Chemical Engineering Fluid Dynamics and Heat Transfer

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Lecture - 38
One-dimensional Heat Conduction (Contd.)

Hello, everyone and welcome back once again in another lecture of chemical engineering fluid dynamics and heat transfer. We are in one-dimensional heat conduction section where we have spoke about the concept of composite wall, how we solve composite wall problems.

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And we will see now with one example how those equations or the relations that we found out to from the thermal resistance or the thermal circuit equivalent thermal circuit concept how it is applied or can be applied to a problem solving skills.

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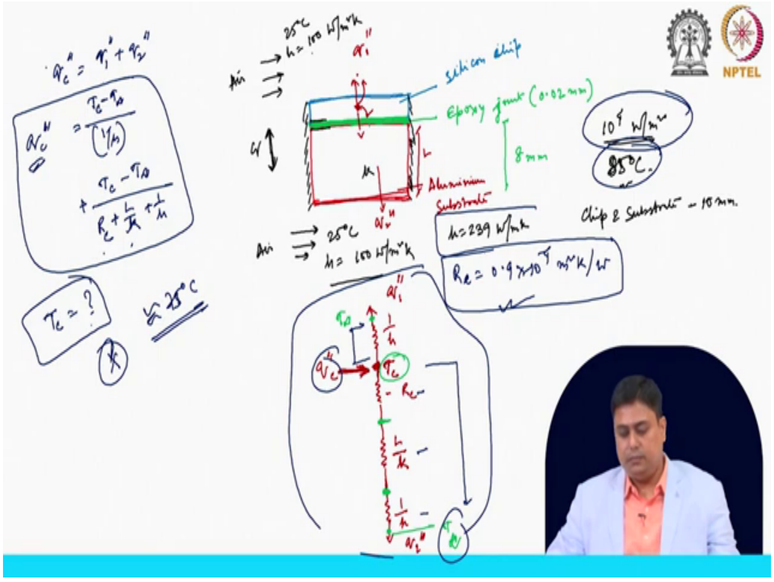

Now, let us say a problem is there where we have a thin chip, we have a thin chip in which say we draw the schematic; we have a substrate on which there is a joint between this substrate and a silicon chip. So, say I give this joint a certain thickness. So, it becomes clearer to you that there is another material. On top of which there is a substrate this chip that also has a certain thickness.

So, usually what happens this chip is fabricated on a substrate and here say the material the substrate material is say aluminium, this is the silicon chip and this is a joint by epoxy. So, the chip is glued to the substrate with a heap of epoxy. Now, this say is having a thickness of 0.02 mm, very thin. This substrate has a thickness of 8 mm.

Now, the question is there is air that is flowing on both of it is sides. This is at 25 ºC and the h value of air is 100 W/m^{2}K. It is the same 25 ºC. These sides are insulated. So, that means, the heat transfer that is happening is in the one direction that is this direction Q is the transfer.

The state problem statement is something like this that a thin silicon chip and this aluminium substrate, these are separated by an epoxy joint of 0.02 mm thickness. This chip and substrate are 10 mm on sides and their exposed surface are cooled by air which has a temperature 25 ºC and convection heat transfer coefficient is 100 W/m^{2}K.

Now, if the chip dissipates 10^4 W/m^2 this amount of heat under normal condition, the question is will it operate below a maximum say allowable temperature I would say maximum allowable temperature of 85 ^{0}C. So, that means, the maximum allowable temperature of the chip is 85 ^{0}C and under normal condition this amount of heat it dissipates.

If the air has 25 ^{0}C and the convection heat transfer coefficient of this value flows on both the sides would this chip work or not because it can reach a maximum of or it can operate at a maximum allowable temperature of 85 ^{0}C.

So, which means we have to find out what is under this circumstance what is the temperature of the silicon chip. If it is beyond 85 ^{0}C the answer would be that it cannot operate under this condition, something has to be done. The cooling rate has to increase, then it is temperature can drop down to the value that is below 85 ^{0}C.

So, if this is the question, then how we proceed on this? So, now here the thing that are known is that all the dimensions that we know heat dissipation and maximum allowable temperature of this chip is known, the thickness of the substrate epoxy joint it is known, and the convection that is happening from the surfaces that surface area we can calculate because it is mentioned that the chip and substrate the chip and substrate are having each at 10 mm on the sides.

So, the value that we need is the K value of aluminium substrate and say it is given as 239 W/mK. So, we assume this example or this case to be one-dimensional, steady state conduction and the sides are insulated it is already mentioned in the problem statement. So, there is no chance of heat leakage or heat transfer from the sides of this composite.

