Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 36
One-dimensional Heat Conduction

Hello everyone. Welcome back once again on the another lecture of Chemical Engineering Fluid Dynamics and Heat Transfer.

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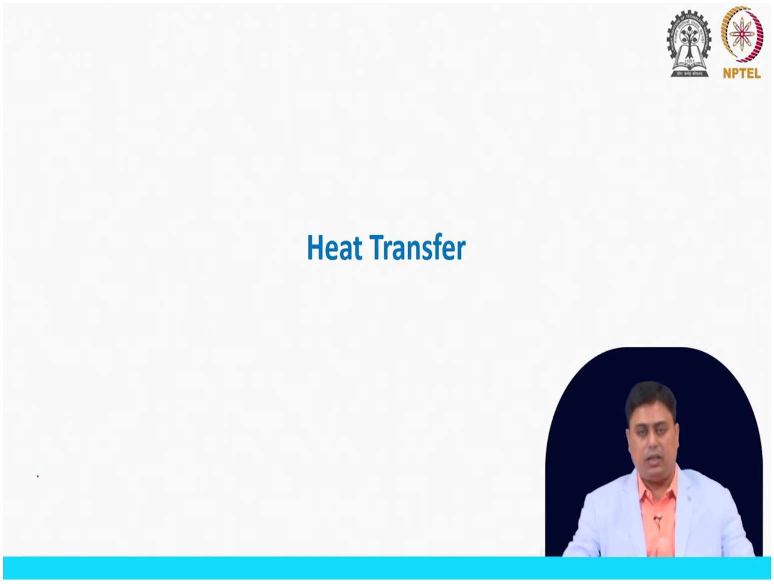

As we are aware that we are in the heat transfer section, and today we will discuss about One-dimensional heat conduction and that too in steady state condition. So, by now we have seen the general heat equation or the heat diffusion equation in all three-dimensions.

Now, for particular case if we consider that heat transfer is happening only in one direction, that generic heat transfer equation or generic form of heat transfer equation or the heat diffusion equation can be simplified dropping the terms in the other 2 directions.

So, further to that, we here in this couple of lecture, in the coming fourth coming couple of lecture, what we will do. We will further simplify that heat diffusion equation into steady state form. That means, the transient term in that generic expressions is also dropped.

In addition to that, for initial couple of lectures, we will look into the heat conduction, rate transfer rate as well as the temperature distribution in a domain, when there is no generation or sink of the heat. So, which means the equation, the generic form of heat diffusion equations is further simplified to just only one direction, steady state, and without heat generation.

So, this couple of lectures would be devoted towards that to understand how we find out temperature distribution in a domain at steady state. And from there how we calculate the heat transfer rate or the heat flux.

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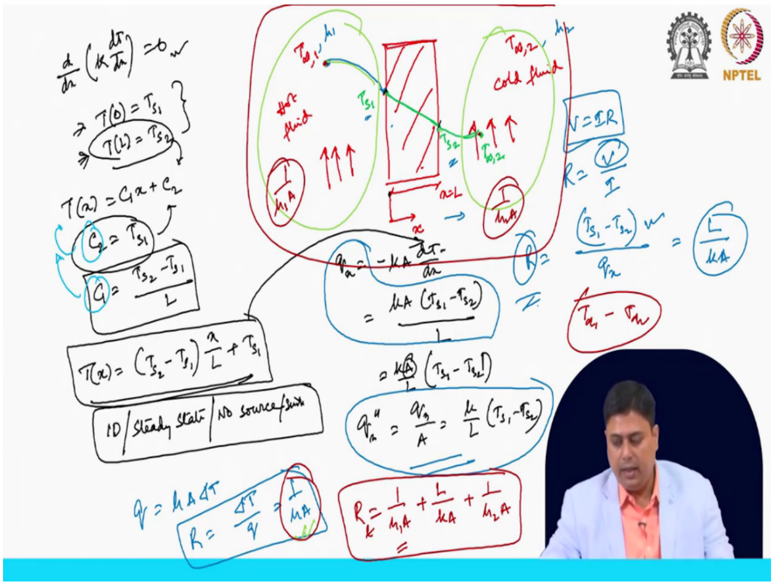

So, considering say for example, we have a domain or a solid body which separates two fluids on its two sides. So, (Refer Time: 02:41) for example, here we have say hotter hot fluid. So, temperature is $T_{\infty 1}$, and here say we have cold fluid that has a temperature $T_{\infty 2}$. So, side 1, side 2 or left side and the right side.

