Chemical Engineering Fluid Dynamics and Heat Transfer
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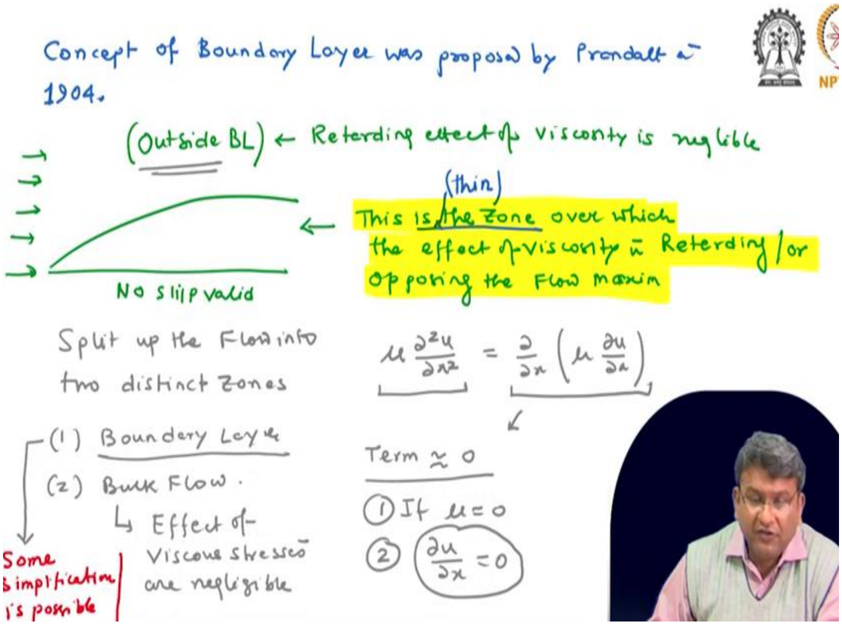
Lecture - 15
Boundary Layer Analysis 1

Welcome back, we are going to discuss the Boundary Layer Analysis.

We have used the condition of no slip boundary condition in the last lecture; that means, the adjacent fluid layer to a surface does not flow or it has the same velocity as that of the surface to which it is contacting. Viscous resistance or the viscosity effects tries to retard or oppose the flow. So, because of the no slip condition the adjacent layer does not flow. And because of the viscosity this layer tries to hinder the flow of the next layer and subsequent layer. The zone or the thickness up to which the effect of viscosity or the opposing effect of viscosity towards flow is significant that thickness or zone is called the boundary layer. The concept of boundary layer was proposed by Prandtl in 1904. So, if there is a flow over a flat surface where no slip condition is valid, there will be boundary layer separation or the zone over which the effect of viscosity in retarding or opposing the flow is maximum. Beyond this outside boundary layer, the retarding effect of viscosity is negligible.



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If you investigate the R.H.S of the Navier stokes equation for a Newtonian fluid,the term $\mu\frac{\partial^2 u}{\partial x^2}$ can be also written as,

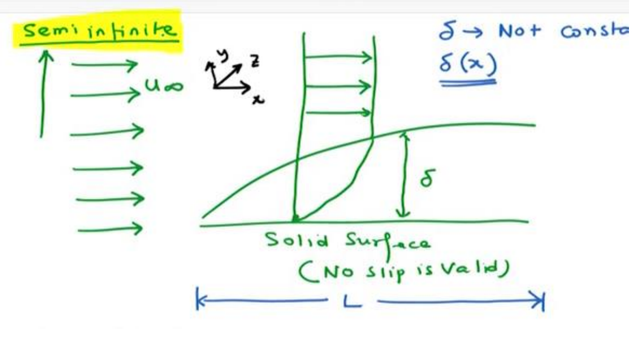
$$\mu\frac{\partial^2 u}{\partial x^2}=\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)$$

This term can be zero (or insignificant) in two ways, either

1.viscosity $\mu=0$: inviscid flow or there is no effect of viscosity.

2.Velocity gradient $\frac{\partial u}{\partial x}=0$: region for a viscous fluid the velocity gradients are 0.

Based on this, Prandtl split up the flow into two distinct zones. One is the boundary layer and other zone is the bulk flow. In bulk flow the effect of viscous stresses is negligible. So, Euler equation valid in the bulk flow region. And within the boundary layer region, you can solve the boundary layer equation and get the shear stress profile and the velocity distribution within the boundary layer by making some simplifications. And beyond boundary layer, the velocity gradients are negligible because the effect of viscous stresses are negligible. So, there is no velocity gradient, no stresses and Euler-Stokes equation can be used to solve. We are using order of magnitude analysis to solve the boundary layer equation.



Consider semi-infinite Viscous potential flow hitting a solid surface and there is boundary layer separation occurs. So, this liquid layer can stretch up to infinity. So, outside the boundary layer the velocity remains constant. And there is a solid surface at which no slip condition is valid. Over the thickness of the boundary layer, the velocity gradually changes from u=0 at y=0 to $u=u_\infty$ at $y=\delta$. L is the length of the plate over which boundary layer separation taking place. Boundary layer thickness (δ) is not constant and its function of x i.e., $\delta(x)$. The thickness of the boundary layer changes along the length of the plate.

