Chemical Engineering Fluid Dynamics and Heat Transfer

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Lecture - 25
Turbulence 03

So, welcome back,

Reynolds decomposition of turbulence,

$$u(y,t) = \bar{u}(y) + u'(\Gamma, t)$$

Time average, $\bar{u}^t(x_0) = \lim_{t_1 \to \infty} \frac{1}{t_1} \int_0^{t_1} u'(x_0, t) dt$

And we are considering stationary turbulence, therefore all the means are going to be zero.

$\overline{u'} = 0, \overline{v'} = 0, \overline{w'} = 0, \overline{p'} = 0$

$\int_0^t u'v'dt = \overline{u'v'} \neq 0; \quad \overline{u'v'} = -ve$

Basic rules about averages:

1. $\overline{f + g} = \bar{f} + \bar{g}$

2. $\overline{f.g} = \bar{f}.\bar{g}$

3. $\bar{\bar{f}} = \bar{f}$

As per Reynolds decomposition, we can say $\mathrm{u} = \bar{\mathrm{u}} + \mathrm{u}'$

If we are taking time average,

$\bar{\mathrm{u}} = \overline{\bar{\mathrm{u}} + \mathrm{u}'} = \bar{\bar{u}} + \overline{u'}; \overline{u'} = 0$

Therefore, $\bar{\bar{u}} = \bar{u}$

So, we are going to analyses the continuity equation for turbulent flow, Consider the flow is to be incompressible and Newtonian fluid (viscous fluid), then the continuity equation becomes,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Introduce Reynolds decomposition to the above equation,

$$\frac{\partial(\bar{u}+u')}{\partial x}+\frac{\partial(\bar{v}+v')}{\partial y}+\frac{\partial(\bar{w}+w')}{\partial z}=0$$

$$\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{v}}{\partial y}+\frac{\partial\bar{w}}{\partial z}+\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0$$

We now take the time average of the above equation,

$$\overline{\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{v}}{\partial y}+\frac{\partial\bar{w}}{\partial z}+\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}}=0$$

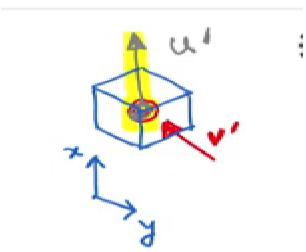
$$\overline{\frac{\partial\bar{u}}{\partial x}}+\overline{\frac{\partial\bar{v}}{\partial y}}+\overline{\frac{\partial\bar{w}}{\partial z}}+\overline{\frac{\partial u'}{\partial x}}+\overline{\frac{\partial v'}{\partial y}}+\overline{\frac{\partial w'}{\partial z}}=0$$

$\overline{\frac{\partial u'}{\partial x}},\overline{\frac{\partial v'}{\partial y}},\overline{\frac{\partial w'}{\partial z}}$ all the terms are going to be zero. Therefore, we can write,

$$\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{v}}{\partial y}+\frac{\partial\bar{w}}{\partial z}=0$$

$\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{v}}{\partial y}+\frac{\partial\bar{w}}{\partial z}=0$: the mean component of velocity obeys equation of continuity. Also,

$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0$: the mean and fluctuating components of turbulent velocity independently obeys equation of continuity.



Consider a 2D flow for the time being,

Continuity equation for mean component velocity

$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}=0$: $\frac{\partial u'}{\partial x}=-\frac{\partial v'}{\partial y}$

Due to fluctuation, some fluid particles moved out. That means there is +ve u' at a particular location, fluid particles from the adjoining area come to fill up any possible void that may create. So, if for this situation,

u' →+ve and v' →-ve,

Therefore,$u'v' = -ve$

Next we will consider the momentum balance equation,

x component Navier Stokes equation:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

We consider that this is a steady state flow, apparently a steady state flow because as we have discussed it is very difficult to achieve formal steady state in turbulence because the fluctuation component is always going to be time dependent. But anyway, for simplicity we consider the term, $\frac{\partial u}{\partial t} = 0$.We will take the conservative form of the above equation an we will continue in the next clas.

Thank you very much.