Now, here this is our solid body, this is the x-direction, the length of this plate or bar whatever you consider, the width of this bar or the depth of this plane is (x = L). Now, in this case, as this is a fluid on both the sides, accordingly we will have its convection heat transfer coefficient say $h_{\infty 1}$, $h_{\infty 2}$.

Or say instead of making it in this complex scenario, we just level this as h_1 and h_2, h_1 and h_2, which means these are the convection heat transfer coefficient of the hot fluid and the cold fluid, respectively. Now, if this is say a temperature $T_{\infty 1}$, from here to the surface temperature of this side of the solid bar, there will be a temperature profile. Say for example, this.

And then on the other side since it is on the cold in contact with the cold fluid, what will happen the temperature on this side would be lower than this temperature. So, if I consider now this temperature as T_{S1}; that means, surface temperature on the side number 1 or the left hand side, and here it is T_{S2} on this side.

So, there will be a temperature distribution inside the domain for conduction mechanism. I mean this is the result of the conduction. And here again, from here to the temperature which is $T_{\infty 2}$, there will be another profile of the temperature. So, inside this, now here there is no generation or the sink of heat is there. So, there will be a temperature profile between T_{S1} and T_{S2}.

Now, how that would look like and why I have drawn this as a straight line. We will see that now. It could have been a different profile, but it will be a straight line. The temperature variations would be linear in absence of any generation or depletion. Now, why that is that we will see through its derivation.

Now, if this is the case, that if we consider that in only one direction heat transfer is happening and that too in the x-direction, the equation that we can write from the generic form of the heat equation of the heat diffusion equation is that, since it is in one direction, we simply write:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

This would be the appropriate form in such scenario provided this is one-dimensional heat transfer, heat conduction specifically, steady state situation, the steady state temperature profile that we are looking into and there is no generation term or the heat source term in the domain.

So, now if we try to solve this, we can analytically solve this, we would need two boundary conditions with respect to x. And those conditions are, that we can easily write that:

$$T(0) = T_{S1}$$

Because this is our origin and this is in the direction of x.

$$T(L) = T_{S2}$$

If these two are known values, we can find out a solution of this expression.

Now, how do we do that? It is simply that we find out what would be the solution of this differential equation. So, in this case:

$$T(x) = C_1 x + C_2$$

By integration, we can find out the simple expressions. And to evaluate the values of C_1 and C_2, we now use these boundary conditions. If we do that:

$$C_2 = T_{S1}$$

Again further, if we replace now this expression after and this one together, we can find the value of C_1 which would:

$$C_1 = \frac{T_{S2} - T_{S1}}{L}$$

So, now again if we replace the value of C_1 and value of C_2 in this expression, we find its analytical form.

$$T(x) = (T_{S2} - T_{S1})\frac{x}{L} + T_{S1}$$

So, what we get, temperature profile inside this solid bar or the solid slab is:

$$T(x) = (T_{S2} - T_{S1})\frac{x}{L} + T_{S1}$$

The constraints are that this is one-dimensional, steady state, no source or sink of heat. So, what we see? When this is the criteria, the temperature distribution between the two ends or the two boundaries vary linearly in the direction x. And that is why the profile that I drew earlier is the linear.

Now, so, if we apply the Fourier's law here that heat transfer rate. So, heat transfer rate that we write:

$$q_x = -kA\frac{dT}{dx}$$

And once we do this derivative, we apply or replace T from T(x) expression and computing $\frac{dT}{dx}$, the resultant is:

$$q_x = kA(T_{S1} - T_{S2})\frac{x}{L}$$

.So, what we see that the heat transfer rate or the conduction heat transfer rate is minus k A dT by dx from the Fourier's law.

Once we replace this expression, the temperature distribution of the temperature profile, the q_x has a form which is:

$$q_x = kA(T_{S1} - T_{S2})\frac{x}{L}$$

Where, A is the area normal to the heat transfer direction. That is very important to note and understand. So, if somebody tries to calculate now the flux, which is:

$$q_x'' = \frac{q_x}{A} = \frac{k}{L}(T_{S1} - T_{S2}) \qquad (1)$$

So, these two equations that we have now seen, it clearly shows that the heat transfer rate and the heat flux by conduction is independent of x. There is no x term in both the equations. Those are constant and independent of x. So, in this case, the simplifications, the further simplifications that one can do is that to evaluate this T_{S1} and T_{S2}, which are typically unknown in such problems, one assumes that it is of the same temperature that is of the fluid.

