Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 27
Turbulence 05

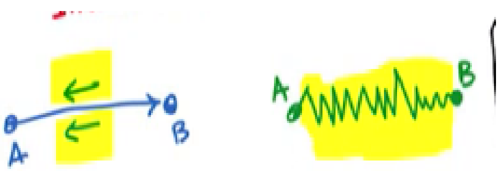
So, welcome back.

Time averaged Reynolds decomposed x component equation:

$$\underbrace{\rho\left(\frac{\partial}{\partial x}\bar{u}^2+\frac{\partial(\overline{uv})}{\partial y}+\frac{\partial(\overline{uw})}{\partial z}\right)}_{\text{Inertial terms}}+\rho\left(\frac{\partial}{\partial x}\overline{u'^2}+\frac{\partial}{\partial y}(\overline{u'v'})+\frac{\partial}{\partial z}(\overline{u'w'})\right)=\underbrace{-\frac{\partial(\bar{p})}{\partial x}}_{\text{Pressure}}+\underbrace{\mu\left(\frac{\partial^2(\bar{u})}{\partial x^2}+\frac{\partial^2(\bar{u})}{\partial y^2}+\frac{\partial^2(\bar{u})}{\partial z^2}\right)}_{\text{Skin friction term}}$$

$$\underbrace{\rho\left(\frac{\partial}{\partial x}\bar{u}^2+\frac{\partial(\overline{uv})}{\partial y}+\frac{\partial(\overline{uw})}{\partial z}\right)}_{\text{Inertial term}}=\underbrace{-\frac{\partial(\bar{p})}{\partial x}}_{\text{Pressure}}+\underbrace{\mu\left(\frac{\partial^2(\bar{u})}{\partial x^2}+\frac{\partial^2(\bar{u})}{\partial y^2}+\frac{\partial^2(\bar{u})}{\partial z^2}\right)}_{\text{Skin friction term}}\underbrace{-\rho\left(\frac{\partial}{\partial x}\overline{u'^2}+\frac{\partial}{\partial y}(\overline{u'v'})+\frac{\partial}{\partial z}(\overline{u'w'})\right)}_{\text{Interaction between the different fluctuating components}}$$

So, if a particle wants to travel from point A to point B in laminar flow field, it will move straight and spatial variation of velocity the particle depends on the viscous shear stresses



But on the other hand if the particle is moving in a turbulent flow field will follow a tortuous path. So, this the particle has to spend more amount of energy because of the fluctuations. The presence of the fluctuation in the flow field imparts additional resistance to flow. So, now we understand that this interaction between the fluctuation terms also oppose the flow and therefore, it imparts additional resistance to the flow. Therefore, this can be considered as an additional stress term. This term known as turbulent stress term.

$$\underbrace{\rho\left(\frac{\partial}{\partial x}\bar{u}^2+\frac{\partial(\overline{uv})}{\partial y}+\frac{\partial(\overline{uw})}{\partial z}\right)}_{\text{Inertial term}}=\underbrace{-\frac{\partial(\bar{p})}{\partial x}}_{\text{Pressure}}+\underbrace{\mu\left(\frac{\partial^2(\bar{u})}{\partial x^2}+\frac{\partial^2(\bar{u})}{\partial y^2}+\frac{\partial^2(\bar{u})}{\partial z^2}\right)}_{\text{Skin friction term}}\underbrace{-\rho\left(\frac{\partial}{\partial x}\overline{u'^2}+\frac{\partial}{\partial y}(\overline{u'v'})+\frac{\partial}{\partial z}(\overline{u'w'})\right)}_{\text{Turbulent stress term}}$$

$$\rho\left(\frac{\partial}{\partial x}\bar{u}^2+\frac{\partial(\overline{uv})}{\partial y}+\frac{\partial(\overline{uw})}{\partial z}\right)=-\frac{\partial(\bar{p})}{\partial x}+(\mu+\mu_T)\left(\frac{\partial^2(\bar{u})}{\partial x^2}+\frac{\partial^2(\bar{u})}{\partial y^2}+\frac{\partial^2(\bar{u})}{\partial z^2}\right)$$

