Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 37
One -dimensional Heat Conduction (Contd.)

Hello everyone welcome back once again, an another lecture of Chemical Engineering Fluid Dynamics and Heat Transfer.

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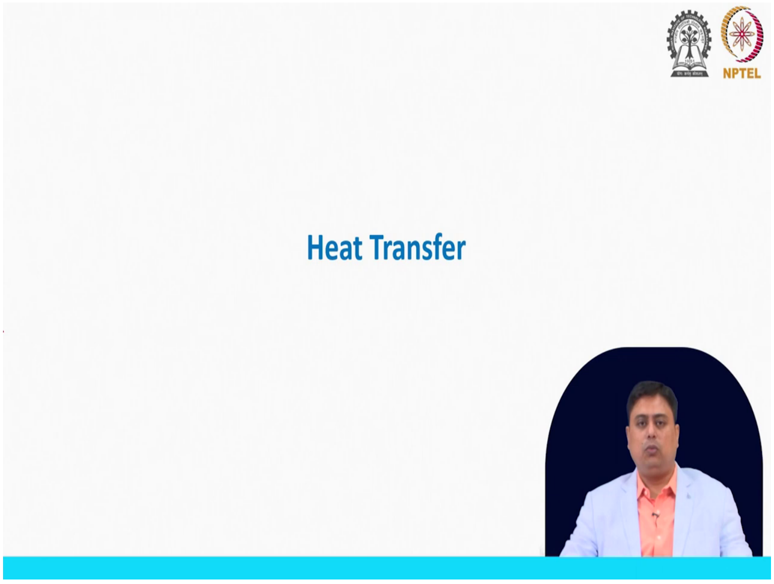

We are in heat transfer and specifically in one dimensional conduction heat transfer, steady state scenario. In the last class, I spoke about the thermal circuit and the thermal resistance concept.

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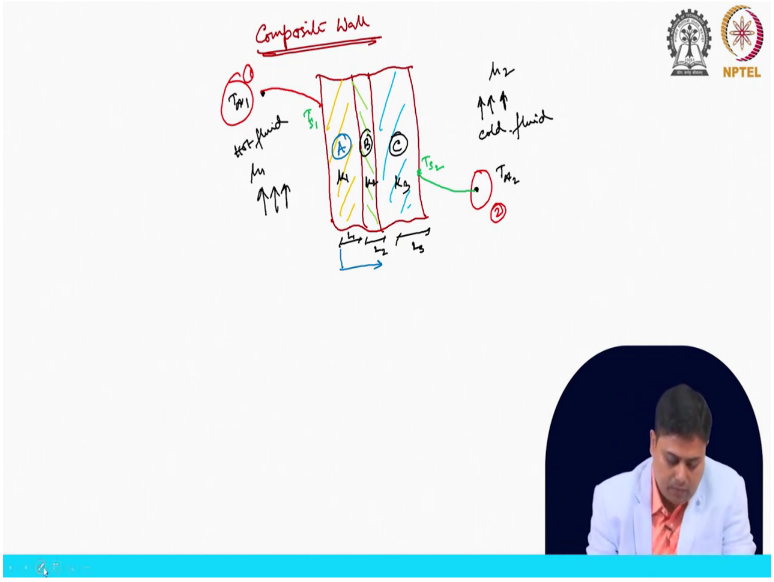

Now, taking that forward, we also introduced composite wall scenario. Now, here what is there is that we considered three different materials, material A, material B and material C as shown in the schematic of different thickness, which are pasted together for certain application set.

Now, on one side of this composite wall or slab, we had a temperature $T_{\infty 1}$, which is higher than the other side fluid temperature $T_{\infty 2}$. Now, at steady state, what would be the temperature profile or temperature distribution for this whole problem? That means, from point 1 to point 2.

As we have seen in the previous case, when there was single wall or a single slab or single material slab, we had a profile that we drew is this that is your temperature of the wall T_{S1} and from T_{S2}, we had this profile because this fluids remain same. Now, inside this, for material 1, say thermal conductivity is K_1, this is K_2 and this is K_3 so, in such cases or if I instead of getting it confused with the 1, 2, 3 because 1 is the fluid that we have mentioned.

