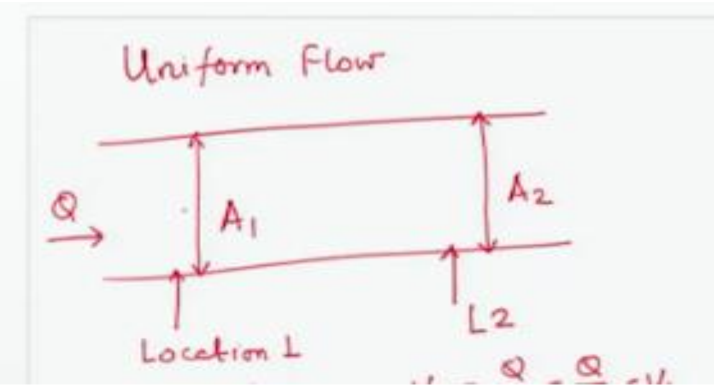
Chemical Engineering Fluid Dynamics and Heat Transfer
Prof. Rabibrata Mukherjee
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 03
Kinematics - 03

Welcome back to the third lecture now. In the previous lecture, we introduced the concept of uniform and nonuniform flow. Steady state flow is a type of flow where the velocity or the linear velocity at a particular location is not a function of time. We also talked about an interesting concept of 'acceleration'.

So, if we have an incompressible fluid (which means density is constant) flowing through a tube of uniform cross section at both locations let us say L_1 and L_2 where the velocities are V_1 and V_2. And now so, basically V_1 is equal to V_2 and both are time independent. then this will be the steady uniform flow.

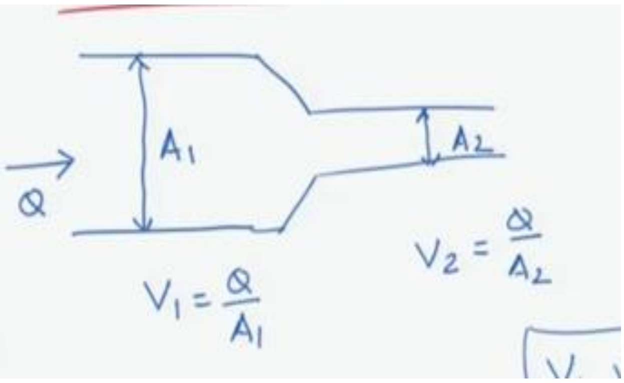


At Location 1 (L_1), $V_1 = \frac{Q}{A_1}$

At location 2 (L_2), $V_2 = \frac{Q}{A_2} = \frac{Q}{A_1} = V_1$,

Therefore $V_1 = V_2 \neq f(t) \rightarrow$ steady uniform flow

However, if we consider the flow of an incompressible fluid through a conduit or a tube of varying cross section such that the upstream cross-sectional area is larger than the downstream cross-sectional area. Therefore, $A_2 < A_1$ and since the same amount of fluid has to flow, then V_2 is going to be greater than V_1.



At Cross sectional area A_1, $V_1 = \frac{Q}{A_1}$

At cross sectional area A_2, $V_2 = \frac{Q}{A_2}$

Since $A_2 < A_1$, therefore $V_1 > V_2$

Therefore $V_1 = V_2 \neq f(t)$

Here V_1 and V_2 are also time independent. Even though, it is not an unsteady flow; however, there is an increase in velocity or change in velocity. Because of the change in the velocity, there is a non-zero acceleration. For a fluid, the velocity may not change with time, but still, you can have acceleration.

Now, let us try to understand this concept better with a bit of mathematics and investigate the concept of substantial derivative or material derivative. Consider a fluid particle in the flow field which is initially at position (x, y, z) at time t and its components of velocity are u (x, y, z, t), v (x, y, z, t) and w (x, y, z, t).