The assumptions that we are considering for the boundary layer analysis are

1.Steady state flow

2.No pressure gradient in z direction (open channel flow); that means, that there is no flow in z direction.

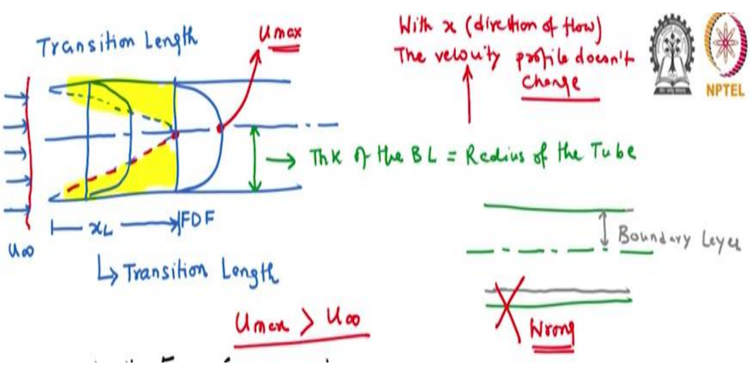
we are considering z along the depth to be very wide. So, there is no flow in Z direction and no variation of any quantity in z direction. Therefore, $w = 0 : \frac{\partial}{\partial z}() = 0$. So, this system becomes 2D flow.

3.Incompressible fluid

4. Newtonian fluid.

5.We are going to consider Semi-infinite pool of liquid that is that the liquid layer stretches up to infinity, there is no upper bound.

<u>Transition length:</u> Consider a potential flow without any velocity profile enters the pipe or tube of uniform cross-sectional area. When it hits surface, boundary layer separation starts from the edge and the thickness grows. After particular distance, the velocity profile develops full. Beyond this point the flow becomes fully developed flow, that means the thickness of the boundary layer becomes equal to the radius of the tube. Further downstream the thickness of the boundary layer cannot change, and this length is known as the transition length. Transition length is the length over which the boundary layer thickness still grows. And since it is a finite geometry the growing thickness of the boundary layer becomes equal to the radius of the tube. So, this fully developed flow means that with x or the direction of the flow the velocity profile does not change.



So, let us investigate the continuity equation for 2D incompressible flow.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

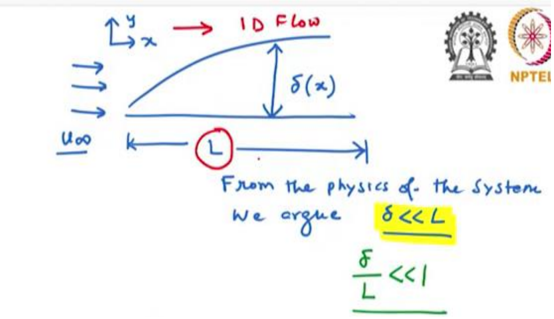
Based on the order of magnitude analysis, that for every variable what is the maximum value or the maximum possible value of that variable it can take that is called the order. Then based on the figure we can write,

$O(u) = u_\infty$

$O(x) = L$

$O(y) = \delta$

$O(v) =$ Not Known



From the physics of the system, we argue that $\delta \ll L$: $\frac{\delta}{L} \ll 1$. So, boundary layer separation takes place over the length of the tube. And in this the zone over which the effect of viscosity in retarding or opposing the flow is maximum.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y}$$

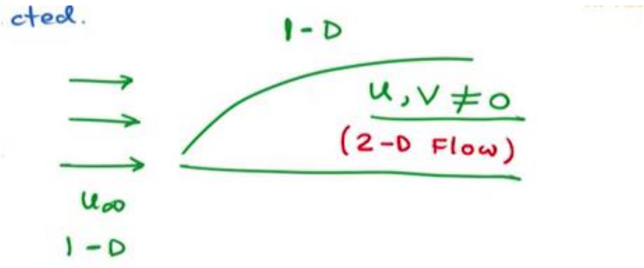
In order of magnitude analysis, we can neglect the sign. Therefore,

$$\frac{O(u)}{O(x)}=\frac{O(v)}{O(y)}$$

$$O(v)=\frac{O(u)}{O(x)}.O(y)=\frac{\delta}{L}.u_\infty$$

Since $\delta \ll L$: $\quad \boldsymbol{O(v) \ll O(u)}$

For order of magnitude analysis, no term can be neglected in any equation that has its origin in mass balance. Since the continuity equation has its genesis in mass balance, we cannot neglect the term $\boldsymbol{O(v)}$.And if we consider the geometry, flow outside the boundary layer is 1D with the velocity u_∞ inside the bulk. And from the order of magnitude analysis, we got flow within the boundary layer is 2D, the value of u and v both are non-zero inside the boundary layer.



As per the order of magnitude analysis,

$$O\left(\frac{\partial u}{\partial x}\right) = \frac{O(u)}{O(x)}$$

$$O\left(\frac{\partial^2 u}{\partial x^2}\right) = O\left(\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)\right) = \frac{O(u)}{O(x)O(x)} = \frac{O(u)}{\left(O(x)\right)^2}$$

Thank you very much.