**Chemical Engineering Fluid Dynamics and Heat Transfer**
**Prof. Arnab Atta**
**Department of Chemical Engineering**
**Indian Institute of Technology, Kharagpur**

**Lecture - 43**
**Transient Heat Conduction**

Hello, everyone. Welcome back once again with another lecture on Heat Conduction in the course Chemical Engineering Fluid Dynamics and Heat Transfer.

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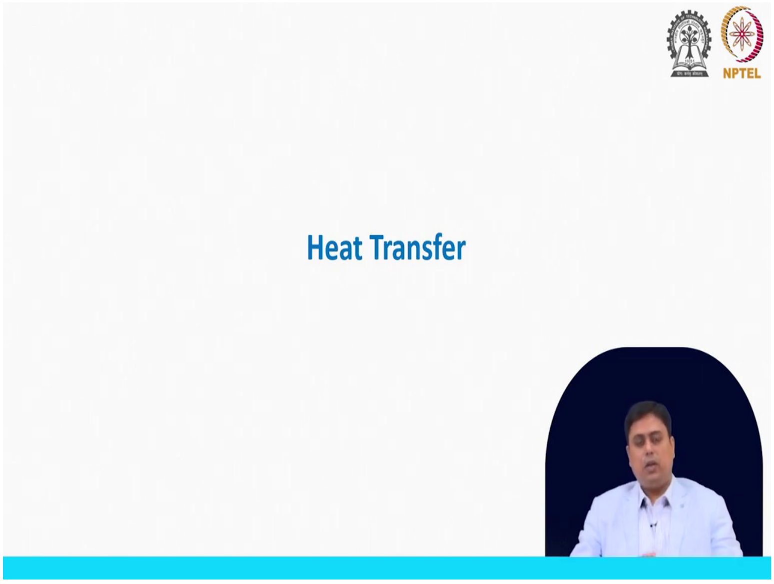


This lecture onward we will discuss about the Transient Heat Conduction. So, whatever we have discussed till now, slowly we are complicating the system. Initially, we started with a simple one-dimensional steady state heat conduction without any thermal energy generation. We saw it is with some example, couple of problems we have tried to do that.

And, then what we did we incorporated thermal energy generation. With thermal energy generation what can be its implication, we have seen it and further today we will now introduce the transient variation of heat conduction. So, now till this before this lecture whatever we discussed, we considered that there is no change in temperature with respect to time.

In transient heat conduction which is more natural that you see in everyday life that say for example, your cup of tea or coffee in this season or winter season gets colder or cooler with time if you leave it outside. So, another simple example you take a metal ball heat it and you then quench it a pool of liquid. The ball temperature would eventually try to cooler with the liquid pool temperature with time.

So, that means, along with the temperature profile that we have seen in case of steady state there is a spatial temperature distribution. Now, along with that spatial temperature distribution there is a change of spatial temperature distribution with time that we did not consider in a last in the last lectures.

So, now as we understand this is now a bit more complex that the temperature distribution is varying in space as well as the temperature distribution is varying with time.

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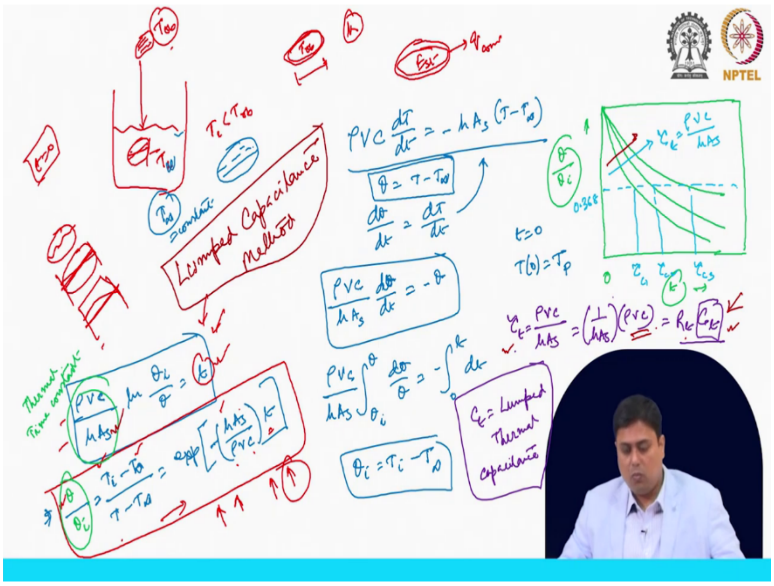


