Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 39
One-dimensional Heat Conduction (Contd.)

Hello everyone. Welcome back once again with another lecture on heat transfer in Chemical Engineering Fluid Dynamics and Heat Transfer. We were discussing about One-dimensional Heat Conduction without any generation term and steady state condition.

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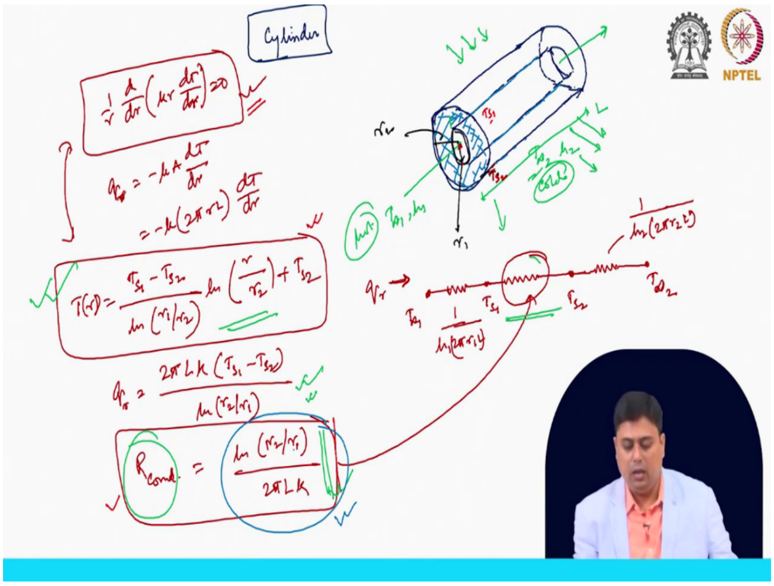

So, here in this part we in the last lecture what we discussed about the cylindrical coordinate system and the one-dimensional heat conduction equation without any heat generation part. So, this slide has been shown to you in the last class where what we did we solved one-dimensional heat conduction or the heat diffusion equation under steady state and without any heat generation or the sink term.

So, from there with the analogy of circuit resistance circuit the thermal resistance that we have got for this case this cylindrical coordinate is that this resistance that, $\ln\frac{r_2}{r_1}$ considering the fact if you look at the schematic once again is that r_1 was the inner radius of those cylindrical coordinate.

And the cylindrical system and r_2 was the outer radius where this blue marked region is the portion where heat conduction is happening and through the inner tube a hot fluid was flowing or we considered the hot fluid is flowing and at the outside surface that has radius r_2 a cold fluid is flowing over it.

And for this condition if we assume that the surface temperature the inner surface temperature of the inner fluid the surface temperature of the inner tube is T_{S1} and the surface temperature of the outer cylinder is T_{S2}. Then we found also a temperature distribution equation that is given by this expression.

$$T(r_1) = \frac{T_{S1} - T_{S2}}{\ln\left(\frac{r_1}{r_2}\right)} \ln\left(\frac{r}{r_2}\right) + T_{S2}$$

And when we have this, we apply this after having temperature distribution if we apply Fourier's law, we can get the heat transfer rate in the one-dimension in this case and that also has an expression like this one that is given here. And again, from there when we found that what would be the resistance to conduction in this cylindrical coordinate that has an expression of this.

Now the thing that we have now noticed is that in such cases or in this coordinate system when even irrespective of say heat generation or heat sink it is completely absent in this case, but even then the temperature distribution inside this domain is not linear, but it is varying with the logarithmic profile.

Unlike that was happening in Cartesian coordinate that we have seen earlier that if we had no heat generation the heat conduction across the media had a temperature profile that is linear. Now, for this case the equivalent circuit that we thermal circuit that can be drawn is also shown to you which is given here where this resistance which is inside the domain or in the medium in the solid medium where conduction is happening that resistance is this value which we have derived.

Now, this concept we can further add in the case of multiple layers on this tube say for example, a system is given where this hot fluid is flowing in order to heat this material. Now, outside there is substantially a cold fluid flowing, but our task is not to lose much of the heat that is being transferred from this hot fluid to the solid medium.

