Chemical Engineering Fluid Dynamics and Heat Transfer
Prof. Rabibrata Mukherjee
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 30
Turbulence 08 and Final Wrap-up

So, welcome back

In the last lecture we got the equation in wall coordinate system,

$$\left(1+\frac{\epsilon}{\gamma}\right)\frac{\partial u^{+}}{\partial y^{+}}=1$$

In this ratio $\frac{\epsilon}{\gamma}$:$\varepsilon \rightarrow$ eddy diffusivity and $\gamma \rightarrow$momentum diffusivity.

Eddy diffusivity is a flow property, but momentum diffusivity is a fluid property. We have some limiting cases depending on the magnitude of the ratio $\frac{\epsilon}{\gamma}$.

Case-1:

If $\epsilon \ll \gamma$, $1+\frac{\epsilon}{\gamma} \approx 1$, $\frac{du^{+}}{dy^{+}}=1$

$$u^{+}=y^{+}+C_1$$

That means low intensity turbulence,

In case of turbulent flow very close to the wall, u = 0 and v = 0, the fluctuation components are zero, u’ = 0 and v’ = 0. That means low intensity turbulence means the flow is tending towards a laminar flow. So very close to the wall the fluctuation components are nearly zero and therefore eddy diffusivity will be zero.

$$\frac{du^{+}}{dy^{+}}=1$$

Case-2:

If $\epsilon \gg \gamma$,

This is possible in the turbulent core or that is at a region that is far away from the wall.

Therefore, $\frac{\epsilon}{\gamma} \ggg 1$

$$1 + \frac{\epsilon}{\gamma} \approx \frac{\epsilon}{\gamma}$$

Away from the wall,

$\frac{\epsilon}{\gamma}\frac{du^+}{dy^+} = 1$

Case 1: (this case is valid very close to the wall), u' and v' are 0.

$\epsilon \ll \gamma,\ \frac{du^+}{dy^+} = 1$

Integrating the equation

$u^+ = y^+ + c_1$

By using boundary condition, At y = 0, ū = 0, y^+ = 0, u^+ = 0

Therefore, the value of c_1 = 0, Hence, $u^+ = y^+$

That means, that even for a turbulent flow very close to the wall there is no fluctuation. This region is known as the viscous sub layer. And the equation is valid till the edge of the viscous sublayer, where the height is y^+_{VSL}.

Case 2: Turbulent core or away from the wall

$\frac{\epsilon}{\gamma}\frac{du^+}{dy^+} = 1$

$$\epsilon = L^2\left(\frac{\partial \bar{u}}{\partial y}\right)$$

Prandtl mixing length, $L = Ky$

hence,

$$\epsilon = k^2 y^2 \frac{\partial \bar{u}}{\partial y}$$

And we know that from wall coordinate system,

$$y = \frac{y^+ \gamma}{u_*}$$

$$\frac{\partial \bar{u}}{\partial y} = \frac{(\, u_*)^2}{\gamma} \frac{\partial u^+}{\partial y^+}$$

If we substitute the value of $\frac{\partial \bar{u}}{\partial y}$ in the expression for eddy diffusivity, we will get.

$$\epsilon \;=\; k^2 y^2 \frac{(\, u_*)^2}{\gamma} \frac{\partial u^+}{\partial y^+}$$

$$= k^2 \left(\frac{y^+ \gamma}{u_*} \right)^2 \frac{(\, u_*)^2}{\gamma} \frac{\partial u^+}{\partial y^+}$$

$$\epsilon \;= k^2 \gamma (y^+)^2 \frac{\partial u^+}{\partial y^+}$$

and the equation for turbulent core or away from the wall, $\frac{\epsilon}{\gamma} \frac{du^+}{dy^+} \;= 1$

if we substitute for ϵ in the above equation, we will get.

$$\frac{k^2 \gamma (y^+)^2}{\gamma} \frac{du^+}{dy^+} \frac{du^+}{dy^+} \;= 1$$

$$k^2 (y^+)^2 \left(\frac{du^+}{dy^+} \right)^2 \;= 1$$

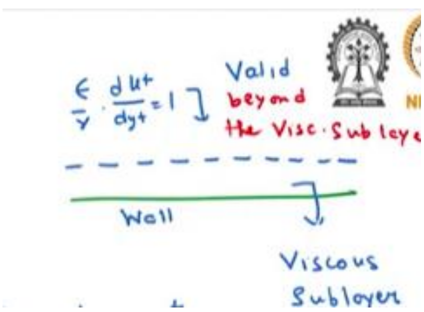
$$k y^+ \frac{du^+}{dy^+} = 1$$

$$du^+ = \frac{1}{k} \frac{dy^+}{y^+}$$

$$=> u^+ = \frac{1}{k} \ln y^+ + c_2$$

To evaluate c_2,we will use the boundary condition,

We have viscous sublayer very close to the wall.



Hence $\frac{\epsilon}{\gamma}\frac{du^+}{dy^+} = 1$ is valid beyond the viscous sublayer.

For this equation starts at y^+_{VSL}

$u^+|_{VSL} = \frac{1}{k}\ln y^+_{VSL} + C_2$

At the edge of the boundary layer, $u^+|_{VSL} = y^+_{VSL}$

Therefore, the value of C_2 becomes,

$$C_2 = y^+_{VSL} - \frac{1}{k}\ln y^+_{VSL}$$

Therefore, the equation becomes,

$$u^+ = \frac{1}{k}\ln y^+_{VSL} + y^+_{VSL} - \frac{1}{k}\ln y^+_{VSL}$$

This equation is known as the Prandtl–Taylor law of wall. The above equation if we write in the generic form,

$$u^+ = A\ln y^+ + B$$

And experimentally the values have been measured A=2.5 and B=5.5.

<u>Universal velocity profile:</u>

viscous sublayer is valid for, 0<y^+<5→ $\boldsymbol{u^+ = y^+}$

Transition layer/Buffer layer,5<y^+<30 → $\boldsymbol{u^+ = \ln\left(\frac{y^+}{5}\right)}$

Turbulent layer, 30<y^+<400→ $\boldsymbol{u^+ = 2.5\ \ln y^+ + 5.5}$

We have discussed that close to the wall, where u and v are 0. And this starting equation that your wall shear stress or the apparent shear stress within that region is not a function of y, it is equal to the effective stress is equal to the wall shear stress is actually valid within that sub layer. Which actually is the viscous sub layer where your velocities or the inertial effects are negligible, the liquid is almost stagnant and naturally as the liquid is almost stagnant even if you have a core boundary layer, core turbulent flow right the fluctuations are negligible.

Thank you very much.