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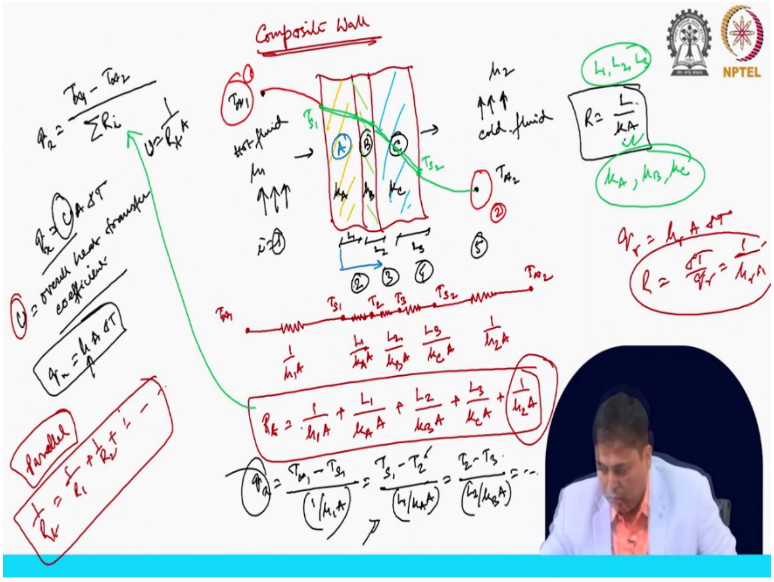

Let us differentiate it with the value K_A, K_B and K_C. Material A, material B, material C respective thermal conductivity values. So, depending on the value; because now we know that $R = \frac{L}{kA}$, the conductive heat resistance. So, based on the value of respective L, the thickness and K values, because A here is constant for all the cases or all the materials, we will have certain amount of resistance.

So, if in this case, we consider that K_A is greater than K_B, which is greater than K_C or say material A is more thermal conductive or it has a better conductivity, so that means, resistance is low. If resistance is low, then the value, it would definitely be linear, but its slope may not be steeper than in the case where K_B is low value than the K_A.

If material B as a lower conductivity; that means, the resistance would be higher in material. So, the temperature difference between the two sides would be larger than what would be there in the case of material A. And similarly, from here for this material C, it is having another linear profile, but with a different slope.

So, these all are the linear profiles depending on the value of corresponding K_A, K_B and K_C as well as L_1, L_2 and L_3 values. This is again a schematic not to the scale. So, depending on this L_1, L_2, L_3 and K_A, K_B, K_C this profile, the temperature profile and its slope would be determined accordingly.

But the take home message is that our q_x or the heat transfer is equal to the heat transfer rate is eventually the complete temperature difference for the whole system, the total temperature difference divided by the summation of all R's ($\sum R_i$), where i is of different system, different material starting from if I consider i is 1 is the first fluid, this is i is 2, i is 3, 4 and this is the 5th for i.

So, all the resistances in series here that we see so, what are those resistances if we try to now write clearly, that is for 1 first case; so, again if I try to draw a thermal circuit in this case, the thermal circuit would look like or the equivalent thermal circuit is that this is the value that we had $T_{\infty 1}$. Here we had a convective heat transfer resistance and this will reaches till T_{S1}, this value is $\frac{1}{h_1 A}$, particularly.

Till a distance of L_1 we have an intermediate temperature which say, if I designate as T_2 in between material A and material B. And then again, in between material B and; inside the material B we had another resistance and which leads to temperature 3 say T_3 and inside medium C or the material C we have another resistance for two conduction and this is T_{S2} and outside which there is another resistance that takes to $T_{\infty 2}$.

So, in between T_{S1} and T_2, the resistance is $\frac{L_1}{k_A A}$, between T_2 and T_3 we had $\frac{L_2}{k_B A}$. In between T_3 and T_{S2} we had, $\frac{L_3}{k_C A}$. And here again I have $\frac{1}{h_2 A}$, this is for the convection resistance. So, the summation of Ri or the R total; which means:

$$R_t = \frac{1}{h_1 A} + \frac{L_1}{k_A A} + \frac{L_2}{k_B A} + \frac{L_3}{k_C A} + \frac{1}{h_2 A}$$

After I replace it here after evaluating each and every values, in this expression I can find out my heat transfer rate by conduction.

Or the other way in electrical circuit that we have also done is that at every joint or in between these two points we can also write that:

$$q_x = \frac{T_{\infty 1} - T_{S1}}{\left(\frac{1}{h_1 A}\right)} = \frac{T_{S1} - T_{S2}}{\left(\frac{L_1}{k_1 A}\right)} = \frac{T_2 - T_{S2}}{\left(\frac{L_2}{k_2 A}\right)} \ldots \ldots \ldots ..$$

And similarly if we can write for the material C as well as for the fluid B.

What it means that you can find out once either one of these values are known that is the surface temperature, fluid temperature, material properties everything are given, you can find out what could be the interface temperature between material A and B from such expression. That if you know the heat transfer rate which does not depend on the direction x, it is independent as we have seen earlier by using the value of q_x you can find out what would be the temperature T_2, T_3 wherever it is required. This helps in several critical scenario for designing various problems.

