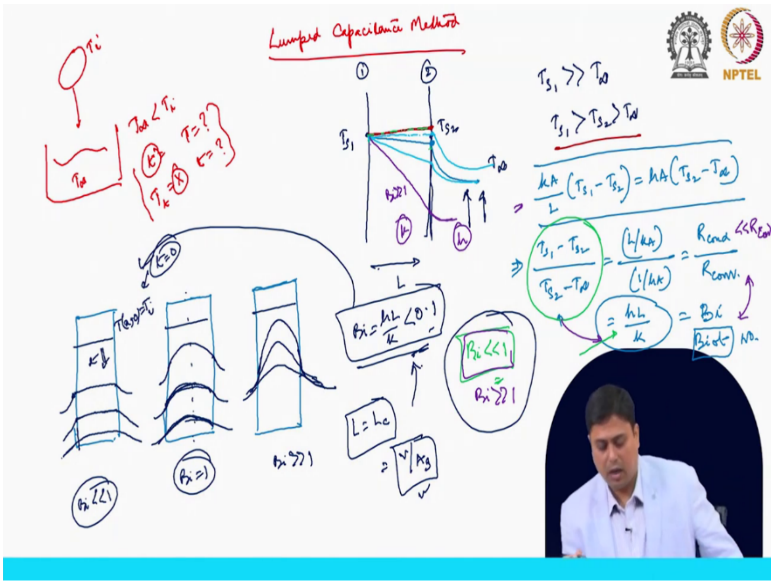

In the last class what we discussed is about the lumped capacitance method. There we developed or we derived one expression which can be applied to calculate either of the two that if the θ or the temperature instead of θ let me write it as a temperature.

Say an object is there initially at a temperature T_i. There is a liquid pool in which it is dipped, the liquid pool temperature is T_∞. If $T_\infty < T_i$, the hot or object would cool down on quenching. If the question is that at time, t what would be the temperature? We have seen that expression or the other thing can be that now T is given at a particular time, but it is asked that what is that time at which it would reach a certain temperature.

Both of these have been addressed in the last lecture. That how we have derived that expression and how we can use it. And we have seen how simple it is to understand or to implement. Now, for this reason this lumped capacitance method is extremely popular in transient heat conduction, but the point is that it is not valid for every situation the application of such method or application of such procedure.

Because one of the basic assumption there is that the temperature inside the object is not spatially varying or there is no temperature distribution inside the object, but when that is the case or how logical is that assumption. As I mentioned if you look at the Fourier's law that assumption requires thermal conductivity to be extremely high or say infinite value. And then only if we consider say a slab or a wall say this wall, we kept the temperature T_{S1}. This wall temperature would be something different if this side there is a flowing fluid at a temperature T_∞. Now, if the condition is that $T_{S1} > T_\infty$ then the temperature on this side if this is my wall 1 and this is the width of a wall certain L and this is the other side which is the demarcated by line 2. So,

$$T_{S1} > T_{S2} > T_\infty$$

Now, in lumped capacitance analysis what we are assuming is that this temperature is not varying. So that means, T_{S1} and T_{S2} there is no temperature variation this is the assumption.

Now, this is the real fact. So, the point is that the profile in absence of any thermal energy generation would be something similar to like this. At this point the T_{S2} would be there and from here there will be the other variations due to convection. This kind of thing we have already understood. So, if we look at it and try to have the energy balance the surface energy balance. The thing that we can write for steady state condition.

$$\frac{kA}{L}(T_{S1} - T_{S2}) = hA(T_{S2} - T_\infty)$$

That a steady state condition surface energy balance can be written as this in this case. Which means the way we can if we rearrange this one:

$$\frac{T_{S1} - T_{S2}}{T_{S2} - T_\infty} = \frac{\left(\frac{L}{kA}\right)}{\left(\frac{1}{hA}\right)} = \frac{R_{cond}}{R_{conv}}$$

If we further simplify this eliminating A from both the sides what we have $\left(\frac{hl}{k}\right)$. This is a dimensionless number and is called Biot number.

$$Biot\ no. = \left(\frac{hl}{k}\right)$$

This has significant influence on the understanding of this assumption that we made in lumped capacitance analysis. So, here what happens it involves surface convection as well as conduction inside the body. Now, if we look at it this Biot number provides an understanding of the temperature drop. If we look at the latent side. So, this Biot number is essentially is related with the temperature drop. In between the body T_{S1} to T_{S2} and T_{S2} to T_∞ the steep of the flowing fluid nearby.

Now, what we are assuming here if we want this kind of profile to be assume. This red line the strain there is no variation; that means, (T_{S1} -T_{S2}) or this Biot number has to be much much lesser than 1 ($Bi \ll 1$). If this condition somehow, we attain then what will happen the numerator that means in order to maintain this criteria what will happen this difference inside the temperature difference inside the solid body must be much lesser than the outside temperature difference; that means, its outer surface to the flowing fluid. And that can happen if you adjust the material properties accordingly or if that can be adjusted.