So, to start with as the example that I mentioned the example that I told you that we have a pool of liquid and an object an object that is at a higher temperature $T_{object}$ and this is the $T_{liquid}$. So, $T_L < T_{object}$, once it is dipped into the pool of liquid and if we start counting the time from them that once it is dipped at that point if we consider T is equals to 0 the temperature of this object would also change with time.

Now, to start with what we will consider this complex scenario is that the temperature of this body is uniform which means we are neglecting on the first hand that there is any spatial variation of temperature inside the body that we have seen earlier. In steady state what we determine basically is the temperature variation inside the body it can be linear if there are two surfaces at two different temperature or it can be a parabolic or a having different profile if there is a thermal energy generation inside the domain.

But, now for the time being what we consider for the sake of simplicity that inside the body there is no temperature distribution. So, that means, a body if I say the temperature is $T_{object}$ that means, everywhere inside the body the temperature is fixed or constant. Now, this is difficult to apprehend because technically speaking a body will have its own thermal conductivity value the value K.

Now, until and unless this K is infinite is very very high there will be resistance inside the body to heat conduction which led to the temperature distribution inside the body specially because of the Fourier's law if you remember. Now, the situation is that once we drop that body here the mode of heat transfer that is happening is essentially the conduction inside the body as well as the heat transfer by convection at the solid-liquid interface or the surface.

Now, this assumption of considering thermal conductivity is very high or the resistance to heat conduction is very small compared to the resistance that is outside which is for the convection would be reasonable if this contrast is huge or say contrast is large. In that case we can consider that the resistance inside the body is much lesser than the resistance outside the body with the liquid.

So, it is a comparative analysis based on which we are assuming that the body can have an uniform temperature if the resistance to conduction is much lesser than the resistance for convection. We will see that analysis in details here. So, for the sake of simplicity what we are considering that the body is of a uniform temperature. Now, if this is the case then what happens, the amount of energy that is being dissipated is actually gained by or it is being distributed or dissipated by convection.

So, now if we consider a control volume of this object say this object that we are considering if we consider a control volume what is happening is that the energy that is stored in this object because we consider that this temperature is to be higher than the liquid pool temperature. So, the energy that it contains is essentially dissipated by the $q_{convection}$ ($E_{stored} = q_{convection}$). So, the energy stored is essentially the energy that is being dissipated by the convective mode of heat transfer or the convection heat transfer.

So, in that case what we can write. So, essentially what happens here there is no input of energy to the object because we are not considering what is happening before (t=0). We are setting (t=0) when the body is dipped or immersed in the liquid pool. So, at that point what my energy balance would be the energy balance would be that the energy that is stored is eventually being dissipated by convection.

So, what we would write here is that the energy that is being dissipated and the energy that is stored, the energy stored what we can write in this case it is the:

$$\rho V C\left(\frac{dT}{d\tau}\right) = -hA_s(T - T_\infty)$$

So, this becomes my energy balance, the overall energy balance for the system considering again that the object is having a uniform temperature there is no special gradient of temperature inside the body. So that means, we are eliminating the consideration of conduction inside the material.

Now, so, in this case if we try to simplify this and we define a variable θ which is: $\theta = (T - T_\infty)$ Then what we can write?

$$\frac{d\theta}{dt} = \frac{dT}{dt}$$

And, if we consider by this process this immersion the liquid pool is significantly large that this temperature $T_\infty$ is not changed and this remains constant.

Then what we can write we can write:

$$\frac{\rho V C}{hA_s}\frac{d\theta}{dt} = -\theta$$

Now, if we try to solve this by separating variable and integrating it from the initial condition which is at (t = 0) our temperature was $T(0)=T_P$ objective or say if I say this is the say the $T_P$ which is the particle in this case we can consider.