Now, when we combine everything all these resistances are combined, this also we will see later that we write in a form which is:

$$q = UA\Delta T$$

are particularly in one direction this is one dimensional case that we are talking about is ($UA\Delta T$), where U is called the overall heat transfer coefficient, that has an analogy with ($hA\Delta T$) in Newton's law of cooling for convection. But for convection this h was only convective or convection heat transfer coefficient, but here it includes everything means convection, conduction as well as the radiation.

Once if you are wondering about how the radiation resistance we would write, if you remember we talk about that radiation the heat transfer due to radiation or heat transfer rate due to radiation we can express $q_r = hA\Delta T$. The temperature of the body and the surrounding this temperature difference so, radiative heat transfer coefficient or radiation heat transfer coefficient.

So, similarly here, once you try to find out the resistance it is, $R = \frac{\Delta T}{q_r} = \frac{1}{h_r A}$. So, if the radiative heat transfer coefficient or radiation heat transfer coefficient is known to you, you can find its resistance as well. So, again if it includes radiation, if the problem says it is also emitting energy and the radiation heat transfer you have to take into account, then that radiation and this convection in this case are happening in parallel.

It is not now in series. From here if we consider radiation as well then it happens in parallel and we know how to calculate the total resistance when that is happening in parallel. That means, in that case when this is in parallel, we know the

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

So, in the last case, we will have 2 resistances that are in parallel and we have to find the total resistance a sub total resistance rather for the case of convection and conduction together and add it as the last point. I hope this methodology is becoming clearer to you because this provides an important platform to solve the problem that we will be now talking about. So, this U is the overall heat transfer coefficient that includes convection, conduction and radiation heat transfer coefficient.

So, now this, so that means; this U has a relation, if you try to find out or if you try to write it in terms of resistances. This U that we can write is essentially $U = \frac{1}{R_t A}$, where R total is just the term that we have calculated. For the composite cases or compound heat transfer cases where we have to consider all the modes of heat transfer or multiple mode of heat transfer mechanism.

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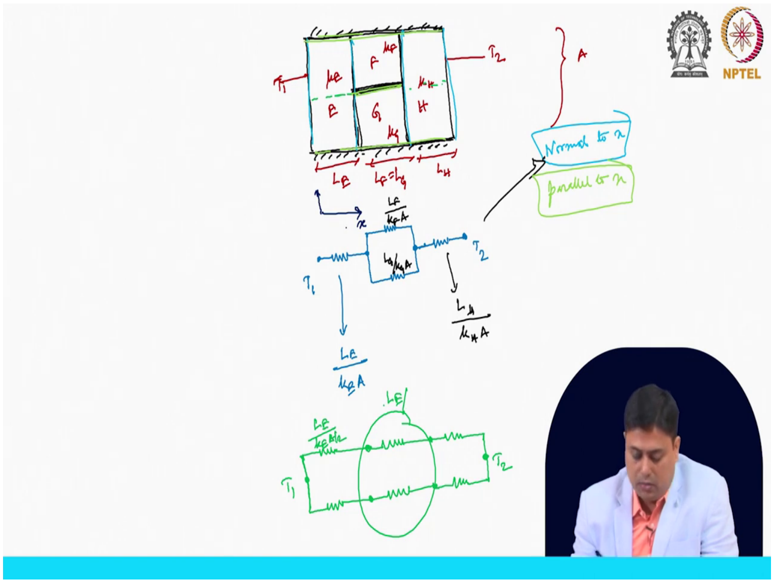

So, considering this scenario let me draw a schematic and we will see how thermal circuit helps in understanding the problem. Say we have 3 materials, but here we have another 2 materials inside it. Now, the temperatures that are here on this side is say T_1. So, that is of the surface temperature let us consider the surface temperature is T_1, this surface temperature is T_2, area is A the length of its is say L_1.

And say we have a material that is say E this is the material say E, F this is G and this is H. So, respective thermal conductivity is k_H, k_G, k_F and k_E. Here the length say consider L_E this is $L_F = L_G$ and this length is L_H. The direction is of heat transfer is in this directions which is the x.

Now, in this case consider further that we have insulations that the heat transfers due to this 1 dimensional that we are considering these 2 sides are the insulated. So, heat transfer is happening in only 1 dimensional. Now, how we draw an equivalent thermal circuit for such case. Because in the other 2 cases or other case composite wall case we have seen it is in 1 in series.

But the thing that I was talking about that can also happen in parallel like this example. So, depending on how these boundary conditions are defined; that means, particularly this vertical that is the lines that are parallel to x these are the boundary conditions parallel to x and normal to x. So, parallel to x also includes this line this interface between F and G.