After time $t = t+\Delta t$, the particle from its initial position moved to its new location $(x+\Delta x, y+\Delta y, z+\Delta z)$. At this point, the velocity components are $u+\Delta u$, $v+\Delta v$,$w+\Delta w$.

$$u+\Delta u = u\,(x+\Delta x, y+\Delta y, z+\Delta z, t+\Delta t)$$

$$v+\Delta v = v\,(x+\Delta x, y+\Delta y, z+\Delta z, t+\Delta t)$$

$$w+\Delta w = w\,(x+\Delta x, y+\Delta y, z+\Delta z, t+\Delta t)$$

The only assumption considering here is that Δt should be infinitesimal. Consider the Taylor series expansion of the first equation approximated up to the first term (neglecting the higher order terms).

$$u+\Delta u = u\,(x+\Delta x,\ y+\Delta y,\ z+\Delta z,\ t+\Delta t)$$

$$u+\Delta u = u\,(x, y, z, t) + \frac{\partial u}{\partial x}.\Delta x+ \frac{\partial u}{\partial y}.\Delta y + \frac{\partial u}{\partial z}.\Delta z+ \frac{\partial u}{\partial t}.\Delta t$$

Rearranging the terms,

$$\frac{\Delta u}{\Delta t} = \frac{\partial u}{\partial x}.\frac{\Delta x}{\Delta t}+ \frac{\partial u}{\partial y}.\frac{\Delta y}{\Delta t} + \frac{\partial u}{\partial z}.\frac{\Delta z}{\Delta t}+ \frac{\partial u}{\partial t}$$

The particle from the initial location (x, y, z) is moving to its new location at ($x+\Delta x$, $y+\Delta y$, $z+\Delta z$). Therefore, over a time of Δt, the amount of distance by which particle has moved in x direction is $u\Delta t$.

$$\Delta x = u.\ \Delta t$$

$$\Delta y = v.\ \Delta t$$

$$\Delta z = w.\ \Delta t$$

$$\frac{\Delta u}{\Delta t} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}+ \frac{\partial u}{\partial t}$$

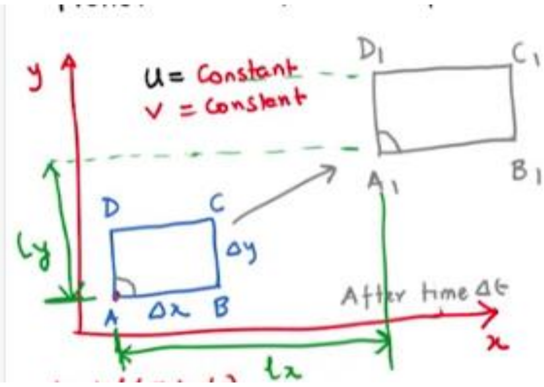
$$\frac{Du}{Dt} = \lim_{\Delta t\to 0} \frac{\Delta u}{\Delta t} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

This term is known as the material or substantial derivative with respect to time in a convective flow field. It has two terms.

Term 1 $\rightarrow$ Temporal derivative and Term 2 $\rightarrow$ Spatial derivative

So, we can understand that a particle in a flow field can have temporal as well as spatial acceleration. Next, let us look into the condition of how exactly the particle undergoes a motion or a deformation in a flow field where the velocities are itself dependent on the spatial coordinates. Let us take a 2D flow field so that you understand it well.

Let us consider a rectangular particle whose sides are Δx and Δy and let's mark the particle as ABCD. Let's consider a particular case where the fluid particle ABCD is in a 2D flow field with u and v are both constant.



$u \neq (x, y, z, t)$

$v \neq (x, y, z, t)$

since u and v are both constant, all points over this particle move with the same x component and y component velocity. After time t = t + Δt, the particle moved into a new location. since all the points have the same x component and y component velocity, A_1B_1=AB

A has moved by certain amount in the x and y direction, B has also moved by that exact same amount in the x and y direction. So, there is no change in their relative orientation as well as the distance between them.

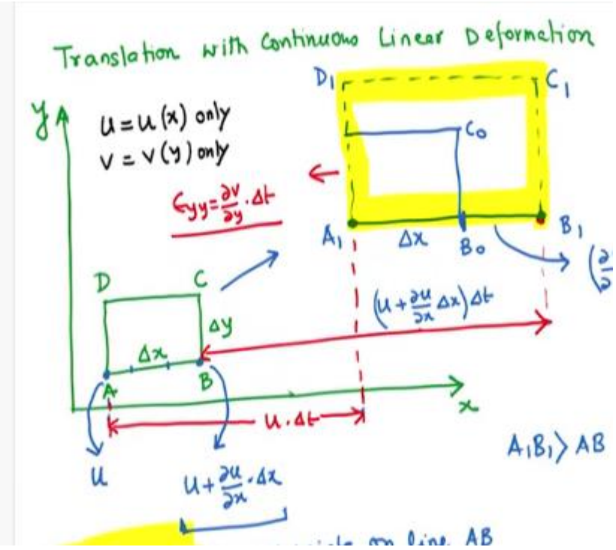
$l_x = AA_1|_x = u.\Delta x$

$l_y = AA_1|_y = v.\Delta x$

There is also no change in the angle of each sides of the particle w.r.t the x or y direction. So, if A B was horizontal before movement, A 1 B 1 will also be horizontal at the new location.