So, then what happens the material property we are considering these are constant values are not changing. So, from here what we get:

$$\frac{\rho VC}{hA_s}\int_{\theta_i}^{\theta}\frac{d\theta}{\theta} = -\int_0^t dt$$

$$\theta_i = T_i - T_\infty$$

So, we considered we started with this theta. So, at T at time T is 0 at the initial case temperature we assume that this becomes the $\theta_i$ because this is the $\theta$, $T_i$ of the object.

Now, if we try to integrate it if we do the integration and solve for it the expression that we get is:

$$\frac{\rho VC}{hA_s}\ln\frac{\theta_i}{\theta} = t$$

or this can further be written in a different form that:

$$\frac{\theta_i}{\theta} = \frac{T_i - T_\infty}{T - T_\infty} = \exp\left(-\frac{hA_s}{\rho Vc}\tau\right)$$

So, both the forms we can write it for this solution. So, this expression tells that we can estimate a time if a desired temperature is set. That means, if we try to find out that initially the temperature was 40 °C at which time the temperature would drop to 30 °C or 20 °C if the cooling fluid temperature is 10 °C. The time we can estimate from this expression directly if we replace all the numerical values, provided these material properties are known which are typically given.

The other question can be for the same problem that if it is there for this much amount of time or this much time it spends on the liquid pool what would be its temperature at that time that we can find out from this expression. In that case on the right hand side we place the temperature the time at which time we require the value of $\theta_i$ and the temperature T we can evaluate from this expression.

So, for that we in both the cases we needed the information of what is the initial temperature of the object which is $T_i$, the liquid temperature or the quenching fluid temperature for this example, the fluid convection heat transfer coefficient, the surface area of the object, the density of the object, volume of the object and the heat capacity of the object.

These are mostly the fluid and material property. If it is asked that at a given time what would be the temperature we can use this expression and if it is asked that at which time it would attain this much temperature, then we can use this expression. Is just the rearrangement of the same expression.

Now, that means, in this case what we see that the solid and fluid temperature eventually would decay with time; with time if we see that this temperature of the solid must decay exponentially as T approaches 0. It is apparent from this expression. As T approaches infinity this temperature of the object and the fluid temperature must decay. It goes to eventually 0 at an infinite time.

Now, this actually can be shown by schematic. So, this is $\frac{\theta}{\theta_i}$. So, if I plot or if we try to plot $\frac{\theta}{\theta_i}$ on y-axis and t in the x-axis then what happens the curves would look-like something like this. Now, it could look like that this quantity that we have which is $\frac{\rho VC}{hA_s}$ in this from this expression it looks like that this is a quantity that can be defined as thermal time constant thermal time constant.

It is an analogous to your resistor capacitor diagram where we had the time constant value. So, which means the time constant in this case is something that we draw here and this schematic and this value is 0.368 the time constant that it takes to reach the 36.8 percentage of the original value. So, this would be your time constant say $\tau_{c1}$ this is $\tau_{c2}$ this is the $\tau_{c3}$ this kind of value would be there and this is the time constant which is $\frac{\rho VC}{hA_s}$.

Apparently this

$$\tau_c = \frac{\rho VC}{hA_s} = \left(\frac{1}{hA_s}\right)(\rho VC) = R_t C_t$$

So, it is the resistance to convection and this is a capacitor part. Basically, this $C_t$ is called here in the thermal analysis is the lumped thermal capacitance.

So, what we see here? We are representing the time constant as the resistor capacitor analogous to the voltage circuit diagram, voltage current diagram and this $C_t$ here in case of thermal analysis this is the lumped thermal capacitance and this whole analysis for which the basic assumption was that the body does not have any temperature distribution inside it, this is called the lumped capacitance analysis.

We say this method has lumped capacitance method for calculating the time it takes to reach a certain temperature or if a given temperature is there how much time it takes to reach that temperature such kind of analysis. So, it is called the lumped capacitance method and this is the lumped parameter that we have here.

So, what we see that any increase in R resistance or this capacitance would lead to a larger time constant value as you see from this graph. Which means, that material would respond slowly to any change in the temperature because once R or C any of these increases the time constant value increases and the material responds slowly to any change that that would occur in that varies with time.