This joints that we discussed in the last class this joints on both the sides has negligible contact resistance; that means, the joint between silicon and epoxy and epoxy to this aluminium substrate. So, this has negligible contact resistance. So, we consider that this is an isothermal chip as well that this silicon has a uniform temperature throughout. This is also we assume that we do not have any temperature gradient inside the chip.

So, silicon chip has uniform temperature and the constant property as well as say no radial no radiation heat transfer is happening. We are neglecting the chances or the calculations of radiation heat transfer here. So, if this is the problem, then what we have to do? At first, we have to draw the equivalent thermal circuit by applying whatever we have learnt previously.

So, in this case what is happening is that if we look at this thermal circuit starting from this point that this heat is being dissipated by the silicon chip on both the sides. It is exposed on this side to the air and that heat also would be transferred in the material the substrate part as well through the epoxy joint and it would also go in this direction because it is exposed by a flow of air.

So, such in such case so, that means, if we consider that we have the point from where the tip this chip temperature is changing or say uniform temperature of the chip is say T_c, from here it is going towards air through the convection resistance. So, that means, here it is $\frac{1}{h}$ this resistance and then while coming it through the chip it happens ideally.

So, this is the immediate contact resistance. If I say this is the R_c, this is again through the material of depth L that is happening through $\frac{L}{k}$ and this one again it is this side of this substrate is exposed to air. So, it is $\frac{1}{h}$. So, here the amount of heat that is or the heat flux that is going say in this direction is $q_1^{"}$ and it is $q_2^{"}$. The heat that is being dissipated is from the $q_c^{"}$.

So, this flow of heat flux is this is the in towards the system in means from here the energy is being transferred from both the directions because the other two dimensions or the other two directions or other directions we consider this as insulated. So, one component is going this direction the other component that is going out in this direction.

So, this is the chip temperature; this temperature is basically the interface temperature between the joint and the solid substrate the aluminium substrate. This temperature is the bottom surface temperature of the aluminium substrate and here the temperature is the temperature of the air or the T_∞ that we typically mentioned, this is also T_∞.

So, in short heat dissipated by chip is transferred to the air on its top through the exposed surface and also it happens from the bottom through the epoxy joint and the aluminium substrate. So, if we now try to do a balance energy balance of these two the thing that we can write is basically:

$$q_c'' = q_1'' + q_2''$$

$$= \frac{T_C - T_\infty}{\left(\frac{1}{h}\right)} + \frac{T_C - T_\infty}{\left(R_c + \frac{L}{k} + \frac{1}{h}\right)}$$

R_c this contact resistance values are usually given.

Now, for in this problem say this value is mentioned as 0.9×10^{-4} m^{2}K/W. These values are typically given in the appendix of the text books for a given material if it is a common material otherwise it is experimentally determined or estimated.

So, now if we replace this values and we calculate the value of T_c from here that is the overall task because all the values are known to us the numerical values. So, from this q_c''= is having 10^4 W/m^2; T_∞= 25 ^{0}C; h = 100 W/m^{2}K; R_c value is given; L is mentioned; k value is given 239 W/mK and h is the same as of the previous h. Everything is known except the value of T_c.

We calculate this value say we found a value X and in this problem if you replace the numerical values and do this calculations you would see that the value would be numerically close to 75 ^{0}C which means it is well below it is maximum allowable temperature which is 85 ^{0}C and that means, this chip will operate normally.

So, just a brief summary of this problem solution strategy – we draw the thermal circuit once we have a composite wall or composite material case we identify the flow of energy and we write the conservation equation. After writing conservation equation, we find what are the resistances through which this flow is happening. Is it series, is it in parallel and accordingly we write the R total for the each and every branch of the flow.

The numerical values are usually would be given and we find out our unknown value. So, now if we have understood this thing we then move to because this is what we have understood in Cartesian coordinate.