This motion is simple translation with no deformation. The motion or movement of this fluid particle resembles a solid object when u and v are constant. Let's consider a cuboidal fluid element in a 3D flow field and its velocity components are u, v and w. Here the cuboidal element will keep on moving in the flow field with time without any change in shape or distortion.

Next, we are discussing the translation motion of a fluid particle with continuous linear deformation. Consider the rectangular fluid element ABCD in a flow field, where u is a function of x only and v is a function of y only. In this case, x component velocity of point A and x component velocity of point B are different.



u = u (x) only

v = v (y) only

Let us for the sake of simplicity assume that from A to B it is a function of x and as in the positive direction of x the velocity increases (or changes). At point A, the x component velocity is u and at the point B, the x component velocity becomes (u $+\frac{\partial u}{\partial x}\ \Delta x)\Delta t.$

Since the fluid particle is very small, Δx and Δy are infinite decimal. So, it is quite logical to assume that since A and B are very close to each other, the variation can be linear. So, we can consider the Taylor series approximation and neglected the higher order terms.

since v ≠ f(x), all points over line AB move with the same velocity in y direction. and consequently, line segment AB elongates in the x direction. Since all the points are moving with the same velocity in the y direction, therefore line AB was initially horizontal the line segment A_1B_1 also remains horizontal even after elongation.

So, essentially within the flow field, the particle has moved and undergone a linear deformation, and this happens for a situation where u is a function of x only v is a function of y only and of course, for steady state flow and incompressible fluids.

So, after time Δt, the point A has moved by an amount (u.Δt). Because the velocity of point A was u. And, point B has moved by an amount, (u + $\frac{\partial u}{\partial x}.\Delta x)\Delta t$.

So, the consequence will be $A_1B_1 > AB$.

The additional distance,

$B_0B_1 = A_1B_1 - AB = (u + \frac{\partial u}{\partial x}.\Delta x)\Delta t - u.\Delta t = \frac{\partial u}{\partial x}.\Delta x.\Delta t$

So, the line AB has dilated to a length A_1B_1.

Strain in the x direction is the change in the length divided by the original length.

$$\epsilon_{xx} = \frac{\Delta l_x}{l_x} = \frac{B_0B_1}{AB} = \frac{\frac{\partial u}{\partial x}.\Delta x.\Delta t}{\Delta x}$$

$$\epsilon_{xx} = \frac{\partial u}{\partial x}.\Delta t$$

$$\dot{\epsilon}_{xx} = \frac{\epsilon_{xx}}{\Delta t} = \frac{\partial u}{\partial x}$$

$$\dot{\epsilon}_{yy} = \frac{\epsilon_{yy}}{\Delta t} = \frac{\partial v}{\partial y}$$

strain rate is the strain in the x direction per unit time.

So, these are the important take home messages here. I would like to highlight that since v is not a function of x, all points along the line A B move at the same velocity in the y direction and as a consequence this line segment A B is elongating in the x direction. All the points are moving with the same velocity in the y direction. and therefore, if the line A B was initially horizontal, the line segment A_1B_1 even after elongation remains also horizontal. So, there is no change in its orientation. So, the what is the type of the deformation? So, here you see two cases.

So, we now have looked into the nature of the deformation or the motion of the fluid particle in flow field. So, in the first case what we discussed is that when the velocities were constant, the motion of the fluid particle is like simple translation and there is no deformation of the fluid particle or the fluid element.

In the second case, what was the difference? The difference was u is a function of x and v is a function of y right. So, essentially there are some spatial dependence of velocity, but it's very special as u varies only in the x direction and as a consequence of that what we observe is translation with linear deformation. Obviously, what is the most general case for a 2 D fluid element is going to be when u and v are both functions of x and y.

And that is the most general case and most interesting also and that is what are we are going to take up in the next class.

Thank you very much.