So, if Biot number is much much lesser than 1 then this temperature profile variation can be very minimal. Almost no variation we can consider and that is the logic or the assumption that we made in the lumped capacitance analysis. A slide variation would be there in comparison with what is there at the outside.

If Biot number is 1 ($Bi = 1$) it would have a same slope that is with the outside. If Biot number is much higher than 1 or much greater than 1 ($Bi \gg 1$) then this profile would be more steeper.

The temperature gradient inside the material would be more steeper than it is at the outside. These are the things we simply can understand from this relation. This temperature drop clear. So, the importance of Biot number. So, Biot number is much lesser than 1 this criteria means the resistance to conduction if we look at this definition or this relation. Biot number is much lesser than 1 means resistance to conduction is much lesser than the resistance to convection.

And if it is much smaller it is perfectly to assume that the temperature gradient inside the solid media is negligible compared to what is at the outside. So, Biot number has tremendous influence of the significance in case of transient heat conduction. So, where again this h is the flowing fluid k is for the material that we have seen from this expression and L is the width of the material or the characteristic length of the material. So, how it influence the transient behavior. Let me draw a quick representation of that understanding. So, say I have three slabs or. So, initially in all the three cases this is at $T(x, 0) = T_i$; that means, it is at a certain x and time is equals to 0. Three materials having three different Biot number. In one case the ($Bi \ll 1$), in other case ($Bi = 1$) and in this case ($Bi \gg 1$).

So, in this case with time if time now increases the profile that would look like from here is that it's a nearly flat profile inside the media and then there is a temperature drop at the outside. With time again the flat profile and then it further drops. And eventually the temperature would be as the time grows the temperature of the body and the fluid would be similar. This is the case when ($Bi \ll 1$)due to this scenario.

But when ($Bi = 1$) practically what would happen if this is my plane of symmetry and it is. So, in this case the other assumption was that both sides are identically being heated or cold. Here in this case, it is cold there is a plane of symmetry. So, in this case with time there will be a temperature profile which is symmetric in nature and then there will be the variations of this at the outside for convection whatever happens.

Again, with time the value of this plateau eventually would decrease with the outside and then at this case it is something like this. This is a schematic. So, it gets flatter and flatter, but there will be a significant distribution of temperature inside the media when ($Bi = 1$)

compared to ($Bi \ll 1$) that is a way flatter profile. Almost uniform inside the domain. And in the other case when ($Bi \gg 1$). the temperature profile would look like something much more steeper inside the domain and it would eventually be with time it would change like this.

So, I hope you have understood the importance of Biot number. In this case of transient heat conduction. Now, the point is if we try to apply lumped capacitance method then; that means, the first point that immediately should come to your mind or we should be doing is that to check whether the value of Biot number for the given problem is way lesser than 1 or not. But again, much lesser than 1 we must have a number. How much is the much lesser than 1?

Practically it has been seen that if the Biot number which is $\frac{hl}{k} < 0.1$ then such assumptions are nearly. The maximum value we can consider for Biot number is near about 0.1, If it is anything beyond this you can still apply lumped capacitance method to analyse this amount of time it requires or the temperature it would attain at a certain time. But it would give you a gross result. Some inaccurate result the results with much more error.

The accuracy improves if it falls below this 0.1 value. If it is 0.001 it is more accurate if it is 0.0001 it is further accurate the prediction should be much further accurate. So, this is the limit till which we can consider or we can apply lumped capacitance method to solve a problem. So, now if this is the L in this case we have defined as the width of this wall or this slab.

Usually, this L is called the characteristic length or typically say L_c width name. This L_c is typically defined as the: $L_c = \frac{V}{AS}$. The volume divided by the surface area. This is how we generally calculate what would be the characteristic length of an arbitrary shaped body. Because in practice or in practical cases the bodies may not be of perfectly regular shape like the spherical object, cuboid, hexagon and this kind of thing.

Whatever the shape is we can measure its volume we can measure its surface area and find out what can be the characteristic length to check whether the Biot number is less than 0.1 for that problem or not. Now, if that is the case then what happens?

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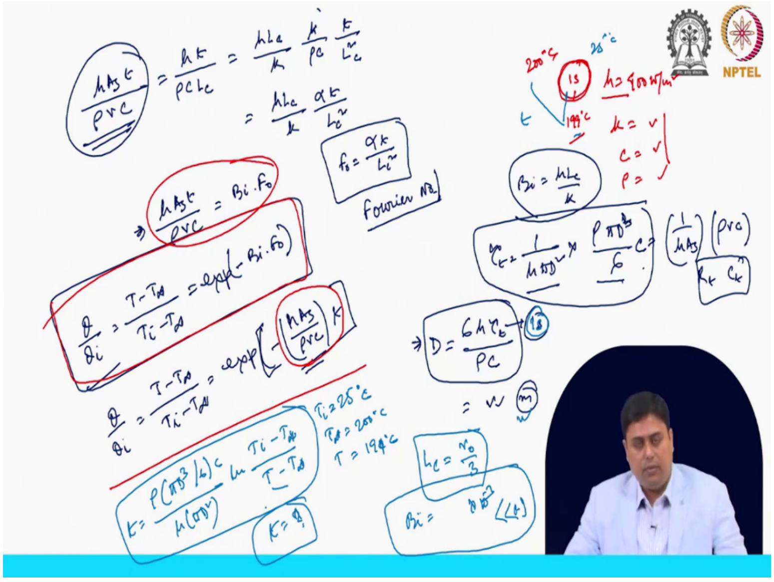

