Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 20
Boundary Layer Analysis 5: Momentum Integral Method 01

So, in the previous lecture we have discussed the Blasius Solution. In this lecture, we will talk about the Momentum Integral Method. And in the last class we have got the ordinary differential equation and also we got series solution or Blasius solution by numerical methods, and the solution is

$$f(\eta) = \frac{0.332}{2!}\eta^2 - \frac{1}{2}.\frac{(0.332)^2}{5!}\eta^5 + \frac{11}{4}.\frac{(0.332)^3}{8!}\eta^8 \,....$$

You can try to solve this equation by numerical techniques. One of the important concepts of boundary layer analysis is how much force the plate is going to experience. So, one of the major utilities of boundary layer analysis is to estimate the wall shear stress. For that Wall shear stress,

$$\tau_w = \mu \frac{\partial u}{\partial y}|_{y=0}$$

$$\frac{\partial u}{\partial y} = u_\infty \sqrt{\frac{u_\infty}{\gamma x}}.\frac{d^2 f}{d\eta^2}$$

To Find, $\frac{\partial u}{\partial y}|_{y=0} = u_\infty.\sqrt{\frac{u_\infty}{\gamma x}}.\frac{d^2 f}{d\eta^2}$,we should get the value of $\frac{d^2 f}{d\eta^2}$ at $\eta = 0$.for that,

$$f(\eta) = \frac{0.332}{2!}\eta^2 - \frac{1}{2}.\frac{(0.332)^2}{5!}\eta^5 + \frac{11}{4}.\frac{(0.332)^3}{8!}\eta^8 \,....$$

$f'(\eta) = 0.332\eta - \frac{5}{2.\ 5!} + \ldots.\eta^4$

$f"(\eta) = 0.332 - \frac{20}{2.\ 5!}\eta^3 + \ldots\eta^6 \ldots$

If we put at $y = 0 : \eta = 0$ to the derivatives, we will get, $f"(\eta) = \frac{d^2 f}{d\eta^2} = 0.332$

Then,

$$\tau_w = 0.332\mu u_\infty \sqrt{\frac{u_\infty}{\gamma x}}$$

$$= 0.332\mu \frac{u_\infty{}^2}{\sqrt{\gamma x u_\infty}}$$

$$= 0.332 \frac{u_\infty{}^2 . \rho}{\frac{\rho}{\mu}\sqrt{\frac{\mu}{\rho} x u_\infty}}$$

$$= 0.332 \frac{u_\infty{}^2 . \rho}{\sqrt{\frac{\rho x u_\infty}{\mu}}} = \frac{0.332 u_\infty{}^2 \rho}{(Re_x)^{0.5}}$$

Wall shear stress, $\tau_w = \frac{0.332 u_\infty{}^2 \rho}{(Re_x)^{0.5}}$

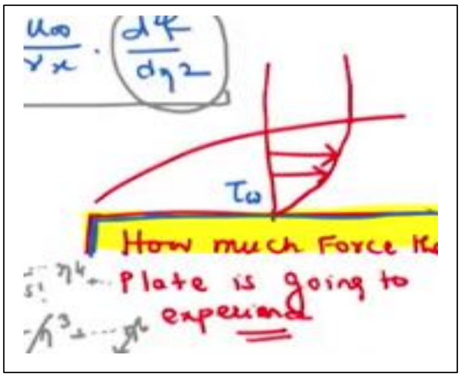
The local coefficient of skin friction,

$$C_{fx} = \frac{\tau_{w|x}}{\frac{1}{2}\rho u_\infty{}^2}$$

$$= \frac{0.332 u_\infty{}^2 \rho . 2}{(Re_x)^{0.5} . \rho u_\infty{}^2}$$

$$= \frac{0.664}{\sqrt{(Re_x)}}$$

Total force acting on the plate, Force per unit depth,



$$F = \int_0^L \tau_w dx$$

$$= \int_0^L \frac{0.332 u_\infty{}^2 \rho}{\sqrt{\frac{\rho x u_\infty}{\mu}}} . dx$$

$$F = \frac{0.664 u_\infty{}^2 \rho}{(Re_L)}$$

So, this is the utility of the Prandtl boundary layer or analysis of the Prandtl boundary layer. We have obtained from this solution that edge of the boundary layer is where u by u_∞ is equal to 1.0.

$$\frac{u}{u_\infty} = 1$$

We have obtained from series solution, $\frac{u}{u_\infty} = 0.992$ for η = 4.92 or 5

$$\eta = \sqrt{\frac{u_\infty}{\gamma . x}} y$$

Therefore, $\boldsymbol{\delta} = \frac{\mathbf{5}}{(\boldsymbol{Re_x})^{\frac{1}{2}}}$

<u>Momentum integral method</u>

Boundary layer displacement thickness (δ^*): It is defined as the vertical distance by which the external potential flow is displaced outward due to decrease in velocity in the boundary layer.

We are assuming a Potential flow approaching the plate and then over which the boundary layer separation takes place. As a consequence of boundary layer separation there will be velocity profile within the flow. At the surface of the plate the velocity of the fluid will be zero, due to no slip condition. And consider that at particular y, u is the local velocity.

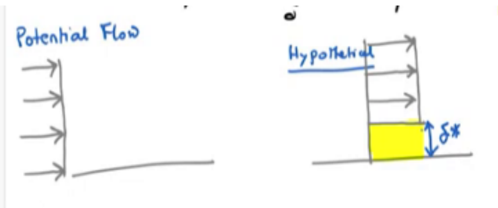
$u_\infty - u \rightarrow$ So, this is the local reduction in the velocity at a particular y due to boundary layer separation.

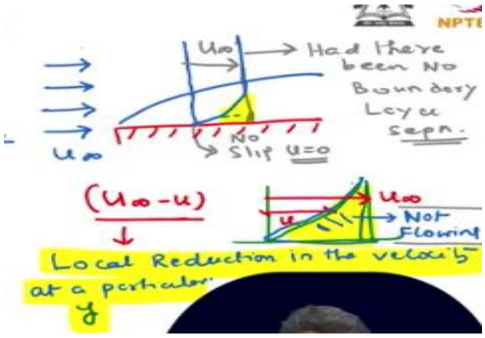
$\int_0^\infty (u_\infty - u) dy \rightarrow$ amount of fluid or amount of mass not flowing through this region due to the formation of the boundary layer.

In order to compensate for the mass that is not flowing through the boundary layer or due to the local reduction in the velocity, as if this potential flow has got shifted by a distance δ^*.

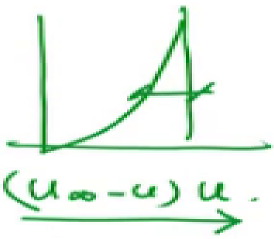
$u_\infty \ \delta^* = \int_0^\infty (u_\infty - u) dy$

Therefore, $\boldsymbol{\delta^*} = \int_0^\infty \left(\mathbf{1} - \frac{\boldsymbol{u}}{\boldsymbol{u_\infty}}\right) \boldsymbol{dy}$





Boundary layer momentum thickness $(\delta^{**}\ or\ \theta)$**:** It corresponds to the distance equivalent that the potential flow gets shifted based on the loss in momentum. The loss in momentum at every level is the loss in the reduction in the local velocitv .



$$\rho u_\infty{}^2 \delta^{**} = \int_0^\infty \rho u \,(u_\infty - u) dy$$

$$\theta = \delta^{**} = \int_0^\infty \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$$

$$H = \frac{\delta^*}{\theta} = Shape\ factor\ of\ the\ boundary\ layer\ or\ shape\ factor$$

Thank you.