But that is not accurate. To have the accurate understanding or to have the accurate calculations, the boundary conditions that we discussed earlier that is the mixed boundary condition, can be applied here. That is the heat gained by this plate from the hot fluid, that energy balance, that heat gained by this hot plate from the fluid and the heat lost by the fluid to the solid plate.

If we write that expression, we find a relation between $T_{\infty 1}$ and T_{S1}. And if all the properties are known, we can find out the value of T_{S1}. Similarly, on the other side, it is the heat lost by the solid plate and the heat gained by the fluid. This energy conservation, if we write this balance, we find out the value of T_{S2}, provided the other properties are known to us.

So, bottom line is that, for 1D steady state or one-dimensional steady state and no heat generation or sink problems, the temperature profile inside the domain by conduction is a linear profile. And as I said earlier, in this most of the conduction problem, our usually the objective would be to find out the temperature distribution in the domain. And once we find it out, we apply Fourier's law to find the heat transfer rate by conduction or the conduction heat transfer rate.

So, now we find an analogy between this thermal resistance and because this is happening, this temperature drop inside the solid domain is because it has a finite thermal conductivity. That means, the opposite of this thermal conductivity is the resistance. Because it has resistance to transfer the heat from one plane or one side to other side, the temperature dropped at steady state and with time also it drops.

So, now if you look at the voltage, current and resistance as well as now the analogy if you draw this with the thermal resistance, we find similarity. Because usually in electric systems, the thing that we understand is this expression. That means, the resistance is typically expressed in terms of the driving force that is the voltage, the difference between the two points and the amount of current that flows.

So, here also, the resistance is quantified in terms of the driving force which is here is (T_{S1} -T_{S2}), this is the driving force, the temperature difference between two point and the amount of heat transfer that is happening that is the q_x here in this case. So, thermal resistance, now this R becomes the thermal resistance of a system, is essentially the temperature drop in that medium by that is the thermal resistance due to conduction specifically.

$$R = \left(\frac{T_{S1} - T_{S2}}{q_x}\right) \qquad (2)$$

That the temperature distribution or the temperature difference between the two points or the two ends, two sides, divided by the amount of heat being transferred. In electrical system, the amount of current that is going through. So, if that is the case and now if you look at this expression further, what you find out from Equation (1) and (2).

$$R = \left(\frac{T_{S1} - T_{S2}}{q_x}\right) = \frac{L}{kA}$$

So, thermal resistance to conduction in a domain or in a material is essentially $\frac{L}{kA}$, where L is the length in the x-directions or in the direction that the heat transfer is happening.

It is specifically here for the one-dimension the width. k is the thermal conductivity of the material, and A is the area normal to the direction of the heat transfer by conduction. So, this analogy actually helps a lot when we do composite wall problem. Because here we have considered a single kind of material.

Now, in insulation, in thermal insulation what we do or even we look around say for example, immediately in the room where you are sitting, the wall of the room is made up of composite materials. It is not only brick, there is cement, chips etcetera, sands. So, there are different materials are there. And when those are combined in series then or in parallel then this analogy of calculating resistance, thermal resistance is extremely helpful to design a system or to understand the system.

Now, this is similar, again, now the similarity also exist between the different modes of heat transfer while calculating this resistance. The thing I have shown you here is for the resistance thermal resistance to conduction. Similarly, we know the Newton's law of cooling that we have introduced earlier for convection that the amount of heat transfer that is happening is $(hA\Delta T)$. There also the R the resistance; again, if we try to find out the $\frac{\Delta T}{q}$, it is essentially $\left(\frac{1}{hA}\right)$.

So, which means here this side of the system, we had convection. The temperature profile here is dipping, here the temperature profile is increasing because if the surface temperature is at a different value than the fluid temperature here.

So, here also the value is dipping. But it happens in a different profile or with a different gradient. This is because here we had again the resistance for this convective heat transfer to happen or the convection heat transfer to happen and on the other side as well. And those resistances are characterized or can be quantified by this expression. So, on this side, we essentially had $\left(\frac{1}{h_1 A}\right)$ resistance and on this side we had $\left(\frac{1}{h_2 A}\right)$ resistance.