And we can say,

$$-\rho\frac{\partial}{\partial x}\overline{u'^2}=\mu_T\frac{\partial^2(\bar{u})}{\partial x^2}$$

$$-\rho\frac{\partial}{\partial y}(\overline{u'v'})=\mu_T\frac{\partial^2(\bar{u})}{\partial y^2}$$

$$-\rho\frac{\partial}{\partial z}(\overline{u'w'})=\mu_T\frac{\partial^2(\bar{u})}{\partial z^2}$$

$\mu_T \rightarrow$ turbulent viscosity or Pseudo viscosity

The ration of viscous stress to turbulent stress is called turbulent viscosity or pseudo viscosity, and if the turbulent viscosity is higher, the turbulent stress also will be higher.

$$\rho\left(\frac{\partial}{\partial x}\bar{u}^2+\frac{\partial(\overline{uv})}{\partial y}+\frac{\partial(\overline{uw})}{\partial z}\right)=-\frac{\partial(\bar{p})}{\partial x}+(\mu+\mu_T)\left(\frac{\partial^2(\bar{u})}{\partial x^2}+\frac{\partial^2(\bar{u})}{\partial y^2}+\frac{\partial^2(\bar{u})}{\partial z^2}\right)$$

$Effective\ viscosity, \mu_E=(\mu+\mu_T)$

Effective viscosity is basically a summation of the dynamic viscosity and the turbulent viscosity.

If we are divide the abov equation by ρ,we will get

$$\frac{\mu_E}{\rho}=\frac{\mu}{\rho}+\frac{\mu_T}{\rho}=\gamma+\epsilon$$

$\epsilon \rightarrow Eddy\ diffusivity, \gamma \rightarrow Kinematic\ viscosity$

$\mu\ and\ \gamma$ are fluid /material property.

$\mu_T\ and\ \varepsilon$ gives the inetnsity of turbulence.

So, if you are considering the flow of water $\mu\ and\ \rho$ are known. But the value $\mu_T\ and\ \varepsilon$ will be unknown because they are functions of the system or the flow conditions.

If you are considering Newtonian fluid,

$$\tau_{xy} = \mu \frac{\partial u}{\partial y}$$

Conservative form of x component momentum balance:

$$\rho\left(\frac{\partial u^2}{\partial x} + \frac{\partial (uv)}{\partial y} + \frac{\partial (uw)}{\partial z}\right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

For turbulent flow,

$$\underbrace{\rho\left(\frac{\partial u^2}{\partial x} + \frac{\partial (uv)}{\partial y} + \frac{\partial (uw)}{\partial z}\right)}_{\text{Inertial term}} = -\frac{\partial p}{\partial x} + \underbrace{\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}}_{\text{Viscous stress term}} \underbrace{- -\rho\left(\frac{\partial}{\partial x}\overline{u'}^{2} + \frac{\partial}{\partial y}(\overline{u'v'}) + \frac{\partial}{\partial z}(\overline{u'w'})\right)}_{\text{Turbulent stress/Reynolds stress}}$$

$$= -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx_E}}{\partial x} + \frac{\partial \tau_{xyE}}{\partial y} + \frac{\partial \tau_{zyE}}{\partial z}$$

From equation we can write,

Effective stress, $\tau_{xyE} = \tau_{xy\gamma} + \tau_{xyT}$

$$\tau_{xy\gamma} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\tau_{xyT} = -\rho(\overline{u'v'}) = \underbrace{\mu_T\left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right)}_{\text{We are writing as per convention.}}$$

Effective stress, $\tau_{xyE} = \tau_{xy\gamma} + \tau_{xyT} = (\mu + \mu_T)\left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right)$

$$= \rho(\mu + \epsilon)\left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right)$$

Note: Turbulent stress are physically not related to angular deformation. Turbulent stresses are not caused by the angular deformation, it gets caused due to the interaction between the fluctuation components.

Consider 1D flow, if $u = f(y)$ only:

We know the relation for shear stress,$\tau_{xy} = \mu \frac{\partial u}{\partial y}$

$$\tau_{xyT} = \mu_T \frac{\partial u}{\partial y} = -\rho(\overline{u'v'})$$

$$\tau_{xyE} = \mu \frac{\partial u}{\partial y} + \mu_T \frac{\partial u}{\partial y} = \mu \frac{\partial u}{\partial y} - \rho(\overline{u'v'})$$

We will discuss about this in the next class.

Thank you very much.