So, this h that the expression that we have written in that earlier case which is:

$$\frac{hA_st}{\rho VC} = \frac{ht}{\rho CL_c}$$

And again, which we can rearranged as:

$$= \frac{hL_c}{k}\frac{k}{\rho C}\frac{t}{L_c^2}$$

Just to find out some non-dimensional numbers in this parameter that we have seen in the lumped capacitance analysis. So, what we see here this is eventually:

$$= \frac{hL_c}{k}\frac{\propto t}{L_c^2}$$

or that we can write above as:

$$\frac{hA_st}{\rho VC} = Bi.Fo$$

Where Fourier number, $Fo = \frac{\propto t}{L_c^2}$. So, Fourier number is essentially an dimensionless time that in combination with Biot number characterizes the transient conduction problem.

$$\propto = \frac{k}{\rho C_p}$$

Above represents the thermal diffusivity value. So, eventually what it becomes the:

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-Bi.Fo\right)$$

So, earlier we had seen this expression and now once we replace it what we find this $\frac{\theta}{\theta_i}$ expression. Because earlier what we found in the previous class if you noted we had this expression. Now, this parameter is essentially this expression. So, what we have seen here that we have now represented the same expression that we have done earlier. Now in terms of the non-dimensional numbers, Biot number and Fourier number.

So, the point is that in certain cases now it is more generic, because it is now, we know that how to calculate the characteristic length of an arbitrary shaped body once we find out we check the validity of lump capacitance method. If it is within the limit that is less than 0.1, we apply lumped capacitance method to analyse transient heat conduction. That is that how much heat can be transferred or how much heat has been transferred by conduction and what is the time what is the profile and etcetera that we can find out.

So, based on this thing if we try to solve a problem that we have a thermocouple junction. Say for example, that is say we consider is the sphere in shape. Now, that thermocouple junction say it has a convection coefficient the surface and the flowing the environment it has a h = 400 W/m^{2}K. The all-material properties which is thermal conductivity, specific heat, density all the values are known to us.

The question is we have to determine the diameter of this junction. That can have a time constant of 1 second. If the dimension junction is at 25 ^{0}C and is placed in a gas stream that is at 200 ^{0}C. So, the second question is that how much time then it takes to reach 199 ^{0}C?

So, initially the junction is at 25 ^{0}C placed in a 200 ^{0}C environment all the material properties are known the flowing fluid or the environment and the solid surface convection heat transfer coefficient is known.

The question is what should be the diameter of this junction to attain a time constant of 1 second and the second part is that how much time it would take for the junction to reach 199 ºC? So, for this what we should do? So, the first is that we have to find out what is the junction diameter. Now, the point that we immediately assume that the junction is at uniform temperature at any point in time or at any instant that means we are going towards the lumped capacitance method.

Since the diameter is not known we cannot check or we cannot calculate basically the Biot number, because this Biot number requires the definition of L_C or the characteristics length. If you do not know the diameter you do not know the volume or the surface area.

So, the assumption is that we are still applying lumped capacitance method we find out the diameter and then recheck whether the assumption was valid or not by recalculating or calculating the Biot number after finding out the diameter.

So, here the time constant is essentially:

$$\tau_t = \frac{1}{h\pi d^2} \times \frac{\rho\pi D^3}{6} = \left(\frac{1}{hA_s}\right)(\rho V C)$$

. So, by rearranging what we can find that diameter D:

$$D = \frac{6h\tau_t}{\rho C}$$

So, we can easily find out what is the diameter should be of this to make, because $\tau_t = 1\ sec$ that is given. Once we find out D we can calculate L_c by the expression that is in this case for a spherical object the $L_c = \frac{r_0}{3}$. If you use that previous formula, you would get it.

So, once it is done, we find out what is the Biot number and we will see that in this case Biot number is in the order of 10^{-3} which is definitely much less than 0.1. So, our assumption of lumped capacitance would be valid. If we now try to find out how much time it would take to reach from 25 ºC to 199 ºC.

Then we apply our previous expression that we have seen and that would be something like that it is:

$$t = \frac{\rho\left(\frac{\pi D^3}{6}\right)c}{h(\pi D^2)} \ln\frac{T_i - T_\infty}{T - T_\infty}$$

We use this expression we use all the numerical values here and we find out how much time it would take to reach from T_i which is 25 ^oC. So, $T_\infty = 200\ ^0C$ is 200 degree centigrade and T = 199 ^oC.

This T is given and accordingly we find out how much time it would take to reach 199 ^oC. So, I hope this example is clear and accordingly the application of lumped capacitance methods and its applicability. In the next class we will see if the lumped capacitance analysis is not valid how we do the solution. Till then.

Thank you for your attention.