Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 22
Boundary Layer Analysis 7: Momentum Integral Method 03

So, welcome back. In the last class we have got the expression for velocity profile.

Boundary layer displacement thickness, $\delta^* = \int_0^\infty \left(1 - \frac{u}{u_\infty}\right) dy$

Boundary layer momentum thickness, $\theta = \delta^{**} = \int_0^\infty \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$

And if we substitute for $\frac{u}{u_\infty} = \frac{3}{2}\eta - \frac{1}{2}\eta^3$:$\eta = \frac{y}{\delta}$: $dy = \delta. d\eta$in the above equation, we will get

$$\theta = \delta^{**} = \int_0^1 \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)\left(1 - \frac{3}{2}\eta - \frac{1}{2}\eta^3\right) \delta. d\eta$$

$$= \delta \int_0^1 \left(1 - \frac{3}{2}\eta - \frac{1}{2}\eta^3\right)\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right) d\eta$$

$$= \delta \int_0^1 \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3 - \frac{9}{4}\eta^2 + \frac{3}{4}\eta^4 + \frac{3}{4}\eta^4 - \frac{1}{4}\eta^6\right) d\eta$$

Upon integration, we will get

$$= \delta \left[\left[\frac{3}{4}\eta^2\right]_0^1 - \left[\frac{1}{8}\eta^4\right]_0^1 - \left[\frac{9}{12}\eta^3\right]_0^1 + \left[\frac{6}{20}\eta^5\right]_0^1 - \left[\frac{1}{28}\eta^7\right]_0^1\right]$$

$$= \delta \left[\frac{3}{4} - \frac{1}{8} - \frac{9}{12} + \frac{6}{20} - \frac{1}{28}\right]$$

$$= \delta \frac{-35 + 84 - 10}{280} = \frac{39}{280}\delta$$

$$\theta = \frac{39\delta}{280}$$

And we got that

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho {u_\infty}^2}$$

In above equation, if we substitute for θ,we will get

$$\frac{39}{280}\frac{d\delta}{dx} = \frac{\tau_w}{\rho {u_\infty}^2}$$

And we know that, $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)|_{y=0}$.And based on Karman-Pahlausen approximate method, we got velocity profile as $\frac{u}{u_\infty} = \frac{3}{2}\eta - \frac{1}{2}\eta^3 = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

$$\frac{\partial u}{\partial y} = \left(\frac{3}{2\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right)u_\infty$$

$$\frac{\partial u}{\partial y}|_{y=0} = \frac{3}{2\delta}u_\infty$$

If we substitute the value of velocity gradient in the wall shear stress expression, we will get

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)|_{y=0} = \mu\frac{3}{2\delta}u_\infty$$

Then,

$$\frac{39}{280}\frac{d\delta}{dx} = \frac{\mu\frac{3}{2\delta}u_\infty}{\rho {u_\infty}^2} = \frac{3\mu}{2\delta\rho u_\infty}$$

$$\delta\frac{d\delta}{dx} = \frac{280 \times 3\mu}{39 \times 2\rho u_\infty}$$

Upon simplification, we will get

$$\delta\, d\delta = \frac{140}{13}\frac{\mu}{\rho u_\infty}dx$$

By integration:

$$\frac{\delta^2}{2} = \frac{140}{13}\frac{\gamma}{u_\infty}x + C_1$$

To evaluate C_1,by using boundary condition,$x = 0: \delta = 0$,therefore value of $C_1 = 0$

$$\delta^2 = \frac{280}{13}\frac{\gamma}{u_\infty}x$$

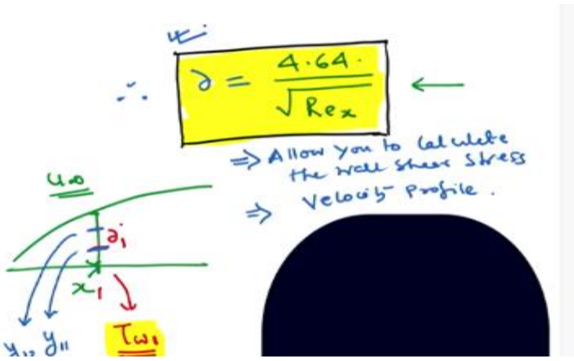
$$\delta^2 = \frac{280}{13}\frac{\gamma}{u_\infty}x = \frac{280}{13}.\frac{\gamma x^2}{u_\infty . x}$$

$$= \frac{280}{13}.\frac{x^2}{\left(\frac{u_\infty x}{\gamma}\right)} = \frac{280}{13}\frac{x^2}{Re_x}$$

$$\delta = \frac{4.64x}{\sqrt{Re_x}}$$

So, this is the boundary layer expression as a function of x. Since the boundary layer thickness is known, therefore we can find out the wall shear stress and the velocity profile,

$$\tau_w = \mu\frac{3}{2\delta}u_\infty : \quad \frac{u}{u_\infty} = \frac{3}{2}\eta - \frac{1}{2}\eta^3 = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$



For any x, u_∞ needs to be known. Because the wall shear stress depends on the magnitude of u_∞ if the flow is faster the stress is going to be higher. From that we can find out the δ.b

Thank you.