In that case we need to wrap this cylinder with some insulation or a material that is having a lower thermal conductivity. So; that means, again the concept of composite wall now in case of the cylindrical coordinate or in the cylinder case.

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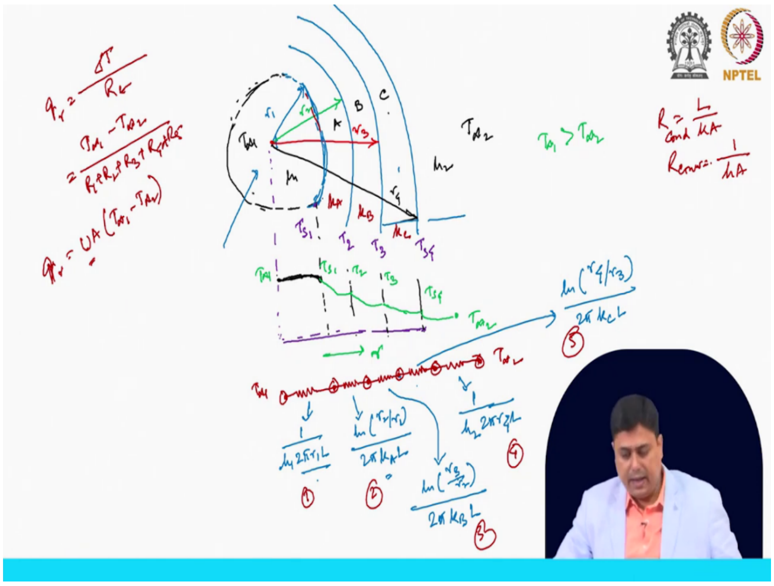

In such case the schematic that we may think of is that a simplified version is that. So, so, say these radiuses we have of different values starting with here we have r_1 this is r_2, this is r_3 and this is r_4, which means the material that we have is say we have material this is A, material B and material C.

So, there is some say hot or the cold fluid that is flowing in this tube which is having and part of which it is drawn here. So, around this there some fluid is flowing and it is wrapped with several different kind of materials. The materials are material A, material B, material C and like that. Now, all these materials will have their different thermal conductivity value those can be written as say k_A, k_B, and k_C.

Each of these interface we can further write the temperature. So, that it becomes more clearer to us, this is the inner tube surface temperature at this point which i write as T_{S1}. And then this temperature which is the interface between material A and material B say this I write as T_2. This is T_3 and this temperature is T_{S4} because this is the other surface temperature which is exposed to the outer fluid or say outer fluid the example can be simple ambient air.

Now, in such a scenario depending on the value of this k_A, k_B, k_C like we have seen in the case of composite wall in Cartesian coordinate there would be a temperature profile. Say for example, from this center point till this part what we can have a temperature profile depending on this value of the thermal conductivity and the h of this fluid. This say h_1 and h_2 inner fluid and the outer fluid.

So, in such case let us say this fluid temperature $T_{\infty 1}$ and this is $T_{\infty 2}$ this is the two different fluids. So, from $T_{\infty 1}$ it is a uniform temperature we consider at steady state all the fluids attain at us radially uniform temperature. From there is a surface temperature that dips a bit which is this point.

So, there are I am drawing this junctions of the interfaces and then what we see here this is my $T_{\infty 2}$. This is T_{S1}, this is T_{S4}, this is T_2 this is T_3. So, there will be similar kind of profile this is schematic not to the scale when we have $T_{\infty 1}$ is higher than $T_{\infty 2}$ this radial direction.

Now, in this case again if we try to have a equivalent thermal circuit for such a system. So, the thermal circuit again from this $T_{\infty 1}$ what we will have a resistance and then the value would be T_{S1} after that again another resistance till T_2 further another resistance T_3 this resistance is R in series until it reaches the and $T_{\infty 2}$.

So, this is $T_{\infty 1}$ and $T_{\infty 2}$. So, these are the temperature points. And the resistances we can write it as $\frac{1}{h_1(2\pi r_1 L)}$. This is part is the surface area across which this convection heat transfer is happening. This expression now this is the resistance for conduction and here this would be $\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$.

