Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 28
Turbulence 06

So, welcome back.

In the concept of turbulence, as per Reynolds decomposition of turbulence there is mean velocity which is time independent and there is a time dependent fluctuating velocity. And then we understood that something that is closest to steady uniform flow would be stationary homogeneous turbulence. And we learnt about the mean how to calculate the time average and the space average and then we moved on with our further discussion to incorporating the Reynolds decomposition into the conservation equations. We also understood that by because of the fluctuation component there is no net transfer of mass, but due to the interaction of the fluctuation in different directions there is transfer of momentum. Then we discussed about flow of a particle, when the particle is trying to flow from one point to another point within the flow field, these fluctuations actually put-up additional hindrance to the flow. These fluctuations contribute turbulent stresses or Reynolds stresses.

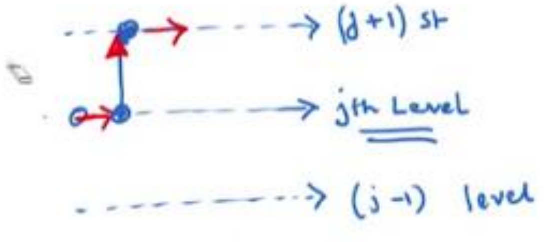
So, if we are looking into the conservation equations of turbulence, there will be total 8 variables and 5 equations. So, we could not solve it and therefore, we have to reduce the number of unknowns. We are reducing the number of unknowns by correlating the fluctuating component of velocity with the mean component of velocity, by using the concept Prandtl mixing length.

8 variables: $(\bar{u}, \bar{v}, \bar{w}, \bar{P}, u', v', w')$

5 Equations: (3 momentum balance equation, continuity equation for mean and fluctuating component of velocity)

<u>Prandtl mixing length:</u>

Consider a fluid particle moving along the jth level. And because of the fluctuation the particle moves form jth level to $(\mathrm{j} + 1\ )st$ level.



Because of this more fluid will flow through the $(\mathrm{j}+1)st$ level and less fluid will flow at the jth level. Assuming the cross-sectional area to be same. This means the linear velocity or the mean velocity at the $(\mathrm{j}+1)st$ level has increased and the mean velocity at the jth level has reduced. It leads to the certain additional velocity gradient between the jth level and $(\mathrm{j}+1)st$ level. This velocity gradient $\left(\frac{\partial \bar{u}}{\partial y}\right)$ is due to the fluctuation component.

Therefore,

$\frac{\partial \bar{u}}{\partial y}$ has got created because of the fluctuation,

$$\frac{\partial \bar{u}}{\partial y} = f(u') \rightarrow u' \propto f\left(\frac{\partial \bar{u}}{\partial y}\right)$$

In most cases,

$u' \propto \left(\frac{\partial \bar{u}}{\partial y}\right) \rightarrow u' = L\left(\frac{\partial \bar{u}}{\partial y}\right)$: the proportionality constant L is called Prandtl mixing length.

$$L = \frac{u'}{\left(\frac{\partial \bar{u}}{\partial y}\right)}$$

We know that Reynolds stress component,

$$\tau_{xyT} = -\rho(\overline{u'v'})$$

And we know that for isotropic turbulence,

$$\left(\overline{u'^2} = \overline{v'^2} = \overline{w'^2}\right)$$

Also $\overline{u'} = 0$:since the fluid is continuum, will get the value of $\overline{u'v'} = -ve$

$$\tau_{xyE} = \mu \frac{\partial u}{\partial y} + \mu_T \frac{\partial u}{\partial y} = \mu \frac{\partial u}{\partial y} - \rho(\overline{u'v'})$$

so, if you are looking for the above equation in mathematical way, the effective shear stress is becoming lower than the viscous stress, but it does not happen because the value of entity $\overline{u'v'}$ is negative. To consider the value $\overline{u'v'}$ as negative, we are assuming that at a particular point the two directions or the two interacting components of fluctuation will have opposite signs.

Therefore, if $u' = L\left(\frac{\partial \bar{u}}{\partial y}\right) : v' = -L\left(\frac{\partial \bar{u}}{\partial y}\right)$

So, if we substitute the value of u' and v' in the Reynolds stress expression, we will get

$$\tau_{xyT} = -\rho(\overline{u'v'}) = -\rho\left(L\left(\frac{\partial \bar{u}}{\partial y}\right)\right)\left(-L\left(\frac{\partial \bar{u}}{\partial y}\right)\right)$$

$$= \rho L^2 \left(\frac{\partial \bar{u}}{\partial y}\right)^2$$

$$\mu_T \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right) = \rho L^2 \left(\frac{\partial \bar{u}}{\partial y}\right)^2$$

For 1D flow, $\frac{\partial \bar{v}}{\partial x} = 0$

$$\rho L^2 \left(\frac{\partial \bar{u}}{\partial y}\right)^2 = \mu_T \left(\frac{\partial \bar{u}}{\partial y}\right)$$

$$L^2 = \frac{\mu_T \left(\frac{\partial \bar{u}}{\partial y}\right)}{\rho \left(\frac{\partial \bar{u}}{\partial y}\right)^2} = \frac{\mu_T}{\rho} \frac{1}{\left(\frac{\partial \bar{u}}{\partial y}\right)} = \frac{\epsilon}{\left(\frac{\partial \bar{u}}{\partial y}\right)}$$

Prandtl mixing length, $L = \epsilon^{\frac{1}{2}} . \left(\frac{\partial \bar{u}}{\partial y}\right)^{-\frac{1}{2}}$

$$L = \left(\frac{\left(\frac{\partial \bar{u}}{\partial y} \right)}{\epsilon} \right)^{-\frac{1}{2}}$$

In the above expression, L,ε, μ_T are not fluid property, they all are flow properties.

<u>Flow over a flat plate:</u>

Prandtl mixing length, $L = Ky$

$K \rightarrow Van\ karmann\ constant: y \rightarrow distance\ from\ the\ wall$

we will discuss in detail about this in the next class.

Thank you very much.