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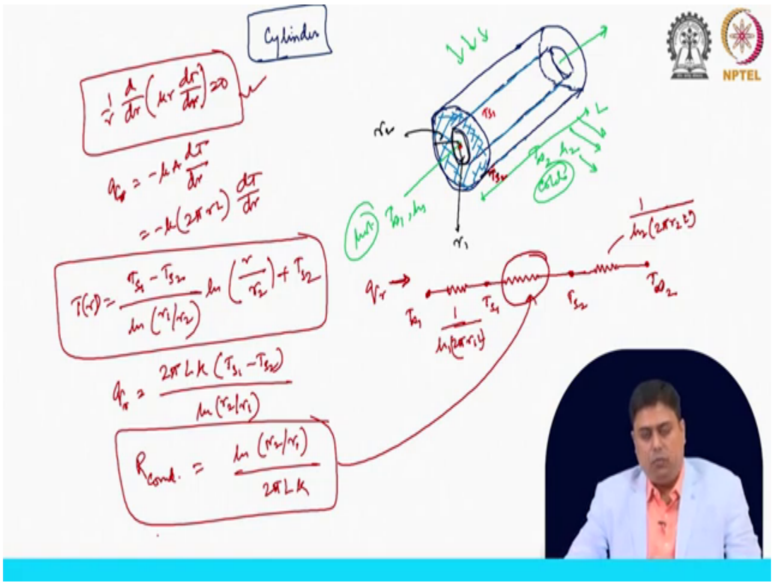

Now, in case of cylindrical or spherical coordinate let us start with the cylindrical one. In cylinder cases in cylindrical coordinate of the schematic that we usually can think of and it is usually a the material through which this happens. So, consider a scenario where we have the material that is there say a hot or a fluid that is flowing inside this pipe that has a certain thickness and outside of this cylinder that is having.

So, this is a temperature that it is flowing is $T_{\infty 1}$ of fluid, h_1 having convection heat transfer coefficient; outside of which we have $T_{\infty 2}$ and h_2. And consider that this is a cold fluid and this is the hot fluid. So, this flow of fluids are like this has a length of L the inner radius is r_1, the outer radius r_2. The surface temperature at the outer wall say T_{S2} and the surface temperature of the inner wall says T_{S1}.

So, in this case again if we try to draw a equivalent thermal circuit what we will realize that the amount of heat being transferred from $T_{\infty 1}$ from this fluid is happening through initially convection and the temperature reaches T_{S1}, the inner fluid temperature. So, here there is a resistance related to the convection heat transfer coefficient.

From T_{S1} it goes through the medium or the thickness of the pipe by conduction. And, then we again we have from the so, this to this it is happening. So, this point is T_{S2} the outer surface temperature and then again there is resistance that leads to $T_{\infty 2}$ that is the outer fluid temperature.

So, the q in the radial directions if we consider that, this is the one-dimensional conduction in cylindrical coordinate. So, q_r is basically is happening from this direction to this point. And, we know from the heat diffusion equation in cylindrical coordinate that in one dimension the governing equation or the heat equation becomes:

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0$$

The simplified version when we drop other directions and considering no heat generation and steady state scenario.

So, in this case that means, we understand that:

$$q_r = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{dT}{dr}$$

And, we again solve this equation with the boundary conditions that we have solved like the in the previous case that is:

$$T(r_1) = \frac{T_{S1} - T_{S2}}{\ln\left(\frac{r_1}{r_2}\right)}\ln\left(\frac{r}{r_2}\right) + T_{S2}$$

This is the kind of profile that we get. So, the temperature profile when it happens in radial direction in cylindrical coordinate, the temperature variation is not linear. It is logarithmic variation. It is not linear and like in the case of Cartesian coordinate when we saw that was a linear temperature profile we drew inside the media in case of non-generation of heat or no sink of heat. For conduction there was a linear profile, but in case of cylindrical profile that is not the case.

If the temperature distribution is now, we use for this heat transfer rate to calculate by Fourier's law what we will get if we replace this we get an expression:

$$q_r = \frac{2\pi kL(T_{S1} - T_{S2})}{\ln\left(\frac{r_2}{r_1}\right)}$$

Now, from this result what we can find out similar to the Cartesian coordinate that the:

$$R_{conduction} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$$

And, in other case it is simply for the convection that we have already seen depending on the heat transfer coefficient value it is:

$$R_{convection} = \frac{1}{h_1(2\pi r_1 L)}$$

Similarly, this resistance in this case is:

$$R_{convection} = \frac{1}{h_2(2\pi r_2 L)}$$

But this one for conduction we have this expression. In Cartesian coordinate, we had $\frac{L}{kA}$, but here it is a logarithmic relation.

So, I will stop here today. We will take this concept further to explore the concept of critical insulation thickness. We will start from here in the next class and we will see how and why critical insulation thickness is necessary for such pipe flow problem. Till then, I my request is that go through this material, understand or there are few steps that I have skipped here which you can do it yourself.

And, we will take it from here in the next class for solution of couple of problems related to or maybe one problem related to critical insulation thickness and then we move to the extended surface problem in conduction.

With this, thank you for your attention and we will see you in the next class.