Because the thing that we have derived last time is this the resistance that happens for conduction in the radial directions that is $\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$ and the k of the material through which it is happening. And here this material is k_A specifically because we have multiple material or the composite wall.

Similarly, this expression the resistance expression in this case would be $\frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_B L}$. This resistance because this is now happening through material C in this zone it will have an expression $\frac{\ln\left(\frac{r_4}{r_3}\right)}{2\pi k_C L}$.

And finally, this resistance is here where the temperature drops further due to convection from the surface T_{S4} to $T_{\infty 2}$, this T_{S4} is the outer surface temperature which is not identical with the fluid temperature that is flowing. And, this is due to the convection heat transfer and associated resistance. And that expression that we know is $\frac{1}{h_2(2\pi r_4 L)}$ where h_2 is for different fluid.

This is how we write the resistance or the thermal resistance circuit when these are in parallel or for the composite wall in series that for the composite wall not for the parallel. So, here what we have seen that the resistance circuit we can draw and write even for the cylindrical coordinate or the cylindrical systems which are pretty much prevalent in practice and those expressions are like this.

So, if once these parameters are known because mostly these are the design parameters that are which are radius or the diameter of the tube and the thickness of the insulations of this material or if these are not insulator, but the different material the thickness is (r_2 - r_1), (r_3 - r_2) these values are typically known. By those numerics, we can find out this total resistance of the system.

The total resistance R_{total} is eventually the summation of all these individual terms. Once we find out those what we do? As I mentioned in these cases in most of the cases our objective is to find out how much heat transfer is happening or; that means, the rate of heat transfer and that:

$$q_r = \frac{\Delta T}{R_{total}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_1 + R_2 + R_3 + R_4 + R_5}$$

The q_r is essentially as we discussed earlier is ($UA\Delta T$) where U is the overall heat transfer coefficient that encompasses, conduction, convection and since we have not considered radiation here these two heat transfer coefficients.

And subsequently we can also find an expression for the U and in that case if we define that U in terms of we have to now define this if you try to find out the value of U this is the overall heat transfer coefficient and that is happening with respect to certain area. Now, here we have multiple areas means that we can define U based on the inner tube surface area or the outer tube extreme outer tube surface area.

Based on that we can find out our relation between the U and this resistance inverse term because U is the heat transfer coefficient and here these are the resistances. So, opposite of resistance would be our conductivity or say the heat transfer coefficients values.

In case of pure conduction these are the conductivity values inverse of the resistances divided by the area because for that for conduction again if you recapitulate the value, it is $\frac{L}{kA}$, this is for conduction, for convection its $\frac{1}{hA}$. So, I hope it is now clear to you that how this can be applied and how we can find out a equivalent thermal circuit even in case of a cylinder.

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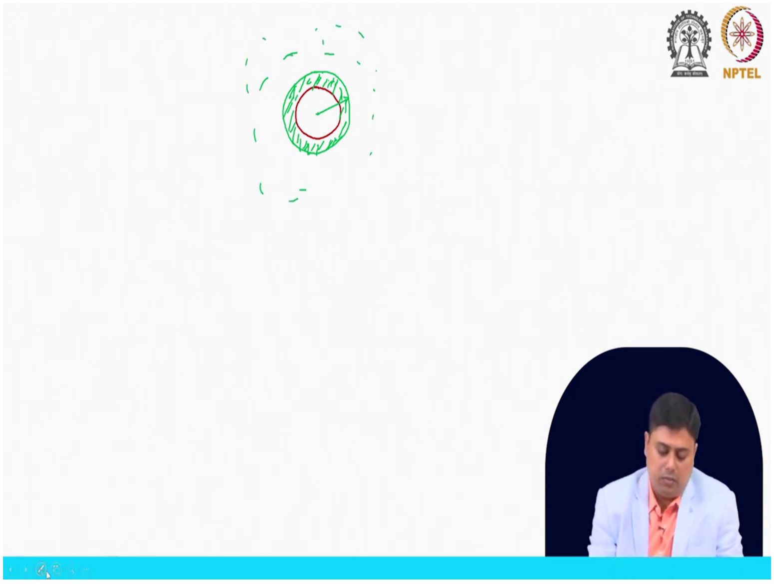

Now, say we apply this understanding for a problem in order to find out a very interesting parameter say we have a situation that there is a thin tube or thin walled tube rather that the tube is having a very thin wall and that is why the thickness is not drawn here. So, very thin this assumption or this means that we are assuming that the thermal conductivity or the temperature profile in the wall is not there, there is uniform temperature inside the material.

