Chemical Engineering Fluid Dynamics and Heat Transfer
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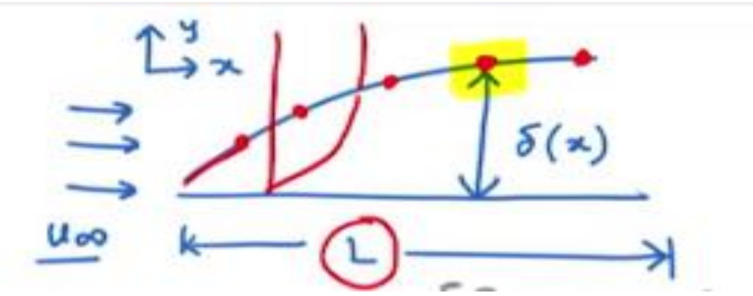
Lecture - 18
Boundary Layer Analysis 4: Blasius Solution 1

Welcome back. In this lecture we are going to discuss, how to solve the boundary layer equation or, basically the momentum balance equation that for the x direction that we have obtained after the simplification. There are basically two classical methods to solve the boundary layer equations. One of them is called the similarity solution or the Blasius solution and the 2nd one are known as the momentum integral method.



x component balance equation can be solve by ,

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\gamma\left(\frac{\partial^2 u}{\partial y^2}\right)$$

1.Blasius solution or the similarity solution

2.Momentum integral method

<u>Blasius solution</u>

In Blasius solution, first we will convert this PDE into an ODE. In this PDE equation, you have two dependent variables u and v, and two independent variables x and y. So, we have to find out one combined independent variable and one combined dependent variable.

The boundary conditions are:

1.At $y = 0, u = 0$ and $v = 0$:for all x, (due to the no slip condition and solid impermeable wall).

2.at $y = \delta, u = u_\infty$(at the edge of the boundary layer, where the velocity becomes 99.2 percent of the free stream velocity)

This boundary line is essentially locus of that value of y at which at that particular x, the velocity within the boundary layer equals the free stream velocity. So, basically, you join all points, you will get the edge of the boundary layer.

We defined Stream function as,

$$u = \frac{\partial \psi}{\partial y} : v = -\frac{\partial \psi}{\partial x}$$

If you replace u and v with their respective stream functional forms and we can write x component balance equation for the boundary layer:

$$\left(\frac{\partial \psi}{\partial y} . \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} . \frac{\partial^2 \psi}{\partial y^2}\right) = \gamma \left(\frac{\partial^3 \psi}{\partial y^3}\right)$$

And from the boundary condition:

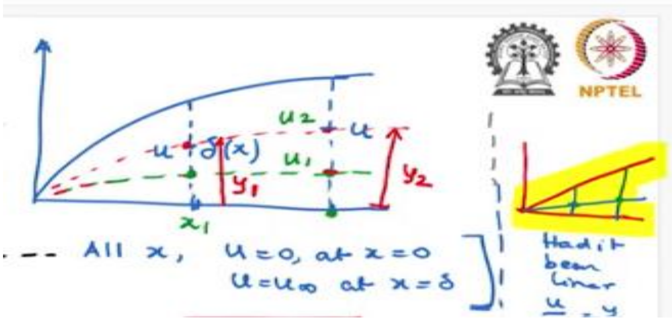
1. $y = 0, u = 0: u = \frac{\partial \psi}{\partial y} = 0: \psi \neq f(y)$

2. $y = 0, v = 0: v = -\frac{\partial \psi}{\partial x} = 0: \psi \neq f(x)$

That means, that on the surface or on the boundary, ψ is not a function of x and y. And therefore, on the boundary ψ is constant.

$$\psi = \int u dy + C$$

At y=0, u=0 that means C=0: this means $\psi = 0$ at the surface.



We know that film thickness of the boundary layer is a function of x and at all x=0, u=0: and at $y = \delta, u = u_{\infty}$. The boundary line is essentially the locus of all points at different x, where along the depth of the boundary layer have the same velocities. So, if we are looking at particular value of u at different x,$(0 \le u \le u_{\infty})$,we can normalise the points by the relation,

$$\frac{u}{u_{\infty}} = \bar{f}\left(\frac{y}{\delta}\right) = \bar{f}(\eta) \colon \eta = \frac{y}{\delta}$$

Now we have, $\eta = \frac{y}{\delta}$

$$\delta = x(Re_{,x})^{-\frac{1}{2}}$$

$$\delta = x^{\frac{1}{2}} \frac{\gamma^{\frac{1}{2}}}{u_{\infty}{}^{\frac{1}{2}}}$$

$$y = \left(\frac{\gamma x}{u_{\infty}}\right)^{\frac{1}{2}} . \eta$$

$$\eta = y\sqrt{\frac{u_{\infty}}{\gamma x}}$$

Now we look into the order of magnitude analysis of the equation,

$$u = \frac{\partial \psi}{\partial y}$$

$$O(u) = \frac{\psi}{y}$$

$$O\left(\frac{u}{u_\infty}\right) = \frac{\psi}{yu_\infty} = \frac{\psi}{\left(\frac{\gamma x}{u_\infty}\right)^{\frac{1}{2}}.\eta.u_\infty}$$

$$O\left(\frac{u}{u_\infty}\right) = \frac{\psi}{\sqrt{\gamma x u_\infty}.\eta}$$

We can substitute, $\frac{u}{u_\infty} = \bar{f}\left(\frac{y}{\delta}\right) = \bar{f}(\eta)$

$$\frac{\psi}{\sqrt{\gamma x u_\infty}} = \eta.O\left(\frac{u}{u_\infty}\right) = \eta.O\left(\bar{f}(\eta)\right)$$

$$\frac{\psi}{\sqrt{\gamma x u_\infty}} = \bar{f}(\eta)$$

$$\psi = \sqrt{\gamma x u_\infty}.\bar{f}(\eta)$$

Next, we will transform the partial differential equation to ordinary differential equation by Falkner skan transformation

<u>Falkner skan transformation</u>

$\psi = \sqrt{\gamma x u_\infty}.\bar{f}(\eta)$

$\frac{\partial\psi}{\partial\eta} = \sqrt{\gamma x u_\infty}.\frac{df}{d\eta}$

$\eta = \sqrt{\frac{u_\infty}{\gamma x}}.y$

$\frac{\partial\eta}{\partial x} = \left(-\frac{1}{2}\sqrt{\frac{u_\infty}{\gamma}}\,x^{-\frac{3}{2}}\right)y = \left(-\frac{1}{2}\sqrt{\frac{u_\infty}{\gamma.x}}.\frac{y}{x}\right) = \left(-\frac{1}{2}\frac{\eta}{x}\right)$

$\frac{\partial\eta}{\partial y} = \sqrt{\frac{u_\infty}{\gamma x}}$

This transformation is called the Falkner-Skan transformation.

Thank you very much.