If we consider that the surfaces that are normal to the x direction are isothermal, normal to the x direction means the surfaces are theses normal to x direction. In those in that case an equivalent thermal circuit the way that we can draw starting from say T_1 if we consider this is the temperature T_1, we have a resistance here, till the length that is L_E.

From there we have 2 resistances in parallel. This is from T_1 to T_2. So, this resistance is essentially $\frac{L_E}{k_E A}$,the parallel resistances here in this case is $\frac{L_F}{k_F A}$ and $\frac{L_G}{k_G A}$ and this resistance is $\frac{L_H}{k_H A}$ when this condition is specified that the lines, that the planes ok the surfaces normal to the x-direction are isothermal.

Now, the other condition that can happen is that that the planes that are parallel to x-direction, the planes that are parallel the surfaces that are parallel to the x-directions are adiabatic like it is given here this one and this green one parallel to x, those planes are adiabatic. In that case this equivalent circuit resistance changes to its different form and it becomes that from T_1.

Now, it can also for the sake of simplicity we consider that it is happening just like this is T_2, this is T_1 and this is $\frac{L_E}{k_E A/2}$. As if we have considered there is an imaginary line although the material properties are identical, but in this case for the second F and G material in order to make it consistent.

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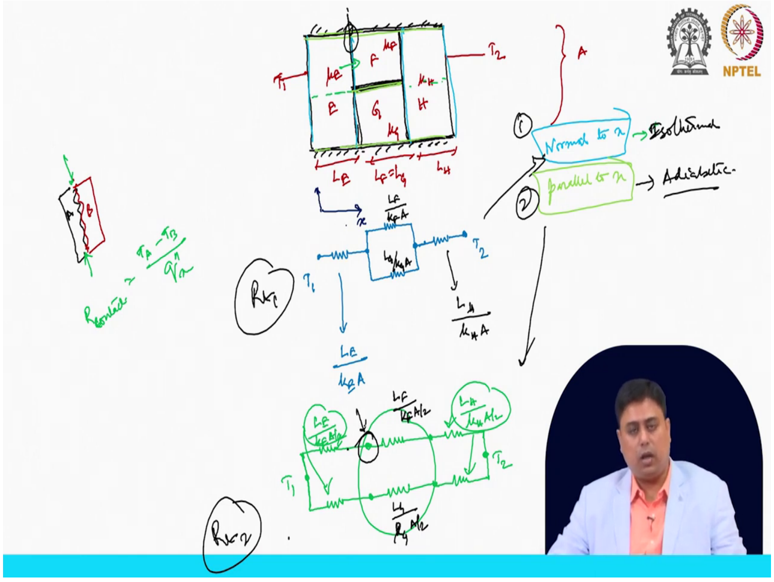

So, this is from the very beginning we have considered that here it becomes $\frac{L_F}{k_F A/2}$ and $\frac{L_G}{k_G A/2}$ and the other two remains similar that we have seen. So, this value remains same for this resistance and $\frac{L_H}{k_H A}$ this also has the same resistances. So, these are the equivalent thermal circuits, in one case we had the planes of the surfaces that are normal to x are isothermal.

The planes that are parallel to x is the second case are adiabatic. So, depending on this conditions of the boundary conditions our equivalent thermal circuit can change and will change, because this gives a different R total value than this one. And so the temperature profile would change, because we have seen earlier that the q and the R relation.

So, this is the concept for composite wall, either in parallel or in series, which we will apply in the one or two problems that will be solving in the next class. But before we close this one there is another concept let me briefly elaborate that when we have this kind of composite materials; that means, two materials are pasted or in contact with each other, in between these two material due to the surface, roughness or the pasting material that we are using or we will use.

The contact in between these two can have a different resistance. That means when a material A and say material B are in contact with each other. So, material A has been pasted against the other material say material A and this is material B. So, in between these two, either there is some pasting material or there can be gap due to surface resistance.

Now, that leads to certain amount of contact resistance, which are sometimes added to the overall resistance. So, this contact resistance again similar to the concept of what we have understood, the R contact is:

$$R_{contact} = \frac{T_A - T_B}{q_x^{"}}$$

And this is then added to this system in between two resistances. So, if there is a contact resistance mentioned in the problem, then in between this at this point at this junction we add that contact resistance in series between the two material and usually this contact resistance are experimentally measured and those values are provided in the problem when we talk about.

So, that means, composite wall we have understood its concept how we solve such problem in addition to that when two materials are in contact there are possibilities of contact resistances. If it is not explicitly mentioned we generally neglect that, because its contributions are minor, but sometimes it may be significant if it is of problem specific nature.

So, in those cases contact resistances are then added in the series between the two material this is the thing that you have to remember. So, with this I stop here today, in the next class we will solve a problem in order to understand these processes of the calculations; till then.

Thank you for your attention.