There is no thermal gradient in the wall in the material itself, in the radial direction. Now, this is wrapped with another material the similar concept that we have seen earlier. Now, the question is if we had to maintain the certain temperature how much thickness if we consider that outer wrapping is the insulator of lower thermal conductivity.

Now, what would be a thickness necessary thickness in order to maintain a desired temperature? That means what would be my optimum the question can be phrased as the what can be optimum thickness of an insulator around it? If we try to maintain a certain temperature of this flowing fluid that is flowing inside it so, if i insulate it with a material of certain thickness.

What would be my thickness necessary thickness in order to maintain a certain temperature? Now, say we consider this is a steady state condition this is the assumption or on the section that we are working. So, it is a logical assumption that we are analyzing a system that is at steady state. There is only one direction of the heat transfer that is one dimensional heat transfer is happening.

As I mentioned the thin wall tube; that means, it is having a negligible thermal resistance in the wall and we have say constant properties of this insulating material which is the green shaded portion. And also, we assume since we have not gone into the details of radiations, we assume that the radiation heat exchanger between this insulator and the outer surface or the outer surroundings, which is here the ambient temperature the ambience is negligible.

So, now based on these assumptions we analyze this system in order to find out that how much thickness of this material is needed in order to have a fixed or a controlled temperature of this inner fluid.

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Now, in this case if we try to draw a equivalent thermal circuit that we have seen already is that say in this case in this case as I mentioned that say this temperature that we have is outside temperature is T_∞ this is the insulation of a material that is having a thermal conductivity k.

This inner surface temperature is T_i ok the radius of it of this thin wall tube is a r_i and this r is unknown because that we are trying to find out that what could be the maximum or the optimum value of r for which we can have a constant temperature or the minimum or the maximum thickness that we can put on it in order to maintain a certain temperature of the fluid.

Now, in such case so, what is what can be done is that here we know that the thermal circuit in this case starting from the center of it what we have a resistance for this convection. So, again say here we consider the fluid is having at a uniform temperature. If we do so then what we can do is that we need not consider this resistance for convection that is there in the fluid as well as from the fluid to the surface temperature.

Because this is the assumption that we have made that this thin wall this material has very negligible thermal resistance. So, which means this fluid temperature is quickly attained by this thin wall tube this thin tube quickly attains the same temperature that is of the fluid it contains in that case we can again redraw this concept is that from T_i.

Now, this is the point where I had the T_i temperature from here there is resistance due to conduction of this insulator, I mean resistance to the conduction by this insulator material from here it reaches the outside temperature of the surface and from here and again we have a resistance that is for convection this is from the surface of the insulator to the outer fluid temperature which is T_∞.

So, which means we consider this as $\frac{1}{2\pi rh}$ if this is the fluid property that is outside fluid property and this would be $\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k}$ like we have done earlier. So, this what we can find out? Per unit length here what we can write these are the two expression that we can write for these two cases. In this case this is the convection, this is the conduction.

Similarly, per unit length of this material avoiding the value of r_l the length of this pipe we can write these two expression. So; that means, what we have to now do? The process is that we consider this as the total resistance of the system and we see that how to minimize or to maximize this resistance once we do a double derivative of that resistance.

We will see whether it is a positive or the negative value and then accordingly we will realize that whether it is the maximum or the minimum value of the resistance. And accordingly, we either put on or decrease the value or the insulation thickness in order to maintain the inner fluid temperature at a fixed value or we will try to maintain that for a fixed value by keeping or wrapping it with the insulator.

So, we will continue on this concept and we will see that derivations, but that would be done in the next class. So, in the next class we will take it forward from here and we will see what would be the expression when we will try to find out whether it is a maxima or the minima.

Till then thank you for your attention.