So, this resistance if it increases, it actually causes a significant temperature drop because the material is less responsive or the fluid and the solid interface is less responsive to the temperature change because once it is dipped a certain high temperature is being dipped or is dipped already in the liquid pool and it is cooled by the other liquid.

Or this capacitance that lumped thermal capacitance which is this part, that is the $(\rho V C)$, the heat capacitance part, physically it implies that this is higher means the heat capacity of the material is higher. It can store higher amount of thermal energy. So, the dissipation would be lesser for that material.

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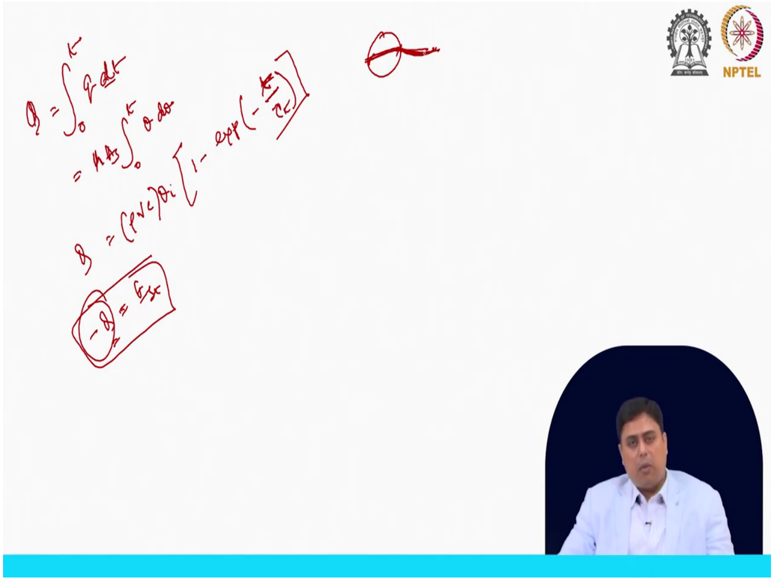


So, if we understood this then eventually the question becomes so, how much heat is being transferred what is the q that we are looking for, the amount of heat being transferred. Now, here instead of q we write it as Q because this amount of heat is being transferred from 0 to time t, it is a transient operation now. It is not at a particular time.

So, from time is 0 to the time you are calculating how much energy or how much heat has been transferred if this is the question what we do? We integrate:

$$Q = \int_0^t q dt$$

$$= hA_s \int_0^t \theta d\theta$$

and if we replace the values of $\theta$ here and calculate it the amount the expression that would be:

$$Q = (\rho VC)\theta_i \left[1 - exp\left(-\frac{t}{\tau_t}\right)\right]$$

So, what we see this quantity Q is essentially related to the change in the internal energy of the solid and this Q for this example is essentially is the energy storage (-Q) is essentially the energy storage because here it is quenching in this example.

If similarly if a cooler object or a cold object is dipped into a hot fluid, the sequence would be that Q would be in that case is positive and the analysis remains same. So, this part the whole this actually the Q in that case when the colder fluid is dipped in there in that case the Q is essentially the negative value and this whole expression becomes positive in that case. But, for quenching Q is positive. The solid is experiencing a decrease in energy.

So, what we have seen in this case that by simplest or to make it simplified the assumption that we made is that there is low temperature gradient inside the solid body because the resistance inside it the assumption is that the resistance to conduction is much lesser than the temperature that is the variations that is there in the outside that the resistance to convection is much larger than this resistance to conduction.

Based on that the process that we adopted is called the lumped capacitance method. We can see that it is extremely easy when it comes for the implementation in case of transient case. But, that means, we have to analyze in which situation or what are the situations that these assumptions are logical or valid. So, in the next class, what we will discuss is about the validity of this assumption this lumped capacitance method. And when it is appropriate to apply such method to analyze the transient heat conduction.

So, till then, I would ask you to go through this material that what we have seen here, what is time constant, rehearse your memory or refresh your memory with your voltage circuit diagram resistance circuit diagram. It is analogous and you would quickly recollect this discussions there the relevant discussions there and how it is similar for the thermal energy case.

With this I will stop here today, and we will be back in the next class for analyzing that when lumped capacitance method is applicable and we will see an example of that.

Till then, thank you for your attention.