So, which means the total resistance of this system if we try to now estimate that now is in series. That means, the total resistance here that we can write, R total (R_t) for this whole system is essentially

$$R_t = \frac{1}{h_1 A} + \frac{1}{kA} + \frac{1}{h_2 A}$$

This you have to understand. R_t is the total resistance of this whole system that we talked about. This is in series, initially from this point to this point the temperature profile or the ΔT is essentially for the whole system is $T_{\infty 1}$ and $T_{\infty 2}$, $(T_{\infty 1} - T_{\infty 2})$ this is the ΔT for the whole system. For this whole system, the total resistance is in series the convection part, conduction part and again the convection part.

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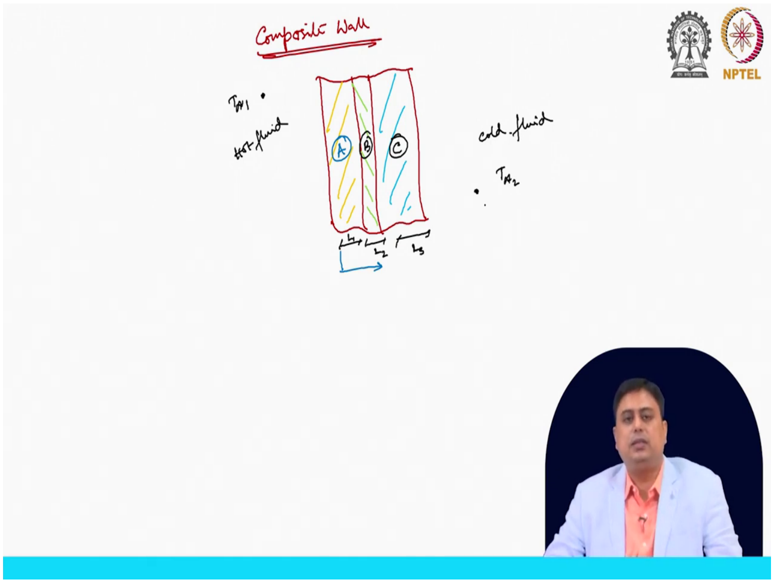

And this greatly helps in systems that we will talk in details is about the composite wall. In composite wall, the situation is that; so, in this case, this is a schematic of material A, material B and material C. So, there is an insulation or a slab or a wall of 3 different material pasted together in series. Each have its own width say L_1, this is L_2, this is L_3. 3 different thickness of the material.

Again, say on this side we have a fluid which is $T_{\infty 1}$ and this side we have a fluid $T_{\infty 2}$. This is the hot side and this is the cold slide. So, how the temperature from $T_{\infty 1}$ to $T_{\infty 2}$, how the temperature profile would look like? This can be quantified or can be evaluated by this concept of thermal circuit that includes the thermal resistance.

So, for example, let me tell you. So, here the thermal circuit that the term that I have used consisting of thermal resistances.

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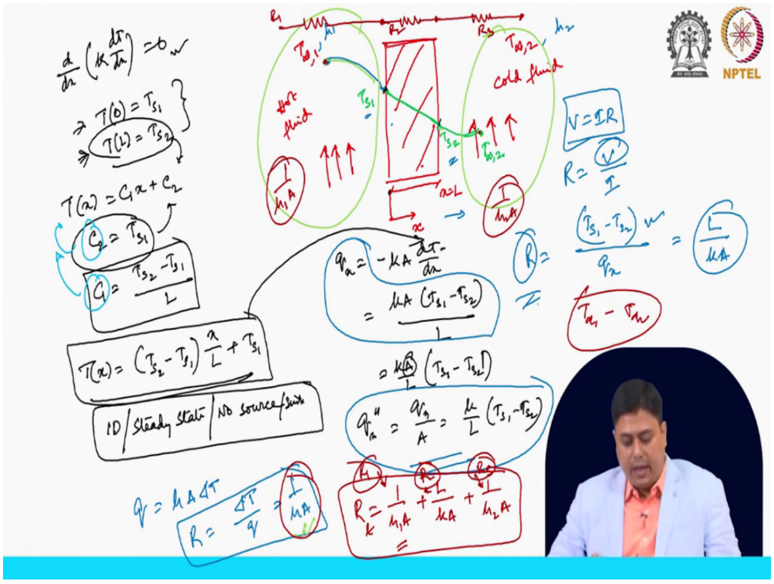

And that resistance is something like this, that we have a resistance due to convection, then here we have resistance for conduction, and again resistance for convection. So, this resistances are essentially R_1, R_2 and R_3. So, again, this concept will be elaborated in the next class that is on the Composite Wall. So, we will see you with this concept with its elaboration with solution of a problem.

And until then please look into those parts in details, so that we can carry forward with this knowledge and solve some problem.

Thank you for your attention.